Drivers of private grain storage
A computational-economics and empirical approach

Storage is an important instrument for stabilizing food supply. Yet, analysis of carry-over grain stocks is usually done by two methodological approaches: Equilibrium modeling and comparison of price characteristics or econometric analysis. This paper develops a new way to analyze private grain storage combining both approaches. Based on the canonical competitive storage model we derive a reduced-form storage equation for grain stocks in an open economy based on domestic and global supply and income. This approximation allows characterizing grain stocking by a piece-wise linear function for a broad set of parameters and model assumptions. The reduced-form model is tested against the competitive storage model where an extremely high fit is found. Then, the reduced-form model is applied to empirical stock data for 63 countries using a non-linear least-square panel regression. The results provide for the first time a direct confirmation of the competitive storage model based on observed stock data.

Keywords: Competitive storage model, private grain stocks, partial equilibrium model,

JEL codes: Q13, Q17, Q18

Contents

1. Introduction

1.1. Background

High and volatile food prices can have serious consequences, particularly for the poor who spend a large share of their income on food. Trade or storage can help to stabilize volatile food prices (compare Makki, Tweeten, & Miranda, 2001). Especially during times of crisis, governments tend to intervene into markets by controlling trade or storage in one way or the other. During the world food crisis in 2007/08, at least 35 countries sold grains from public stocks, at least 25 banned or restricted exports and at least 43 countries reduced tariffs and custom fees (Demeke, Pangrazio, & Maetz, 2009). However, these public interventions are often criticized for distorting markets and weakening incentives for private market actors to store grains except if they are only supporting market based approaches through promotion of productivity growth as well as private trade and storage (The World Bank, 2005). Being described as "the dominant doctrine" this type of intervention relies on market-based approaches to reduce price fluctuations as well as their impacts, complemented by targeted public interventions in times of crisis to reduce the impacts on the poor (Galtier, 2013). It has been criticized for underestimating important factors such has the influence of price instability on the overall welfare (ibid.). Additionally, when markets are incomplete, government interventions such as holding public stocks can increase overall welfare (Gouel, 2013a). Therefore, the calls for further public stocks, mostly for emergency reserves rather than for buffer stocks, have increased.

To evaluate the influence of public interventions and to analyze to which extent free markets maximize the total welfare, it is important to understand the behavioral determinants of private market actors, especially of stock holders. Many authors have used the competitive storage to model market prices and then compare the price characteristics to empirical data for analyzing private storage of grains and other commodities (compare e.g. Cafiero, Bobenrieth H., Bobenrieth H., & Wright, 2011; Deaton & Laroque, 1992, 1995; Peterson & Tomek, 2005). In these analyses, only the prices and price stability over time have been considered but neither the actual stock levels nor the fundamentals which influence the stock levels (apart from their effects on the prices). Furthermore, many of these models do not explicitly include trade. The results are mixed: While Deaton and Laroque (1992) cannot explain the high degree of autocorrelation in the observed prices with the help of the model, Cafiero et. al (2011) show that it may replicate the high degree of autocorrelation by either using more realistic parameters or by applying a finer grid to approximate the equilibrium price function. To the best of our knowledge, there are no approaches to estimate the drivers of private grain stocks based on the fundamentals which also consider the actual stock levels and which use a nonlinear reduced-form storage equation based on a storage model with rational expectations.

1.2. Research questions and goals

Our paper aims to find drivers of private grain storage and empirically verify the competitive storage model through comparing the dependency of stocks on different parameters in the theoretical simulations and for the empirical data. The theoretical simulations show how to set up the empirical model whereas the empirical model identifies which variables are drivers of private grain stocks in reality. The derived reduced form storage equation can be used in simple modelling exercises eliminating the need to solve the non-linear rational expectations market equilibrium.

1.3. Content

The paper can be structured along four crucial steps: First, we set-up a competitive storage model to generate stock data for a wide set of parameters and supply situations (theoretical benchmark model). The competitive storage model includes one country with supply and income shocks which can store and trade with the rest of the world which also suffers from random supply shocks. The market model is solved for the different input parameters and the dependency of the stock levels on the input parameters is analyzed. Second, we explain the qualitative behavior of grain stocks according to visualizations of stocks and develop a tractable reduced-form storage equation that captures these dynamics. Third, we evaluate the quality of the approximation with the reduced-form equation using the R^2 from non-linear least-square fits on the generated data from the first step. Forth, we apply the reduced-form storage equation with a non-linear least square regression on actual storage and supply data from 63 countries. We sum up our main findings and its implications for policy making and future research in the Conclusion.

2. Theoretical model

2.1. Model equations

The model specification follows Gouel (2011) or Gouel and Jean (2012) but differs in explicitly including the rest of the world as a second country, in including income shocks and excluding public stocks. The model describes a homogeneous agricultural product which can be produced, consumed, and stored in both countries as well as traded between them. Only one second good, the numeraire, exists. It is a partial equilibrium model with discrete, annual time steps.

2.1.1. Stockholders

The behavior of the stockholders is based on the competitive storage model. Each country i $\in \{A, B\}$ has a single representative stockholder who is risk neutral and acts competitively. The stock quantity $S_{i,t}$ is purchased at price $P_{i,t}$ in country i and then carried from period t to period $t+1$, where it is sold for price $P_{i,t+1}$. Storage losses $(1-\delta_i)$ occur and constant marginal storing costs k_i apply. Therefore, the stockholder maximizes his expected profits according to

$$V_i^S(S_{i,t-1}, P_{i,t}) = \max_{\{S_{i,t+j}\geq 0\}_{j=0}^{\infty}} E_t \left\{ \sum_{j=0}^{\infty} \beta_i^j \left[\delta P_{i,t+j} S_{i,t+j-1} - \left(P_{i,t+j} + k_i\right) S_{i,t+j} \right] \right\},$$

where $\beta_i = 1/(1+r_i)$ is the discount factor (r_i the interest rate) and E_t the rational expectation operator conditional on information available at time t. It is possible to express this problem in a recursive form which leads to a Bellman equation:

$$V_i^S(S_{i,t-1}, P_{i,t}) = \max_{X_{i,t}\geq 0}\{\delta_i P_{i,t} S_{i,t-1} - (P_{i,t} + k_i)S_{i,t} + \beta_i E_t[V_i^S(S_{i,t}, P_{i,t+1})]\}.$$

Having two state variables, carry-over stocks and exogenous prices (for the stockholder), this model can be expressed as a complementarity condition. Therefore, the first-order condition on the stocks and the envelope theorem need to be used while the possibility of a corner solution needs to be considered, that is the constraint that stocks need to be greater or equal to zero. As a result one obtains

$$S_{i,t} \geq 0 \quad \perp \quad \beta_i \delta_i E_t(P_{i,t+1}) - P_{i,t} - k_i \leq 0, \tag{1}$$

where the $\perp$ symbol indicates that the two equations are orthogonal, i.e. if for one of them the strict inequality holds, the other one needs to be strictly equal. As a consequence, stocks are zero if expected revenues, considering losses, do not exceed the costs for stockholding, implying that stockholding serves as a stabilizing kind of speculation in this model.

2.1.2. Producers

Each country has one representative producer who is risk neutral and makes his planting decision $H_{i,t}$ one period before the harvest in period $t+1$. Due to random, normally distributed yield shocks $e_{i,t}$ with mean 1 and variance σ_i, embedded by a multiplicative disturbance term, the producer harvests $H_{i,t}e_{i,t+1}$. Maximizing his profits, the producers makes his production decision according to the following equation

$$\max_{\{H_{i,t+j}\geq 0\}_{j=0}^{\infty}} E_t\left\{\sum_{j=0}^{\infty} \beta_i^j [\delta P_{i,t+j} H_{i,t+j-1} e_{i,t+j} - \Psi_{\mathrm{i}}(H_{i,t+j})]\right\}.$$

Here, $\Psi_{\mathrm{i}}(H_{i,t})$ are the costs of planning the production $H_{\mathrm{i,t}}$. Again, this problem can be written in a recursive form yielding the following Euler equation

$$\beta_i E_t(P_{i,t+1} e_{i,t+1}) = \Psi_i'(H_{i,t}). \tag{2}$$

This equation implies that the marginal cost of production equals the expected, discounted marginal profit from one unit of planned production. One would expect the first derivative of the production cost function to be strictly increasing. This is fulfilled by taking a convex, isoelastic function

$$\Psi(H_{i,t}) = h_i \frac{H_{i,t}^{1+\mu_i}}{1+\mu_i}, \tag{3}$$

with scale parameter h_i and the inverse supply elasticity in country i, $\mu_i \geq 0$. To limit the number of state variables, the carry-over stocks and the harvest can be combined to one state variable per country, availability $A_{i,t}$ with

$$A_{i,t} = (1-\delta)S_{i,t-1} + H_{i,t-1}e_{i,t}. \tag{4}$$

2.1.3. Trade

A representative trader uses all spatial arbitrage possibilities and trades competitively between both countries A and B. Trade takes place instantaneously and per unit trading costs θ as well as country-specific import tariffs τ_i (which can also be regarded as export tariff of the other country) apply. Due to the fact that trade is instantaneous, the trader doesn't maximize expected profits but instantaneous profits. As the other equations, the trader's behavior can be expressed as a complementarity problem

$$P_{i,t} - P_{-i,t} + \theta + \tau_i \geq 0 \;\perp\; X_{i,t} \geq 0 \tag{5}$$

where $P_{-i,t}$ represent the price in period t in the country which is not i, and $X_{i,t}$ are the exports from country i to the other country. From equation (5), it follows that $X_{i,t} \geq 0 \perp X_{-i,t} \geq 0$, i.e. there are never exports to and imports from the same country at the same time.

2.1.4. Consumption

Each country has risk-neutral consumers which consume according to an isoelastic demand function

$$D(P_{i,t}, Y_{i,t}) = \gamma_i P_{i,t}^{\alpha_i} Y_{i,t}^{\eta} \tag{6}$$

with normalization parameter γ_i, price elasticity $-1 \neq \alpha_i < 0$, and income elasticity $\eta_i \neq 1$. The income $Y_{i,t}$ is assumed to be constant in the rest of the world, i.e. country B, while in country A it is subject to random, normally distributed shocks with mean 1 and variance σ_i^y. For simplicity, the consumers always consume the current income and do not save which allows ignoring the consumers' savings. Otherwise, these would need to be considered as additional state variables otherwise. Of course, self-insurance of consumers is therefore neglected implying that no maximization problem for the consumer needs to be solved. As a result, the current income in country A, $Y_{A,t}$, is the third state variable of the model besides $A_{A,t}$ and $A_{B,t}$.

2.1.5. Market equilibrium

The shocks are considered at the beginning of each period so that equation (4) and the realization of the income shock in country A provide the state variables. The market equilibrium condition is

$$A_{i,t} - X_{i,t} + X_{-i,t} = D_{i,t}(P_{i,t}, Y_{i,t}) + S_{i,t}. \tag{7}$$

Therefore, when the model is solved numerically, a recursive equilibrium needs to be found, i.e. a set of functions $S_{i,t}(A_{i,t}, A_{-i,t}, Y_{A,t}), H_{i,t}(A_{i,t}, A_{-i,t}, Y_{A,t}), P_{i,t}(A_{i,t}, A_{-i,t}, Y_{A,t})$, and $X_{i,t}(A_{i,t}, A_{-i,t}, Y_{A,t})$ which describes the dependency of these response variables on the state variables. To obtain this set of equations it is assumed that the stockholders, producers, and the trader maximize their profits according to equations (1), (2) and (5), respectively, while the market clears according (7) and the transition equation (4) holds.

2.2. Calibration

Table 1 in the appendix provides the values for the calibration of the model. Some explanations and comments are provided in the last column. The expected value of all shock variables is 1. The model is solved on in 9x9x9 grid of the state variables for each set of the parameters. From all of the parameters, five parameters are varied to test their influence on the response variables. The varied parameters refer all to country A, namely the interest rate, the relative country size, the standard deviation of supply shocks, the demand, and the supply elasticity. The choice of these parameters is based on the availability of cross-sectional data for the later application to real-world stock data. For each of these parameter, three different values have been used which leads to 3^5=243 different sets of parameters. Since for each parameter set the model is solved on a 9x9x9 grid of the state variables, we get $3^5 \cdot 9^3 = 177147$ observations in total.[1] The simulations area conducted in Matlab and to solve the model, the CompEcon toolbox (Fackler & Miranda, 2011) and the RECS solver (Gouel, 2013b) are used.

3. Simulation results

The aim of this paper is to analyze drivers of private grain storage qualitatively, i.e. to obtain a precise but qualitative understanding how private storage generally behaves for a broad set of parameters and model assumptions. As the full model contains three state variables, a graphical visualization that can guide us in deriving a reduced form-equation becomes challenging. We therefore consider simplified models with one or two state variables to derive the reduced form function for the full model.

3.1. One country without income shocks

At first, the case of a single country without income shocks is considered. Figure 1 shows the dependence of the closing stocks and prices in country A on the availability (total supply) which is the production plus last year's closing stocks. The upper panel of Figure 1 represents the storage rule for different assumptions about the variability of domestic harvests. It turns out this storage rule has a kink, so it is zero before a certain threshold and then it increases with availability. The kink varies with the standard deviation of supply shocks but the slope remains nearly constant. Figure 2 again illustrates the dependence of the closing stocks on the availability but this time the storage costs and the interest rate in country A are varied as indicated in the graph. In this case, not only the threshold when storage takes place changes, but also the slope of the storage rule changes. For high storage costs or a high interest rate the slope of the storage rule decreases.[2] This implies that higher availability does not increase the stocks as much as in the baseline scenario which is in line with our expectations.

With the conducted simulations we conclude that storage can be approximated by a straight line above a certain threshold for a wide set of parameters. As a next step, the influence of all of the six parameters on the shape of this piece-wise linear storage rule is illustrated. Figure 3 shows how the intercept, i.e. the position of the kink in the storage rule, changes when the parameters are varied. To obtain the graph, all parameters were set to their standard values and then, consecutively, the different parameters were varied (holding the others constant). The graphs indicates that the threshold increases if the interest rate, the storage costs, the storage losses, or the supply elasticity increase while it decreases if the standard deviation of shocks or the demand elasticity increase. Again, the intercept changes nearly linearly with these parameters, except for the standard deviation of supply shocks where it is closer to a quadratic form.

For each of these models, the OLS estimator was used to estimate the slope coefficient of the strictly positive part of the storage rule. As a result, it is possible to evaluate how the slope coefficient changes when the parameters are varied. Figure 4 illustrates this dependence of the slope in the same way as the previous figure. The slope decreases if the standard deviation of shocks, the

[1] At the time when the paper was submitted, the competitive storage model did not run for the interest rate of 0.02; this parameter was therefore not included in the parameter variation of the following analysis which reduced the number of observations by one third to 118098.

[2] The interest rate behaves basically like the storage costs as it increases the (opportunity) costs of storage.

interest rate, the storage costs, or the storage losses increase or if the supply elasticity or demand elasticity decrease. To evaluate if the piece-wise linear approximation of the storage rule is a good fit, Figure 5 shows the R^2 for the each approximation in the same way as the previous figures. As the R^2 is always above 0.998, the linear line turns out to be a very good approximation for the strictly positive part of the storage rule in all of the scenarios tested here. In Figure 4 the individual graphs are not straight lines but are slightly bended. Nevertheless, a linear approximation performs quite well over an interval of limited width for the different parameters.

If a piece-wise linear reduced form approximation is chosen, the response function for stocks in A can be written as

$$S_A(A_A) = \begin{cases} 0 & \text{if } A_A < \tilde{A}_A = -\frac{b}{a} \\ aA_A + b & \text{if } A_A \geq \tilde{A}_A = -\frac{b}{a} \end{cases} \tag{8}$$

or, in a form to be used by least-square estimation, as

$$S_A = \max(0, aA_A + b) \tag{9}$$

In order to capture the impact of important model parameters on the intercept b and the slope a of the storage rule, we can further substitute linear combinations of structural model parameters for a and b according to $a = a_0 + a_r r_A + a_\mu 1/\mu_A + a_\alpha \alpha_A + a_\gamma \gamma_A + a_\sigma \sigma_A$, and $b = b_0 + b_r r_A + b_\mu 1/\mu_A + b_\alpha \alpha_A + b_\gamma \gamma_A + b_\sigma \sigma_A$. This allows applying the storage model to various contexts that differ in their parameter constellation (e.g. due to crop or country-specific characteristics) by fitting:

$$S_A = \max\big(0, (a_0 + a_r r + \cdots)A_A + (b_0 + b_r r + \cdots)\big) \tag{10}$$

3.2. Two countries without income shocks

Extending the model to two countries makes the response variables dependent on two state variables, availability in A and in B. As this can be illustrated by a three dimensional plot, Figure 6 shows how the storage in country A now depends on the availability in A and B. Again, B represents the rest of the world. Clearly, storage only takes place in regions of excess supply which is in line with our expectations as trade is costly and both regions are self-sufficient on average in this simulation. Therefore, excess supplies will be stored in the region where they are produced to either use them in the same region later without having any trade costs or to use them in the other region later so that trade costs only occur once. If there is little supply in B but excess supply in A, there will also be little storage in A but instead exports to B will be high. In case of excess supply in both regions, the storage rule for one region is (nearly) independent of the level of excess supply in the other region. These observations lead to the following mathematical description of the reduced form storage rule for the case of two countries:

$$S_A(A_A, A_B) = \begin{cases} 0 & \text{if } A_A < \tilde{z}(A_B) \\ aA_A + b\big(1 + \tilde{z}(A_B) - \tilde{A}_A\big) & \text{if } A_A \geq \tilde{z}(A_B) \end{cases} \tag{11}$$

$$\text{with } \tilde{z}(A_B) = \begin{cases} \tilde{A}_A & \text{if } A_B > \tilde{A}_B \\ \tilde{A}_A - \beta(\tilde{A}_B - A_B) & \text{if } A_B \leq \tilde{A}_B \end{cases} \tag{12}$$

and $\tilde{A}_B = \theta = \theta_0 + \theta_r r_A + \cdots$, $a = a_0 + a_r r_A + \cdots$, $b = b_0 + b_r r_A + \cdots$, and $\beta = \beta_0 + \beta_r r_A + \cdots$

The $a, b, \beta,$ and θ do, as before, consist of six terms: One constant term with subscript zero and then one term for each parameter, so e.g. $a = a_0 + a_r r_A + a_\mu 1/\mu_A + a_\alpha \alpha_A + a_\gamma \gamma_A + a_\sigma \sigma_A$. Here and in the subsequent sections, this is not explicitly written for all characters but always indicated by the open sum which starts with the constant and the interest rate.

In order to fit the model, a single equation is needed again and can in this case be formulated as:

$$S_A = \max\left[0, aA_A + b\left\{1 + \min\left(0, -\beta(\tilde{A}_B - A_B)\right)\right\}\right] \tag{13}$$

3.3. One country with income shocks

In the case of one country with income shocks but without trade, there are again two state variables, namely the availability and the income level. Figure 7 shows how the storage rule changes in dependence of the availability and of the income for a fixed set of parameters. The graph indicates that income shocks do not influence the slope coefficient of the storage rule but only the intercept, i.e. the threshold. Additional analysis where the slopes were computed and compared have confirmed this assumption. In general, the results show that the storage rule can now be described by the following equation:

$$S_A(A_A, Y) = \begin{cases} 0 & \text{if } A_A < \tilde{A}_A(Y) \\ (a + \omega Y)A_A + b + \rho Y & \text{if } A_A \geq \tilde{A}_A(Y) \end{cases} \tag{14}$$

with $\omega = \omega_0 + \omega_r r + \cdots$, and $\rho = \rho_0 + \rho_r r + \cdots$.

The single equation which could be used to fit the model can be described as:

$$S_A = \max[0, (a + \omega Y)A_A + b + \rho Y] \tag{15}$$

As described, income only influences the intercept which implies $\omega = 0$.

3.4. Two countries with income shocks in country A

Finally, the full model for two countries with income shocks in country A is set up. As there are three state variables now, the results cannot be plotted as in the other cases. But the previous chapters have shown how the different parameters may influence the storage rule. Combing the previous results, the reduced form approximation of the storage rule can be formulated as:

$$S_A(A_A, A_B, Y) = \begin{cases} 0 & \text{if } A_A < \tilde{z}(A_B, Y) \\ (a + \omega Y)A_A + (b + \rho Y)\left(1 + \tilde{z}(A_B, Y) - \tilde{A}_A(Y)\right) & \text{if } A_A \geq \tilde{z}(A_B, Y) \end{cases} \tag{16}$$

$$\text{with } \tilde{z}(A_B, Y) = \begin{cases} \tilde{A}_A(Y) & \text{if } A_B > \tilde{A}_B(Y) \\ \tilde{A}_A(Y) - \beta(\tilde{A}_B(Y) - A_B) & \text{if } A_B \leq \tilde{A}_B(Y) \end{cases}$$

and $\tilde{A}_B(Y) = \theta + \rho Y = \theta_0 + \theta_r r + \cdots + (\tau_0 + \tau_r r + \cdots)Y$. $\tilde{A}_A(Y)$ can be defined but will cancel out anyway.

In order to fit the model by non-linear least squares estimation, a single equation is needed. Transforming equation 16, the following single reduced form storage rule approximation can be derived:

$$S_A = \max\left[0, (a + \omega Y)A_A + (b + \rho Y)\left\{1 + \min\left(0, -\beta(\theta + \tau Y - A_B)\right)\right\}\right] \qquad (17)$$

This equation is an important finding from this paper which will be used for the subsequent analyses to fit the theoretical model and derive the expected signs of the empirical model. As before, ω is equal to zero thereby slightly reducing the complexity of the equation.

3.4. Testing the reduced form storage rule approximation

Equation (17) is the reduced form approximation of the storage rule for the case of two countries with one of them suffering from income shocks. The previous sections showed mostly qualitatively why this specific form is chosen and is expected to being able to describe the simulation results as a reduced form equation. We now want to evaluate quantitatively how good this approximation is. The simulation results from the theoretical model will be used to estimate the parameters of the model described by equation (17). Using a non-linear least squared estimation, the goodness-of-fit based on the R-squared is assessed.

A two-step procedure is applied here. In the first step, it is shown that for each set of parameters individually, the model is able to describe the functional form of the storage rule. As for each individual 9x9x9 grid all the parameters are the same, only the intercepts where included in the regression, i.e. $a_0, b_0, \theta_0, \rho_0, \tau_0, \beta_0$.[3] For each of the 162 sets of parameters the model was fitted individually and the R^2 was calculated. The mean of the obtained R^2 was .99972 with a standard deviation of .00004 and a minimum value of .99962, indicating a very high fit. This leads us to the conclusion that the functional form as such is an excellent approximation given a specific set of parameters. Table 2 provides the summary statistics for the R^2 and the estimated reduced-form parameters. Furthermore, it turns out that for the very different structural parameter sets, most of the estimated reduced-form parameters change only slightly (as indicated by the coefficient of variation (CV)). The only exceptions are the parameters ρ_0 and τ_0.

In the second step it is tested whether the dependence on the different parameters (interest rate, elasticities, …) can be captured by linear combinations of structural parameters (i.e. $a = a_0 + a_r r_A + a_\mu 1/\mu_A + a_\alpha \alpha_A + a_\gamma \gamma_A + a_\sigma \sigma_A$) in equation (17). Therefore, all the simulation results, i.e. the results for all different sets of parameters, are pooled together and then the regression is conducted again, this time including the full specification with linear combinations for $a, b, \beta, \theta, \rho$, and τ instead of including only the intercepts. Table 3 shows the complete regression results. The results lead to 6 implications: (1) The very high R^2 of 0.9997 indicates that the model is well specified and equation (17) is indeed a very good reduced form approximation of the storage rule which results from the partial equilibrium model without any a priori closed form solution. (2) Most of the parameters are highly significant, i.e. even significant at the 0.1% level which is attributable to the high number of observations. (3) The parameters for the interest rate are all insignificant. However, this conclusion is preliminary as we were forced to reduce the tested values for the interest rate to two as described in set the calibration section. (4) The few other insignificant parameters are $\rho_0, \rho_\mu, \rho_\alpha, \tau_0$, and τ_{mu}. (5) All significant parameters are relevant, i.e. (mostly far)

[3] To ensure that income indeed only influence the threshold and not the slope of the storage rule, the estimations were also conducted with ω included but, as expected, it turned out to be insignificant and close to zero.

above 0.00186 which is not small regarding the model calibration. (6) Most insignificant parameters are small, i.e. (mostly clearly) below 0.00128. From these results, the expected signs of the regression results in the empirical part can be deducted.

Overall, the piece-wise linear approximation turned out to perform very well over the broad set over tested parameters. Not only does it approximate the storage rule well for each individual set of parameters but also it is able to describe the influence of the different parameters on the private carry-over stocks.

4. Empirical estimation

4.1 Description and characterization of data

For the empirical validation of the model, the USDA data for stocks, production and demand from 1990 to 2013 for maize, rice, wheat, soy, and sorghum are used. The data is further complemented with stock data from FAO GIEWS which gives in total 63 countries. GDP per capita were obtained from the World Bank. The stock and production data are de-trended using the consumption trend from Hodrick-Prescott filtering. Hence, domestic and rest-of-the world (ROW) stocks as well as supply (production plus carry-over stocks) are divided by the long-term consumption trend which gives a stationary series that is also of similar magnitude among different countries. Likewise, GDP per capita in real terms is de-trended using the Hodrick-Prescott filter to obtain a series for income-induced demand shocks. The standard deviation of supply shocks σ_A is calculated as the standard deviation of the cyclical components of the Hodrick-Prescott filtered production data. The scaling parameter for country size γ_A is obtained by dividing the domestic consumption trend by the consumption trend of ROW.

At first, the dependency of the stocks on the total availability is plotted for selected countries to visually evaluate the overlap with the theoretical results. Figure 8 shows this dependency for the worldwide stock levels and for China for different crops and the total stocks of the considered crops. In both cases, a nearly linear dependency can be observed for all crops. While overall there is some noise and sometimes far too low stock levels on the world level, in China there is little noise around the nearly linear relationship. The kink, i.e. the threshold cannot be seen clearly in the figure. As there are always working stocks, the stock data would not be expected to fall to zero if the availability is very low. Operational stocks could easily be incorporated by an additive term to the entire storage equation, which can be assumed to also increase with the use trend (as working stocks are a fixed share of grain use). Therefore, it would always amount to a similar level in the de-trended data. Figure 9 shows the same plots for the US and for India. For the US the slope and intercept are very crop specific and the linearity is more visible for wheat and maize than for rice. In India, strong noise renders the stocks much more arbitrary but the general dependency still can see seen. Indeed, the huge governmental stocks in India have been found to depend mainly driven by the Indian minimum support price and not by the total availability (Kozicka, Kalkuhl, Saini, & Brockhaus, 2014), which explains this observation. In all of the considered scatter plots, important covariates like the shocks at the global scale as well as income shocks are omitted which leads to further deviations from the piece-wise linear stocking rule over domestic supply.

4.2 Regression on empirical stock data

To incorporate relevant covariates we run a non-linear least square regression on the closing stocks which is based on the approximation equation (17). In order to account for potential unobserved heterogeneity we include the country and crop-specific mean stocks $\bar{S}_i$ over the considered time horizon:

$$S_{i,t} = \max\left[0, \bar{S}_i + aA_{i,t} + \left(b + \rho Y_{i,t}\right)\left\{1 + \min\left(0, -\beta\left(\theta + \tau Y_{i,t} - A_{-i,t}\right)\right)\right\}\right] + \varepsilon_{i,t} \qquad (18)$$

with $a = \alpha_0 + \alpha_\gamma \gamma + \alpha_\sigma \sigma$ and $b, \rho, \beta, \theta, \tau$ likewise. This gives a fixed-effects-like non-linear panel regression; omitting the mean stocks $\bar{S}_i$ yields a random-effects specification. We consider two different panels: The first uses total grain stocks ('Total Grains') and total grain supply as relevant variables while the second specification uses a panel over countries and additionally over crop types ('Pooled Grains'). The first specification is appropriate if grains are perfect substitutes and only total grains matter for the market equilibrium. The second specification accounts for heterogeneity among grains but misses the substitution effects. As data on some of the structural parameters like the demand and supply elasticity as well as the interest rates are challenging and the interest rate turned out to be insignificant in the theoretical model, we only control for potential impacts of the variation of shocks and the size of the country on the reduced-form parameters. A benchmark regression on the dataset generated under Section 3.4 on only those variables that were used in the empirical data is also added (column "Theoretical Model").

The results of these regressions are shown Table 4. The magnitude of the estimated coefficients from the empirical data and the generated data ("Theoretical Model") are difficult to compare due to scaling issues and therefore these are not of big interest. Instead, the focus lies on qualitative behavior predicted by the theoretical model and whether this is confirmed by the empirical analysis. The grey shaded cells indicate where the empirical model matches the theoretical results. This is the case if either both coefficients are insignificant or if the sign of the coefficients from the empirical model matches the one from the theoretical model in case both coefficients are significant. Most importantly, the slope coefficient a_0 is positive in all specifications, indicating that high supply leads to higher stock-to-use ratios. This effect is stronger for larger countries than for smaller ones, indicated by the positive sign of a_γ which holds for all specifications and is in line with the results from the theoretical model. On the other hand, the intercept b_γ is smaller for large countries in all specifications which, again, is what the theoretical model predicted. Combining both it implies that larger countries tend to start storing later (at higher supply levels), but then have a higher slope, i.e. start to build up stocks more quickly. The influence of the standard deviation of shocks on the slope and primary intercept coefficient b was not observable in the empirical data. For the other parameters in most cases the coefficients of the empirical model are not significant which may be attributable to partly bad data quality or other factors discussed in the next section. Only in very few cases the coefficient is significant but the sign is not in line with the theoretical findings.

4.3 An alternative minimalistic regression on grain stocks

The large number of insignificant coefficients in the empirical regression can have several reasons: One explanation can be that real-world storage is distorted by market failures (e.g. high transaction costs) or policy interventions that follow a different logic than the optimal storage model. The inclusion of storage and trade costs, however, accounts for transaction costs to a certain extent. Likewise, if governments stock when supply is excessive and stock-out when supply is scarce, they follow the qualitatively the same behavior as private stock-holders even if they are not profit maximizing and usually have non-optimal stock levels (in the sense of the model for risk-neutral consumers). Another explanation for the large amount of insignificant coefficients and for some coefficients that have an unexpected sign can be due to the fact that the slope and intercept coefficients hardly change between countries; and if they change, these changes are not attributable to the factors we can control for with real-world data. To consider this possibility, we run a minimalistic reduced-form storage model which excludes the possibility that underlying structural parameters affect slopes and intercepts of the piece-wise linear storage approximation. In contrast, we assume a homogenous response among all countries, i.e. $a = \alpha_0$, $b = b_0$ etc. in equation (18). Since in the individual regressions in Section 3.4 the parameter ρ turned out to be insignificant and also in the full model many coefficients related to ρ are insignificant (see Table 3), ρ was omitted from the regression. To further account for working or operational stocks that are contained in the data, the term w was added to the regression. Hence, our regression model reads

$$S_{i,t} = \max\left[0, \bar{S}_i + aA_{i,t} + b\left\{1 + \min\left(0, -\beta\left(\theta + \tau Y_{i,t} - A_{-i,t}\right)\right)\right\}\right] + w + \varepsilon_{i,t} \qquad (19)$$

The results of the regression are shown in Table 5. A first surprising result is that for the generated data, i.e. the theoretical model, the fit remains extremely high (R^2 equals 99.4 %). Thus, the reduced-form storage model performs well even when no flexibility on intercepts and slopes is allowed indicating a limited influence of the different parameters on the model's general form. In contrast to the full regression in Table 4, we get clear results on the signs of the coefficients which are in all cases consistent with the theoretical model. Only in case of τ, which determines the change of the threshold level for stock-outs under GDP shocks solely, the regression on real-world stock data yields a statistically insignificant coefficient. This suggests that short-term income fluctuations have a very limited effect on carry-over stocks. The statistically significant and high β in the empirical model indicates high market integration: Domestic storage responds strongly not only to domestic supply, but also to international supply. Finally, we estimate operational or working stocks w to be slightly below 11% of domestic consumption which appears to be a reasonable level.

5. Conclusion

In this paper, two completely different methodological approaches were combined to analyze private carry-over grain storage: The competitive storage model which is based on a rational expectations equilibrium and econometric storage regressions which typically lack a theoretical foundation or only look at prices instead of actual stock levels while often excluding trade. The aim was to reconcile the complexity of the competitive storage model which lacks a closed-form solution with econometric modeling of agricultural fundamentals that is often used in applied and policy-related research. By using the competitive storage in a broad setting with trade, GDP shocks and large parameter variations, we were able to generate stock data and find a multidimensional piece-wise linear reduced-form storage equation. The reduced-form model turned out to be extremely precise as well as flexible when applied to data generated by the competitive storage model. It is therefore a useful approximation for storage behavior in future empirical and applied research which does not rely on solving the competitive storage model and thereby reduces the complexity of the model significantly without reducing its precision as long as the model assumptions hold.

The basic qualitative behavior of private grain storage can be summarized as follows: Ending stocks are zero if domestic and global supply are below a certain threshold. This threshold is shifted downwards by positive GDP shocks (positive income shocks lead more likely to stock-outs). If the production in the rest of the world is very low, both the threshold and the slope change and the crop is exported instead of being stored. The threshold and slope are influenced by structural model parameters, in particular, storage costs, interest rates, supply and demand elasticities as well as the variability of domestic harvest shocks and harvest shocks in the rest of the world. If supply within the country and the rest of the world is above the critical threshold, ending stocks depend positively and linearly on domestic and global supply. The slope is again influenced by structural model parameters.

Applying the reduced form model to observed production, stock and income panel data for 63 countries and five crops confirmed the appropriateness of our reduced form-approach. Due to the piece-wise linear storage rule, a non-linear least squared regression was used. The estimated coefficients are largely in line with those expected (i.e. with same sign as in the regression with data generated by the theoretical benchmark model). Structural characteristics of countries and crops, however, seem to have only a small impact on threshold levels and slopes.

Three results are of direct policy relevance: First, operational stocks are roughly 11 percent of domestic consumption, implying that stock-to-use ratios have to be subtracted by 11 percentage points to yield the amount of stocks that are actually available for consumption smoothing. Second, domestic stocks respond strongly to the international supply situation which indicates a high degree of market integration. This underlies the need for multinational agreements and regulations about how to deal with supply shocks in individual countries as well as on the global level. Third, GDP shocks are important in the theoretical model but insignificant in the empirical validation. This might indicate that stockholders do not perform well in anticipating future demand. As a result, private storage levels might not be optimal providing a rational for interventions and information system might need to focus also on demand side factors rather than only on the supply side.

Future research should further explore the role of domestic stabilization policies on private storage. The application of a much simpler reduced-form approach allowed calculating crowding-out effects of public storage on private storage in India (Kozicka et al. 2014). Further research could also focus on the role of policies in leading to higher or lower (private) grain storage than optimal. In addition, more research on the role of public interventions in times of production shocks in a highly integrated multinational environment is needed which should put a particular emphasis on the effects of such interventions on private traders and, especially, stockholders.

6. Appendix

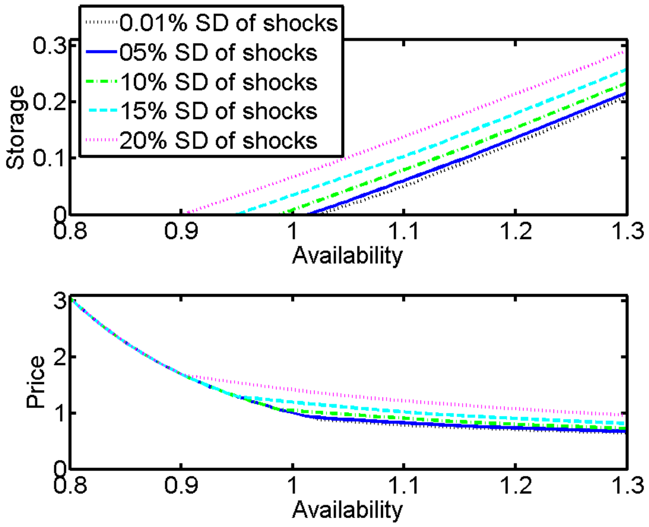

Figure 1: Closing stocks (above) and prices (below) in country A in dependence of availability (total supply) in A for a fixed set of parameters. The different curves represent different values of the standard deviation of harvest shocks in A to see how this affects the storage rule.

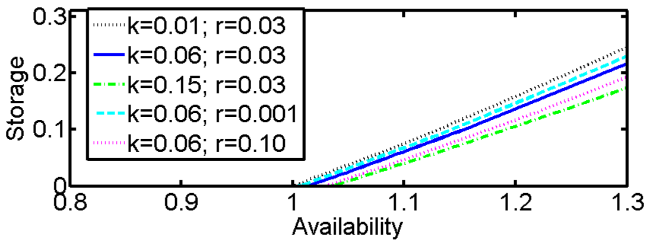

Figure 2: Closing stocks in country A in dependence of availability in A for a different storage costs k and interest rates r.

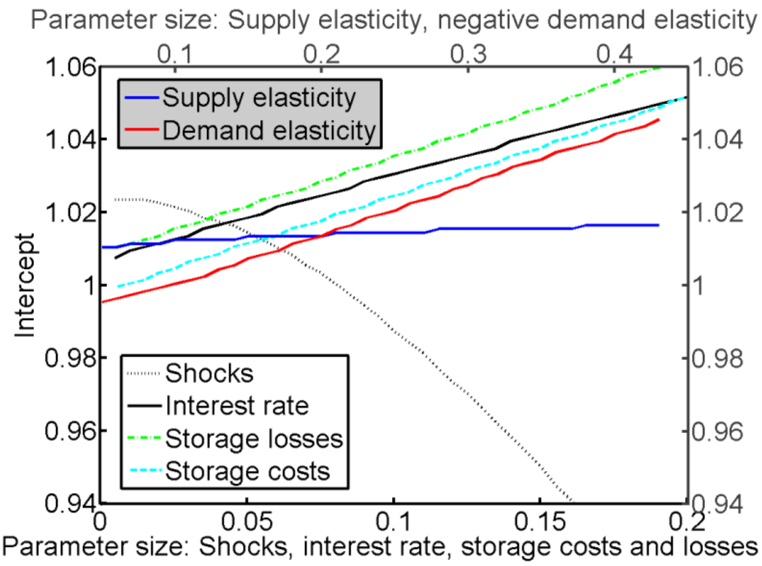

Figure 3: The dependency of the intercept of the storage rule on the different parameters. The SD of production shocks, interest rate, storage costs, and storage losses are plotted using the bottom x-axis, the supply elasticity and negative demand elasticity are plotted using the top x-axis.

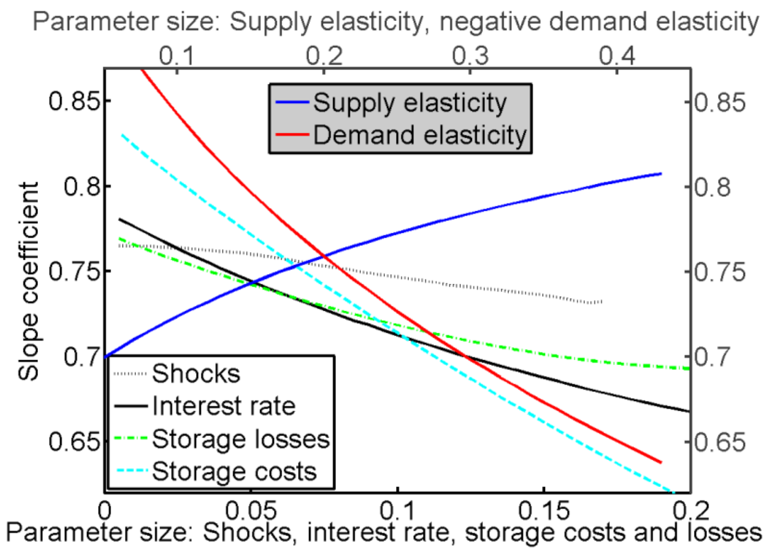

Figure 4: The dependency of the slope coefficient (obtained from a linear OLS fit) of the storage rule on the different parameters. The SD of production shocks, interest rate, storage costs, and storage losses are plotted using the bottom x-axis, the supply elasticity and negative demand elasticity are plotted using the top x-axis.

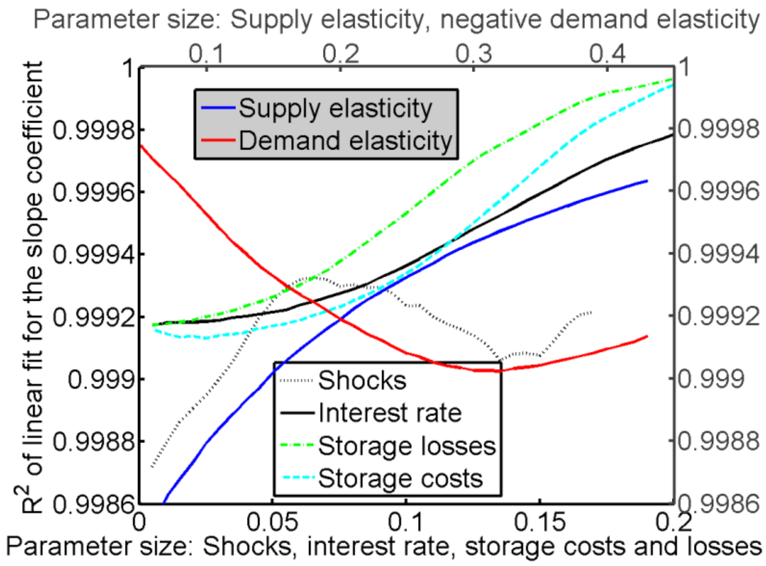

Figure 5: The R^2 of the linear OLS fit for the positive part of the storage rule for the different parameters. Clearly, the fit is always very good and therefore it is reasonable to approximate the storage rule by a linear function with a kink where the position of the kink and the slope coefficient depend on the input parameters.

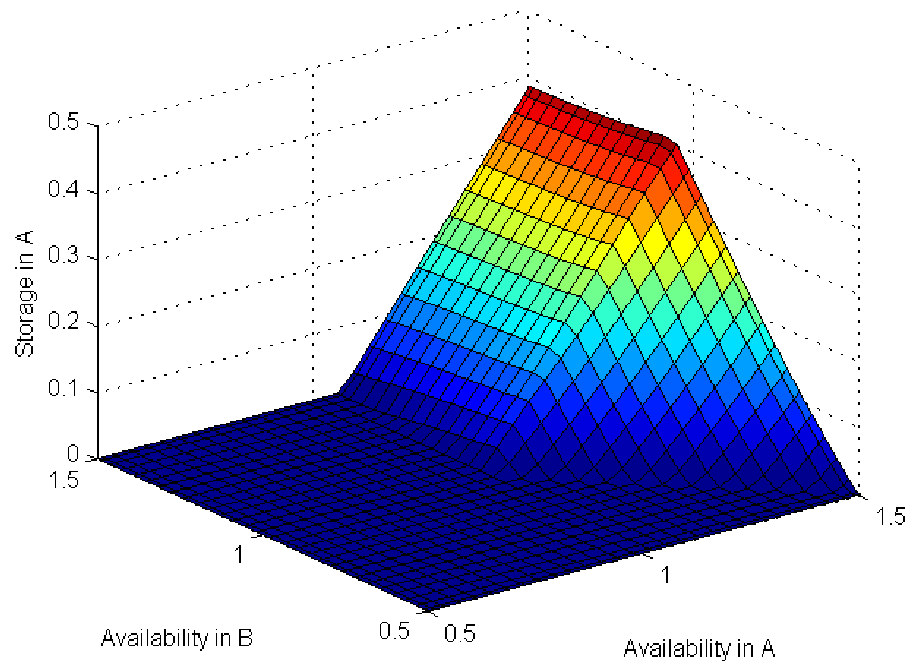

Figure 6: Storage in A dependent on availability in A and in B for the two country model without income shocks. Storages takes only place in regions of excess supply.

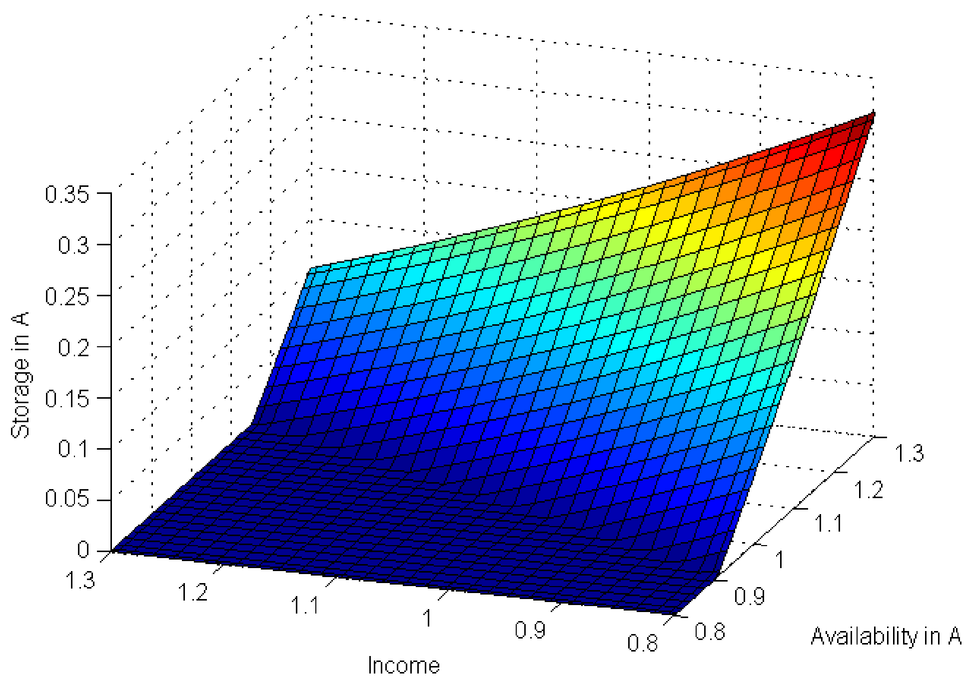

Figure 7: Storage in A dependent on the availability in A and the income level in A. The income only influences the storage threshold but not the slope.

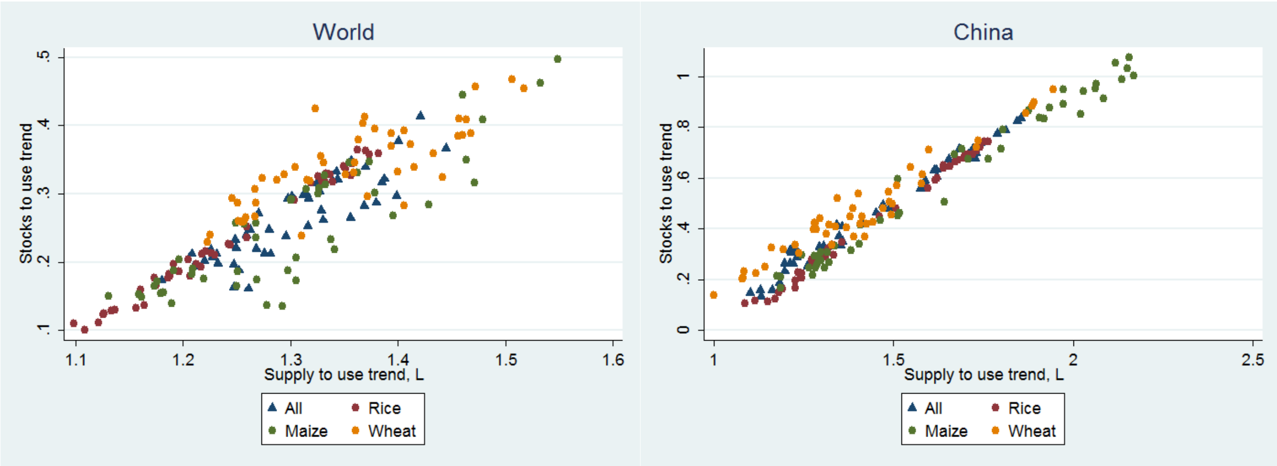

Figure 8: Stocks to use trend dependent on the supply to use trend for the worldwide stock levels (left) and for China (right) of different crops and the total of all considered crops. The nearly linear dependency supports the results from the theoretical model. It turns out that for China, the linear trend is an even better approximation than for the worldwide stocks.

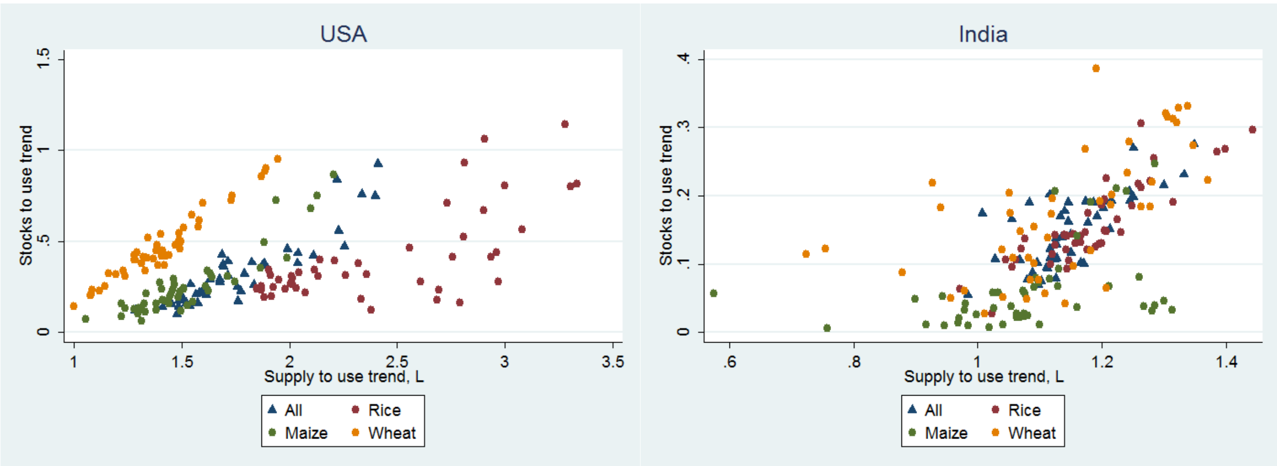

Figure 9: Stocks to use trend dependent on the supply to use trend for different crops in the US (left) and in India (right). While the dependence is nearly linear but very crop specific in the US, it is neither very crop specific nor very well approximated by a kinked line in India.

Table 1: Calibration Parameters for the simulations

Parameter	**Variable**	**Value(s)**	**Comments**
Supply elasticity in ROW	$1/\mu_B$	0.2	Value from Gouel, Gautam & Martin (2014)
Supply elasticity in A	$1/\mu_A$	0.2; 0.3; 0.4	Variation of values to cover range in FAPRI and USDA databases
Price elasticity in ROW	α_B	-0.27	Typical value
Price elasticity in A	α_A	-0.08; -0.16; -0.32	Variation of values to cover range in FAPRI and USDA databases
Storage costs per unit	k	0.06	Common value for such models
Interest rate in ROW	r_B	0.03	Typical rate
Interest rate in A	r_A	0.02; 0.07; 0.15	Variation to cover main range in World Bank data
Trade costs	θ	0.1	Common value for such models
Income elasticity in ROW	η_B	Income is fixed	

Income elasticity in A	η_A	0.5	Common value for such models
Normalization par. in ROW	γ_B	2	Describes relative scaling to parameter in A
Normalization par. in A	γ_A	0.02; 0.1; 0.2	Variation in size: 1%, 5% and 10% of rest of the world size.
SD (standard deviation) of supply shocks in ROW	σ_B	0.065	Estimated for world wheat production from USDA data
SD of supply shocks in A	σ_A	0.02; 0.06; 0.12	Variation to cover main range in USDA data
SD of income shocks in ROW	σ_B^y	Income is fixed	
SD of income shocks in A	σ_A^y	0.035	Estimated from standard deviation of GDP deviations from HP-filtered trend from World Bank database
Production normalization parameter in ROW	h_B	1/(1+r_B)	
Production normalization parameter in A	h_A	1/(1+r_A)	
Parameters for algorithm to solve the model:			
Number of nodes for each shock variable		7	
State variables grid for which the solutions are calculated		9 for each state variable (between 0.7 and 1.3)	This is the grid on which the solutions are calculated for each set of parameters.
Time horizon for convergence to steady state		9	

Table 2: Summary statistics over the separately estimated storage equation (17) (first-step validation).

Variable	Mean	Std. Dev.	CV	Min	Max
R^2	0.9997	0.0000	0.0000	0.9996	0.9998
a	0.7855	0.0105	0.0134	0.7637	0.8054
b	-1.0145	0.0465	-0.0458	-1.1007	-0.9384
ρ	0.0009	0.0065	7.6105	-0.0080	0.0158
β	-0.7767	0.0273	-0.0351	-0.8194	-0.7344
θ	0.7519	0.0527	0.0701	0.6760	0.8148
τ	0.0567	0.0424	0.7486	0.0064	0.1184

Each column shows the summary statistics over the goodness-of-fit (R^2) and the estimated reduced-model parameters using equation (17) for each structural parameter set separately. A non-linear least squares fitting procedure is used.

Table 3: Regression results for the second step of the validation of equation (17).

	Coefficient	Std. Err.	t-value	P>|t|
a_0	0.7623	0.0005	1538.21	0.000
a_r	0.0000	0.0023	0.00	0.999
a_μ	0.1045	0.0011	94.58	0.000
a_α	0.0253	0.0009	27.93	0.000
a_σ	-0.0921	0.0022	-42.04	0.000
a_γ	0.0236	0.0012	19.06	0.000
b_0	-5.3433	0.0144	-372.29	0.000
b_r	0.0001	0.0744	0.00	0.999
b_μ	-1.9393	0.0379	-51.11	0.000
b_α	0.0641	0.0300	2.13	0.033
b_σ	0.9195	0.0727	12.64	0.000
b_γ	-1.7322	0.0418	-41.46	0.000
ρ_0	0.0013	0.0156	0.08	0.935
ρ_r	-0.0001	0.0710	0.00	0.999
ρ_μ	-0.0212	0.0350	-0.61	0.545
ρ_α	-0.0470	0.0286	-1.64	0.100
ρ_σ	-0.2953	0.0693	-4.26	0.000
ρ_γ	0.5663	0.0390	14.52	0.000
β_0	-0.1412	0.0000	-2857.43	0.000
β_r	0.0000	0.0003	0.00	1.000
β_μ	0.0245	0.0002	116.76	0.000
β_α	-0.0040	0.0001	-32.78	0.000
β_σ	0.0019	0.0003	6.41	0.000
β_γ	0.0225	0.0002	92.80	0.000
θ_0	-4.9728	.	.	.
θ_r	0.0000	0.0206	0.00	1.000
θ_μ	-1.4286	0.0146	-98.02	0.000
θ_α	0.2420	0.0085	28.39	0.000
θ_σ	-0.3438	0.0204	-16.86	0.000
θ_γ	-1.4645	0.0152	-96.04	0.000
τ_0	0.0003	0.0023	0.12	0.903
τ_r	0.0000	0.0104	0.00	0.999
τ_μ	-0.0062	0.0051	-1.22	0.223
τ_α	-0.0159	0.0042	-3.81	0.000
τ_σ	-0.0392	0.0101	-3.87	0.000
τ_γ	0.5933	0.0057	103.75	0.000

N=118098. R^2=0.9997. A non-linear least squares fitting procedure is used.

Table 4: Regression results for the empirical estimation

Depended variable: Closing Stocks						
	FE-like			Random Effects		
	Empirical Model		Theoretical	Empirical Model		Theoretical
	Total Grains	Pooled Grains	Model	Total Grains	Pooled Grains	Model
Slope coefficient α						
a_0	.05435***	.03301*	.78951***	.13715***	.10713***	.78933***
	(.00743)	(.01768)	(.00022)	(.02143)	(.02308)	(.00026)
a_σ	1.1e-07	-1.9e-06	-.09622***	6.9e-06	1.4e-05	-.09128***
	(4.8e-06)	(5.8e-06)	(.00221)	(7.8e-06)	(9.9e-06)	(.0027)
a_γ	2.3968***	1.66***	.02303***	3.4466***	1.3489***	.02057***
	(.20357)	(.33364)	(.00125)	(.47684)	(.50377)	(.00153)
Primary intercept coefficient b						
b_0	-.29457***	.08377	-1.2174***	.13311***	.13439***	-.97225***
	(.04563)	(.08088)	(.00098)	(.04819)	(.03354)	(.00119)
b_σ	-4.4e-06	-8.2e-05	.19743***	2.3e-05	6.8e-05*	.18142***
	(1.1e-05)	(.0002)	(.011)	(2.7e-05)	(3.5e-05)	(.01706)
b_γ	-3.0445***	-2.4008*	-.36189***	-7.1081***	-3.9905***	-.52252***
	(.44627)	(1.4118)	(.0084)	(1.3713)	(.89323)	(.00975)
Primary intercept shift due to GDP ρ						
ρ_0	.05984	-.10126	-.00804***	-.06913**	-.00549	-.00902***
	(.08389)	(.09231)	(.00085)	(.03229)	(.02909)	(.00101)
ρ_σ	4.7e-06	-2.1e-05	-.00265	-4.6e-05**	-8.4e-05**	.06366***
	(4.1e-06)	(.0002)	(.00948)	(2.2e-05)	(3.4e-05)	(.0131)
ρ_γ	-.09872	.47525	.10217***	2.768***	1.7023***	.05477***
	(.15251)	(1.5675)	(.00661)	(.76444)	(.61515)	(.00766)
Secondary intercept shift due to trade or GDP β						
β_0	.99233*	-.60655***	-.64529***	-2.0509	7.2103***	-.80509***
	(.50723)	(.13172)	(.00023)	(3.7356)	(1.9406)	(.00039)
β_σ	9.0e-06	3.4e-05***	-.02014***	.00999	-.00449***	-.08197***
	(1.1e-05)	(1.1e-05)	(.00237)	(.0106)	(.00148)	(.00563)
β_γ	-5.5184*	-.89646*	.11852***	-2599.4	11.701*	.33089***
	(3.2132)	(.538)	(.00163)	(2935.8)	(6.162)	(.00277)
Secondary intercept shift coefficient θ						
θ_0	2.1268***	.98857	.83183***	3.0975*	1.1753***	.83348***
	(.42489)	(.79092)	(.00093)	(1.6699)	(.04136)	(.00107)
θ_σ	.00026	-.0002*	-.02429**	.00014	.00029	-.17643***
	(.00027)	(.00012)	(.0106)	(.00026)	(.00045)	(.01635)
θ_γ	-11.982	2.925	-.76895***	-141.62	-2.9538	-.67369***
	(13.146)	(2.9305)	(.00824)	(179.13)	(3.5142)	(.00912)
Secondary intercept shift due to GDP τ						
τ_0	-.00588	-.41938	-.00638***	-1.8162	.04356	-.00642***
	(.14898)	(.74201)	(.00082)	(1.7077)	(.03484)	(.00089)
τ_σ	-.00044	6.3e-05	.00226	-.00017	-3.3e-05	.05413***
	(.00033)	(.00013)	(.00899)	(.00026)	(.00037)	(.01134)
τ_γ	20.33	.29819	.6053***	150.45	.7071	.55924***
	(16.225)	(2.7752)	(.0061)	(187.48)	(2.7469)	(.00658)
R^2	.85179	.77772	.99967	.74533	.63684	.99951
Observations	1262	3796	118098	1262	3796	118098

Non-linear least square regression. Standard errors clustered by country and crop in parentheses. Grey shaded cells indicate that the estimated coefficient has the same sign as expected by the theoretical model.
All independent variables are de-trended as described before.
* $p < 0.10$, ** $p < 0.05$, *** $p < 0.01$

Table 5. Regression results for a minimalistic reduced-form storage model

Depended variable: Closing Stocks		
	Empirical Model	Theoretical Model
Slope coefficient α	.12636*** (.00308)	.78473*** (.00038)
Primary intercept coefficient b	-.12864*** (.03857)	-1.0142*** (.00083)
Secondary intercept shift due to trade or GDP β	-2.3898*** (.76375)	-.77389*** (.00064)
Secondary intercept shift coefficient θ	.80707*** (.16831)	.74762*** (.00095)
Secondary intercept shift due to GDP τ	.11062 (.09892)	.05797*** (.00048)
Working stocks w	.10842*** (.01017)	
R^2	.32314	.99431
Observations	3796	118098

Non-linear least square regression. Standard errors in parentheses. The empirical model is applied to pooled grains. All independent variables are de-trended as described before.
* $p < 0.10$, ** $p < 0.05$, *** $p < 0.01$

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