Modeling Firm Heterogeneity in International Trade: Do General Equilibrium Effects Matter?

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Abstract

This paper contrasts two different formulations of the Melitz model of international trade. The first formulation is based on the original Melitz (2003) model, considering one industry with differentiated products and heterogeneous firm productivity, one primary factor and iceberg transport costs. The second formulation is a computable general equilibrium refinement of the same model, proposed by Roson and Oyamada (2014), where two industries and two factors are considered, alongside intermediate factors and input-output linkages. The two alternative formulations are tested by running similar simulation experiments, with comparable data sets.

Keywords: Computable General Equilibrium Models, Melitz, Firm Heterogeneity, International Trade.

JEL CODES: C63, C68, D51, D58, F12, L11.

1 Introduction

To be written.

2 Two model formulations

2.1 The Melitz basic model

Dixon, Jerie and Rimmer (2013) elaborated the theoretical model of trade introduced by Melitz (2003). We summarize here the main equations of this framework, providing only a brief description of every equation and a discussion

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of its meaning. The interested reader may get more details from the two papers above.

There is an industry, in which several firms produce and sell (to geographically distinct markets) differentiated products. Each firm uses only one input (labour), and each one has a specific labour productivity parameter Φ. This expresses the units of output produced by one unit of labour in that firm.

The consumers have preferences determined by CES utility functions, with a parameter $\sigma > 1$ expressing the elasticity of substitution. Therefore, all goods (both domestic and imported) are regarded as imperfect substitutes.

We indicate with s the region of origin of trade flows, with d the destination market, and with the symbol $^\circ$ values referring to the "average" firm (in terms of productivity) among all those who are serving market d from region s.

The firms have some degree of market power and set their price on the basis of a mark-up rule over marginal cost, where the elasticity of substitution σ determines the price elasticity of individual demand functions. For the average, representative firm:

$$P_{^\circ sd} = \left(\frac{W_s T_{sd}}{\Phi_{^\circ sd}}\right)\left(\frac{\sigma}{\sigma - 1}\right) \tag{1}$$

where $T_{sd} > 1$ is a cost factor expressing "iceberg" transportation/trade costs in the sd link[1], and W_s is labour cost in region s.

In the destination market d, a CES price index is readily built by considering all goods flowing into that market:

$$P_d = \left(\sum_s N_{sd} P_{^\circ sd}^{1-\sigma}\right)^{\frac{1}{1-\sigma}} \tag{2}$$

where N_{sd} stands for the number of firms active in the link sd (a subset total firms N_s). The CES quantity index for sd can be computed on the basis of the output of the average firm:

$$Q_{sd} = N_{sd}^{\sigma/(\sigma-1)} Q_{^\circ sd} \tag{3}$$

The demand for $Q_{^\circ \mathrm{sd}}$ is, in turn, driven by aggregate demand in the destination market and relative prices:

$$Q_{^\circ sd} = Q_d \left(\frac{P_d}{P_{^\circ sd}}\right)^\sigma \tag{4}$$

Profits obtained by each firm active on the link sd are given by the difference between gross sale profits and fixed costs associated with the establishment of a foreign subsidiary in destination d, which requires F_{sd} units of labour. For the representative firm:

[1] In other words, $T_{sd} - 1$ are the units of product necessary to carry one unit of the produced good from s to d.

$$\Pi_{\circ sd} = \left(P_{\circ sd} - \frac{W_s T_{sd}}{\Phi_{\circ sd}} \right) Q_{\circ \mathrm{sd}} - F_{sd} W_s \tag{5}$$

In addition to link-related fixed costs, each firm has general "headquarters" fixed costs (H_s labour units). Like in a monopolistic competition setting, there is free entry in the industry in region s, driving total expected profits to zero:[2]

$$\sum_d N_{sd} \Pi_{\circ sd} - N_s H_s W_s = 0 \tag{6}$$

In the trade link sd, the marginal firm is the one having the minimum level of productivity Φ_{MINsd} compatible with non-negative profits on that link:

$$\Pi_{MINsd} = \left(P_{MINsd} - \frac{W_s T_{sd}}{\Phi_{MINsd}} \right) Q_{MINsd} - F_{sd} W_s = 0 \tag{7}$$

If the random productivity parameter has a Pareto distribution with parameter α [$p(\Phi)$=$\alpha\Phi^{-\alpha\text{-}1}$, $\Phi \geq 1$], it can be shown that the following relationships apply:

$$N_{sd} = N_s (\Phi_{MINsd})^{-\alpha} \tag{8}$$

$$\Phi_{\circ sd} = \beta \Phi_{MINsd} \tag{9}$$

$$\mathrm{Q}_{MINsd} = \mathrm{Q}_{\circ sd} / \beta^{\sigma} \tag{10}$$

where $\beta = \left(\frac{\alpha}{\alpha - (\sigma - 1)} \right)^{1/(\sigma - 1)}$.

Finally, total labour demand is given by:

$$L_s = \sum_d \frac{N_{sd} Q_{\circ sd} T_{sd}}{\Phi_{\circ sd}} + \sum_d N_{sd} F_{sd} + N_s H_s \tag{11}$$

The set of Equations (1)-(11) determines a system where, given cost, distribution and preference parameters, labour cost W_s and aggregate demand Q_d, the following endogenous variables can be computed:

1. The price $P_{\circ sd}$ of the average firm in link sd;
2. The price index in the destination market P_d;
3. The quantity $Q_{\circ sd}$ of the average firm in link sd;
4. The quantity index Q_{sd} in link sd;
5. The profit $\Pi_{\circ sd}$ of the average firm in link sd;

[2] Profits are expected because each firm does not know its realization of the random variable Φ before entering the market. Timing is therefore as follows: (1) a enter/no enter decision is taken, (2) in case of entry, H_s units of labour are employed, (3) the random variable Φ is known, (4) the firms decide on which markets to operate, (5) prices/quantities are set.

6. The number of active firms N_s in the home region s;
7. Demand for labour L_s in the home region s;
8. Number of firms in the sd link;
9. Productivity of the marginal firm in the sd link;
10. Productivity of the average firm in the sd link;
11. Quantity sold by the marginal firm in the sd link.

A reduction in trade costs T_{sd} increases average productivity, therefore efficiency, in both the origin and destination markets. This is a source of trade-related welfare gains, supplementing the conventional sources based on Ricardian comparative advantages, and market-size economies of scale (à la Krugman).

2.2 A multisectoral, general equilibrium framework

Roson and Oyamada (2014) proposed a refinement of the Melitz model, with the aim of making the structure of the model similar to that of a Computable General Equilibrium (CGE) model. The key characteristics of this alternative formulation are:

- There are two industries, named "Manufacturing" and "Services". Manufacturing is an industry composed of heterogenous firms à la Melitz. Services is a conventional industry, which can be modeled through a representative firm;
- Services are needed to generate headquarters services (fixed costs), both in the home and foreign markets, as well as for the transportation of manufactured goods. The amount of services required for the different purposes is determined by technological requirements, and may depend on firm productivity. The services needed to export in foreign markets generate a demand for foreign resources;
- There are two primary factors (labour and capital). Endowments of primary resources are given and typically different between regions, which therefore differ in terms of comparative advantages.

We summarize here one specific version of the model, where it is further assumed that: (a) intermediate factors are not substitutable among themselves (à la Leontief)[3]; (b) services are domestically produced and consumed. They are not inter-regionally traded[4].

Let us indicate with a_d^{hj} the input coefficients for intermediate inputs, that is the amount of factor goods produced by industry h, necessary to produce

[3]However, manufactured factors are differentiated and substitutable inside the CES aggregate.

[4]Nonetheless, foreign services are needed to establish subsidiary branches abroad.

one unit of output in industry j located in d. There is an important difference here between services, which are an homogeneous industry, and manufacturing, which is a differentiated one. "Inputs" and "outputs" refer to physical quantities in homogeneous industries but, actually, to CES quantity composites in differentiated industries.

The demand for differentiated intermediate factors adds to final consumption demand to determine the overall regional demand for manufactured goods, so that:

$$Z_d^{ms} + Z_d^{mm} + Q_d^m = \left(\sum_s N_{sd} Q_{\circ sd}^{(\sigma\text{-}1)/\sigma} \right)^{\sigma/(\sigma-1)} \tag{12}$$

where Z_d^{ms} stands for intermediate demand for manufactured goods generated by services, and Z_d^{mm} for intra-manufacturing intermediate demand. In particular:

$$Z_d^{mm} = a_d^{mm} \left(\sum_s N_{ds} Q_{\circ ds} / \Phi_{\circ ds} \right) \tag{13}$$

$$Z_d^{ms} = a_d^{ms} X_d^s \tag{14}$$

where X_d^s is the output level of the services industry in d, given by:

$$\begin{aligned} X_d^s = Q_d^s + a_d^{sm} \left(\sum_s N_{ds} Q_{\circ ds} / \Phi_{\circ ds} \right) + a_d^{ss} X_d^s + \\ + N_d H_d + \sum_s N_{sd} F_{sd} + \sum_s T_{ds} N_{ds} Q_{\circ ds} / \Phi_{\circ ds} \end{aligned} \tag{15}$$

where Q_d^s is the quantity of services directly consumed by households in region d. T_{ds}, F_{sd}, H_s express the amount of *services* needed to: (1) carry one unit of manufactured good from d to s[5], (2) establish a trade link sd[6], (3) start a business in region s. The demand for primary factors is given by:

$$L_s^i = \sum_d \frac{N_{sd} Q_{\circ sd} A_s^{im}}{\Phi_{\circ sd}} + X_s^s A_s^{is} \tag{16}$$

where A_s^{ij} stands for the amount of primary factor i used to produce one unit of output in industry j in region s.

Final consumption includes manufactured goods as well as services. Manufactured goods are differentiated goods produced by both domestic and foreign firms. Services are domestically produced and are homogeneous.

For both industries, final consumption levels are determined on the basis of utility maximization of the representative consumer, given the budget constraint:

[5]Contrary to the basic Melitz setting, this parameter is no more a multiplicative factor greater than one.

[6]Notice that the demand for services is generated in the destination country.

$$\sum_i L_d^i W_d^i = \sum_j Q_d^j P_d^j \tag{17}$$

For example, if the utility function is linear logarithmic (Cobb-Douglas), then budget shares (ψ^j) would be constant, and consumption levels would be implicitly set by:

$$\psi_d^j \left(\sum_i L_d^i W_d^i \right) = Q_d^j P_d^j \tag{18}$$

The inclusion of differentiated production factors adds a special feature to the model. Any increase in the number of trading manufacturing firms would not only bring about a welfare gain, because of the Dixit-Stiglitz "taste for variety" effect, but also an increase in productivity for intermediate imported factors, like in Fujita, Krugman and Venables (1999). Aggregate productivity effects therefore overlap firm-level productivity effects.

Furthermore, intermediate demand simply adds to final consumption. The quantity bundle on the right hand side of (12) refers to total demand in a region, implying that the internal composition of intermediate and final trade flows (and the associated price index) is the same.

3 Comparing the models' results

In order to highlight what qualitative differences are implied by the different model formulations, we present and comment in this section results of some "parallel" simulation experiments, performed with the two versions of the model. Let us call the Melitz basic single-sector and the extended multisectoral models as "basic" and "extended" models, respectively. Parameters for the two model versions are calibrated to almost identical (although with different aggregation level) artificial data sets, which are described in the Appendix.

We consider four simulation scenarios for each model version. In the first experiment, a 50 percent reduction in trade costs is simulated. We consider both an "asymmetric" case, where only one trade link is taken into account, and a "symmetric" case, where trade costs are lowered in both directions. In the second experiment, we analyze increases in the size of an economy, interpreted as an increase in the endowment of primary production factor(s). Again, we analyze both the asymmetric (one country) and symmetric (both countries) cases.

3.1 Reduction in Trade Costs

3.1.1 Asymmetric Case

Table 1 shows percentage changes of endogenous variables, for the basic model, induced by a 50 percent reduction of trade costs in the A to B link. The main effects can be summarized as follows:

- The cost inclusive price of traded commodities exported from Country A becomes cheaper than that of the one from Country B. Hence, the volume of trade flow from Country A to B gets larger than that from Country B to A.

- Exports from Country A to B increases. However, the total exports from Country B to A also increases, because of equilibrium in the trade balance. Since the factor endowments of both countries have not changed from the base case, so that the production levels do not change very much in both countries, demand for domestic goods decreases in order to expand international trade.

- The reduction in (trade) costs has effects to pull down the prices of output (then those of the input factor). Hence, relatively less competitive domestic firms sell commodities with relatively cheap prices. This appears as depreciations of sales price of domestic goods in both countries.

- The factor (labor) requirements for transportation from A to B decreases, so that resources become available for the expansion of production in Country A.

- The combination of the cheaper price of imports from Country A and depreciated price of domestic goods lowers the relative factor price (wage) in Country B. Since the labor in Country A is set as the numeraire, the relative wage in Country B gets lower.

Furthermore, welfare impacts are greater for Country B than for A, since B can directly obtain relatively cheaper items imported from A. On the other hand, the greater efficiency of the global economy raises the welfare levels in both countries.

Lower trade margins allow new firms to be competitive in foreign markets. Consequently, the average productivity level decreases and the number of exporting firms increases in A. The real depreciation occurring in B also makes firms in that country more competitive, meaning that average productivity level decreases and number of firms increases.

Table 2 displays the corresponding results obtained with the extended model. In addition to the effects discussed above, one can notice the following impacts:

- Services used for transportation from Country A to B can be saved. As a consequence, production in Country A shifts towards the relatively capital intensive manufacturing sector. Therefore, capital appreciates in Country A.

- Services demanded (as fixed cost) to set up local affiliates increase in both countries as the international trade expands. Since the services cannot be saved in Country B unlike A, the production in B shifts from the manufacturing to relatively labor intensive services sector. Then, the wage appreciates in Country B.

- The above two effects contribute to raise output prices in both countries, because of higher input costs.

One interesting point is that the welfare effects are amplified with the extended model. While the Melitz model captures the efficiency gains from the reallocation of resources in the sector having increasing returns to scale, an extension that includes several industries captures additional gains from sectoral adjustment.

3.1.2 Symmetric Case

Then, let us move to verify the effects of reducing the trade costs on both directions, again by 50 percent. Table 3 depicts percentage changes of endogenous variables from the base case, when the basic model applies. Clearly, the effects illustrated for the asymmetric case can be found here at a higher magnitude. The input factor (labor) used for international transportation of commodity decreases so that the production volumes in both countries expand. Also, exports from both countries increase while the consumptions of domestic goods decrease. Competitiveness of the exporting firms become relatively higher than that of the domestic firms, and the number of the exporting firms increases in both countries.

The results obtained with the extended model are shown in Table 4. This time, the production structure in both countries shifts from the services sector to relatively capital intensive manufacturing, so that capital appreciates in both countries.

This is occurring despite the fact that the services demanded (as fixed cost) to establish foreign affiliates, which are also expanding, tend to cancel out the saving effects of trade cost reduction, implying that the effects of sectoral adjustment in the symmetric setting are smaller than those in the asymmetric one. Even if the magnitude of the positive shock is larger under symmetry, sectoral adjustment processes turn out to be less significant than in the asymmetric case.

3.2 Increase in the Size of an Economy

The next scenarios consider increases in the size of an economy, interpreted as increments in the endowment of a primary production factor. In the basic model, there is only one factor (labor). In the extended model, there are two factors (labor and capital), which are considered here separately.

3.2.1 Asymmetric Case

We start with checking the effects of a five percent increase in factor endowment of Country A. Table 5 shows the percentage changes of endogenous variables from the base case, for the basic model version. Three main effects can be noticed:

- Because of the increased availability of the input factor (labor), the volumes of production, exports, and domestic consumption expand in Country A.
- Exports from Country A to B increases. Then, exports from Country B to A also increase, since balanced trade is implied by the macroeconomic budget constraint. As the factor endowment in Country B is fixed, domestic consumption in that country decreases.
- In Country B, prices of both domestic goods and the input factor get lower than the ones in Country A. In other words, a real depreciation occurs.

With the extended model, a sectoral composition effect comes into play. Table 6 displays the impact of a five percent increase in labor endowment of Country A. Here we can notice that:

- The increase in labor endowments in Country A brings about a capital price appreciation. Consequently, production in Country A shifts from the manufacturing sector to relatively labor intensive services.
- In Country B, demand for services becomes stronger because of the expansion of international trade. Consequently, production shifts from manufacturing to services, raising wages relative to the capital rental price in the country.

The sectoral effects work in the opposite direction when capital endowment is augmented (Table 7):

- The increase in capital endowments in Country A brings about a capital price depreciation. Production in Country A shifts to the relatively capital intensive manufacturing sector from services.
- Contrary to the case of labor expansion, exports from both countries shrink because services (labor intensive), required for transportation and establishment of local affiliates, becomes relatively expensive. Then, Country B, whose factor endowments do not increase, faces a welfare loss.
- In Country B, demand for services also declines because of the shrinking international trade. Thus, the production shifts to the manufacturing from services, raising the capital price relative to wage in the country.

3.2.2 Symmetric Case

Let us see the effects of a five percent increase in factor endowment under symmetry. Table 8 refers to the basic model. Because of the increased availability of input factor (labor), the volumes of production, exports, and domestic consumption expand in both countries. In a symmetric world, all quantities proportionally increase (by five percent) while prices stay constant. Higher final consumption implies welfare gains.

Results obtained with the extended model is shown in Table 9, when labor endowment in both countries increase by five percent. The growth in labor endowments brings about wage depreciations in both countries, whereas capital appreciates. Consequently, production in both countries shifts from the manufacturing sector to the relatively labor intensive services. Cheaper services support the expansion of exports, through the increasing availability of services for transportation and fixed costs: international trade in both directions increases, while demand for domestic goods decreases.

Finally, Table 10 depicts the effects of increasing capital endowments. Contrary to the case of labor, the increase in capital endowments bring about depreciations of capital rental price. Then, production shifts to relatively capital intensive manufacturing sector from services. This kind of sectoral adjustment discourages export, international trade in both directions lowers while demand for domestic goods expands.

4 Conclusion

To be written.

Appendix: Benchmark Calibration Data

Tables 11 and 12 show the data sets for calibration of the basic and extended models. The two types of trade flow data at different price levels (cost, insurance, and freight (CIF) and free on board (FOB)) for the extended model only include intra- and inter-national flows of manufactured goods, since the services are assumed to be non-tradable, although they can be bought by foreign firms to establish subsidiaries abroad.

In addition to calibrated parameters, Melitz-type models require information on: total number of firms, number of active firms, Pareto shape parameter, and CES elasticities. The choice of number of firms is neutral and will not affect simulation results as long as we measure effects as changes from a base case. The values of Pareto shape parameter and CES elasticity are taken here from Melitz and Redding (2013).

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number of firms (n)

	a	b
	0.000	0.000

number of firms (an)

	a	b
a	-3.183	16.825
b	21.625	-4.083

trade flow (pdot)

	a	b
a	-0.763	-6.150
b	0.589	-4.884

trade flow (qdot)

	a	b
a	0.764	6.556
b	-4.501	0.988

trade flow (phidot)

	a	b
a	0.767	-3.592
b	-4.503	0.991

trade flow (total quantity)

	a	b	exp
a	-3.186	29.121	1.105
b	21.627	-4.086	-0.671
imp	0.109	0.324	

trade flow (total value)

	a	b	exp
a	-3.925	21.180	-0.241
b	22.343	-8.770	-4.205
imp	-0.070	-4.376	

total fixed cost (h)

	a	b
	0.000	0.000

total fixed cost (f)

	a	b
a	-3.183	16.825
b	21.625	-4.083

total transportation cost

	a	b
a		-35.440
b	21.627	

input-output table (quantity)

a	c
	0.764
l	1.107
l (t+h+f)	-2.953

b	c
	0.988
l	-0.677
l (t+h+f)	1.804

input-output table (price)

a	c
	-0.762
l	0.000
l (t+h+f)	0.000

b	c
	-4.886
l	-3.950
l (t+h+f)	-3.950

input-output table (value)

a	c	total
	-0.004	-0.004
l	1.107	
l (t+h+f)	-2.953	
total	0.000	

b	c	total
	-3.947	-3.947
l	-4.600	
l (t+h+f)	-2.217	
total	-3.950	

welfare

a	b
0.764	0.988

Table 1: Effects of Reducing the Trade Costs on the Link A-B (Basic Model, % var.)

number of firms (n)

a	b
3.640	-2.230

number of firms (an)

	a	b
a	-2.050	15.025
b	18.275	-8.950

trade flow (pdot)

	a	b
a	13.913	6.411
b	13.920	6.397

trade flow (qdot)

	a	b
a	-4.382	5.253
b	-4.382	5.253

trade flow (phidot)

	a	b
a	1.339	-2.423
b	-4.385	1.693

trade flow (total quantity)

	a	b	exp
a	-7.579	24.073	0.000
b	18.279	-5.762	-0.006
imp	-1.388	1.381	

trade flow (total value)

	a	b	exp
a	5.280	32.027	12.265
b	34.744	0.266	9.271
imp	12.975	8.561	

total fixed cost (h)

a	b
3.640	-2.230

total fixed cost (f)

	a	b
a	-2.050	15.025
b	18.275	-8.950

total transportation cost

	a	b
a		-37.963
b	18.279	

input-output table (quantity)

a	m	s	c
m	-1.529	-1.529	0.437
s	0.012	0.004	3.808
s (t+h+f)	-2.098		
l	0.000	0.000	
k	-0.002	-0.002	

b	m	s	c
m	1.643	1.643	3.330
s	0.016	0.004	-1.079
s (t+h+f)	0.597		
l	-0.001	-0.001	
k	0.002	0.002	

input-output table (price)

a	m	s	c
m	12.576	12.576	12.576
s	8.920	8.920	8.920
s (t+h+f)	8.920		
l	0.000	0.000	
k	37.340	37.340	

b	m	s	c
m	7.212	7.212	7.212
s	11.990	11.990	11.990
s (t+h+f)	11.990		
l	18.080	18.080	
k	-2.770	-2.770	

input-output table (value)

a	m	s	c	total
m	10.855	10.855	13.068	12.271
s	8.933	8.924	13.068	10.226
s (t+h+f)	6.635			8.924
l	0.000	0.000		10.784
k	37.337	37.337		
total	12.545	8.582	10.784	

b	m	s	c	total
m	8.974	8.974	10.782	10.131
s	12.008	11.995	10.782	11.617
s (t+h+f)	12.659			11.995
l	18.079	18.079		10.959
k	-2.768	-2.768		
total	9.849	12.348	10.959	

welfare

a	b
1.108	2.434

Table 2: Effects of Reducing the Trade Costs on the Link A-B (Extended Model, % var.)

number of firms (n)

a	b
0.000	0.000

number of firms (an)

	a	b
a	-7.783	41.100
b	41.100	-7.783

trade flow (pdot)

	a	b
a	-1.895	-1.891
b	-1.891	-1.895

trade flow (qdot)

	a	b
a	1.923	1.923
b	1.923	1.923

trade flow (phidot)

	a	b
a	1.928	-7.783
b	-7.783	1.928

trade flow (total quantity)

	a	b	exp
a	-7.788	55.952	0.677
b	55.952	-7.788	0.677
imp	0.677	0.677	

trade flow (total value)

	a	b	exp
a	-9.535	53.003	-0.359
b	53.003	-9.535	-0.359
imp	-0.359	-0.359	

total fixed cost (h)

a	b
0.000	0.000

total fixed cost (f)

	a	b
a	-7.783	41.100
b	41.100	-7.783

total transportation cost

	a	b
a		-22.024
b	-22.024	

input-output table (quantity)

a	c
	1.923
l	0.689
l (t+h+f)	-1.837

b	c
	1.923
l	0.689
l (t+h+f)	-1.837

input-output table (price)

a	c
	-1.887
l	0.000
l (t+h+f)	0.000

b	c
	-1.887
l	0.000
l (t+h+f)	0.000

input-output table (value)

a	c	total
	0.000	0.000
l	0.689	
l (t+h+f)	-1.837	
total	0.000	

b	c	total
	0.000	0.000
l	0.689	
l (t+h+f)	-1.837	
total	0.000	

welfare

a	b
1.923	1.923

Table 3: Effects of Reducing the Trade Costs on the Links A-B and B-A (Basic Model)

number of firms (n)

a	b
2.030	2.030

number of firms (an)

	a	b
a	-11.683	37.300
b	37.300	-11.683

trade flow (pdot)

	a	b
a	4.515	4.519
b	4.519	4.515

trade flow (qdot)

	a	b
a	-0.183	-0.183
b	-0.183	-0.183

trade flow (phidot)

	a	b
a	3.456	-6.744
b	-6.744	3.456

trade flow (total quantity)

	a	b	exp
a	-14.789	46.959	-0.005
b	46.959	-14.789	-0.005
imp	-0.005	-0.005	

trade flow (total value)

	a	b	exp
a	-10.942	53.601	5.914
b	53.601	-10.942	5.914
imp	5.914	5.914	

total fixed cost (h)

a	b
2.030	2.030

total fixed cost (f)

	a	b
a	-11.683	37.300
b	37.300	-11.683

total transportation cost

	a	b
a		-26.520
b	-26.520	

input-output table (quantity)

a	m	s	c
m	-0.002	-0.002	3.957
s	0.000	-0.002	3.450
s (t+h+f)	-1.907		
l	0.000	0.000	
k	0.001	-0.003	

b	m	s	c
m	-0.002	-0.002	3.957
s	0.000	-0.002	3.450
s (t+h+f)	-1.907		
l	0.000	0.000	
k	0.001	-0.003	

input-output table (price)

a	m	s	c
m	3.822	3.822	3.822
s	4.330	4.330	4.330
s (t+h+f)	4.330		
l	0.000	0.000	
k	22.660	22.660	

b	m	s	c
m	3.822	3.822	3.822
s	4.330	4.330	4.330
s (t+h+f)	4.330		
l	0.000	0.000	
k	22.660	22.660	

input-output table (value)

a	m	s	c	total
m	3.820	3.820	7.930	6.450
s	4.330	4.328	7.930	5.458
s (t+h+f)	2.340			4.328
l	0.000	0.000		5.507
k	22.661	22.657		
total	6.451	4.328	5.507	

b	m	s	c	total
m	3.820	3.820	7.930	6.450
s	4.330	4.328	7.930	5.458
s (t+h+f)	2.340			4.328
l	0.000	0.000		5.507
k	22.661	22.657		
total	6.451	4.328	5.507	

welfare

a	b
3.858	3.858

Table 4: Effects of Reducing the Trade Costs on the Links A-B and B-A (Extended Model)

number of firms (n)

a	b
5.000	0.000

number of firms (an)

	a	b
a	5.517	2.275
b	2.650	-0.500

trade flow (pdot)

	a	b
a	0.114	-0.612
b	0.238	-0.496

trade flow (qdot)

	a	b
a	-0.115	0.622
b	-0.615	0.119

trade flow (phidot)

	a	b
a	-0.112	0.622
b	-0.616	0.124

trade flow (total quantity)

	a	b	exp
a	5.513	2.275	5.083
b	2.652	-0.505	-0.086
imp	5.133	-0.136	

trade flow (total value)

	a	b	exp
a	5.634	1.649	5.049
b	2.896	-0.999	-0.427
imp	5.232	-0.610	

total fixed cost (h)

a	b
5.000	0.000

total fixed cost (f)

	a	b
a	5.517	2.275
b	2.650	-0.500

total transportation cost

	a	b
a		2.275
b	2.652	

input-output table (quantity)

a	c
	6.598
l	5.085
l (t+h+f)	4.773

b	c
	0.119
l	-0.083
l (t+h+f)	0.221

input-output table (price)

a	c
	-1.500
l	0.000
l (t+h+f)	0.000

b	c
	-0.487
l	-0.380
l (t+h+f)	-0.380

input-output table (value)

a	c	total
	5.000	5.000
l	5.085	
l (t+h+f)	4.773	
total	5.000	

b	c	total
	-0.369	-0.369
l	-0.463	
l (t+h+f)	-0.160	
total	-0.380	

welfare

a	b
6.598	0.119

Table 5: Effects of the Increase in Factor Endowment of Country A (Basic Model)

number of firms (n)

a	b
16.120	-0.660

number of firms (an)

	a	b
a	13.817	4.525
b	22.050	-2.667

trade flow (pdot)

	a	b
a	77.184	67.686
b	77.189	67.671

trade flow (qdot)

	a	b
a	-17.732	1.496
b	-17.732	1.496

trade flow (phidot)

	a	b
a	0.472	2.503
b	-4.728	0.484

trade flow (total quantity)

	a	b	exp
a	-6.805	3.497	-4.338
b	5.391	-1.686	0.008
imp	-3.885	-0.445	

trade flow (total value)

	a	b	exp
a	65.128	73.551	67.327
b	86.742	64.844	70.563
imp	70.772	67.118	

total fixed cost (h)

a	b
16.120	-0.660

total fixed cost (f)

	a	b
a	13.817	4.525
b	22.050	-2.667

total transportation cost

	a	b
a		3.497
b	5.391	

input-output table (quantity)

a	m	s	c
m	-3.156	7.547	-0.713
s	-4.324	10.833	14.827
s (t+h+f)	14.379		
l	-4.333	10.833	
k	-4.334	10.833	

b	m	s	c
m	-1.192	3.290	0.945
s	-0.006	-0.001	-0.320
s (t+h+f)	0.176		
l	-0.001	-0.001	
k	0.002	0.002	

input-output table (price)

a	m	s	c
m	68.586	68.586	68.586
s	45.770	45.770	45.770
s (t+h+f)	45.770		
l	0.000	0.000	
k	183.240	183.240	

b	m	s	c
m	68.056	68.056	68.056
s	70.190	70.190	70.190
s (t+h+f)	70.190		
l	72.910	72.910	
k	63.580	63.580	

input-output table (value)

a	m	s	c	total
m	63.265	81.309	67.384	68.788
s	39.466	61.562	67.384	58.623
s (t+h+f)	66.731			61.562
l	-4.333	10.833		65.576
k	170.966	213.924		
total	69.673	60.456	65.576	

b	m	s	c	total
m	66.052	73.584	69.645	69.556
s	70.180	70.188	69.645	70.016
s (t+h+f)	70.490			70.188
l	72.908	72.908		69.837
k	63.583	63.583		
total	68.671	71.295	69.837	

welfare

a	b
2.222	0.695

Table 6: Effects of the Increase in Labor Endowment of Country A (Extended Model)

number of firms (n)

a	b
-10.620	0.530

number of firms (an)

	a	b
a	-7.817	-3.750
b	-17.825	2.200

trade flow (pdot)

	a	b
a	-35.470	-31.963
b	-35.463	-31.960

trade flow (qdot)

	a	b
a	22.001	-1.188
b	22.001	-1.188

trade flow (phidot)

	a	b
a	-0.719	-1.726
b	4.857	-0.383

trade flow (total quantity)

	a	b	exp
a	13.280	-3.223	9.329
b	-4.389	1.375	-0.005
imp	9.050	0.274	

trade flow (total value)

	a	b	exp
a	-26.900	-34.156	-28.795
b	-38.295	-31.024	-32.923
imp	-29.876	-31.842	

total fixed cost (h)

a	b
-10.620	0.530

total fixed cost (f)

	a	b
a	-7.817	-3.750
b	-17.825	2.200

total transportation cost

	a	b
a		-3.223
b	-4.389	

input-output table (quantity)

a	m	s	c
m	7.282	-3.979	6.425
s	9.356	-5.830	-9.437
s (t+h+f)	-9.599		
l	9.333	-5.833	
k	9.354	-5.816	

b	m	s	c
m	2.097	-1.914	-0.763
s	-0.004	-0.004	0.256
s (t+h+f)	-0.147		
l	0.004	0.004	
k	0.001	0.001	

input-output table (price)

a	m	s	c
m	-33.005	-33.005	-33.005
s	-21.270	-21.270	-21.270
s (t+h+f)	-21.270		
l	0.000	0.000	
k	-82.860	-82.860	

b	m	s	c
m	-32.080	-32.080	-32.080
s	-32.770	-32.770	-32.770
s (t+h+f)	-32.770		
l	-33.650	-33.650	
k	-30.650	-30.650	

input-output table (value)

a	m	s	c	total
m	-28.126	-35.671	-28.700	-29.700
s	-13.904	-25.860	-28.700	-24.172
s (t+h+f)	-28.828			-25.860
l	9.333	-5.833		-27.993
k	-81.257	-83.857		
total	-29.899	-25.611	-27.993	

b	m	s	c	total
m	-30.656	-33.380	-32.598	-32.335
s	-32.773	-32.773	-32.598	-32.718
s (t+h+f)	-32.869			-32.773
l	-33.647	-33.647		-32.529
k	-30.650	-30.650		
total	-32.128	-33.032	-32.529	

welfare

a	b
3.045	-0.553

Table 7: Effects of the Increase in Capital Endowment of Country A (Extended Model)

number of firms (n)

a	b
5.000	5.000

number of firms (an)

	a	b
a	5.000	5.000
b	5.000	5.000

trade flow (pdot)

	a	b
a	0.000	0.000
b	0.000	0.000

trade flow (qdot)

	a	b
a	0.000	0.000
b	0.000	0.000

trade flow (phidot)

	a	b
a	0.000	0.000
b	0.000	0.000

trade flow (total quantity)

	a	b	exp
a	5.000	5.000	5.000
b	5.000	5.000	5.000
imp	5.000	5.000	

trade flow (total value)

	a	b	exp
a	5.000	5.000	5.000
b	5.000	5.000	5.000
imp	5.000	5.000	

total fixed cost (h)

a	b
5.000	5.000

total fixed cost (f)

	a	b
a	5.000	5.000
b	5.000	5.000

total transportation cost

	a	b
a		5.000
b	5.000	

input-output table (quantity)

a	c
	6.722
l	5.000
l (t+h+f)	5.000

b	c
	6.722
l	5.000
l (t+h+f)	5.000

input-output table (price)

a	c
	-1.612
l	0.000
l (t+h+f)	0.000

b	c
	-1.612
l	0.000
l (t+h+f)	0.000

input-output table (value)

a	c	total
	5.001	5.001
l	5.000	
l (t+h+f)	5.000	
total	5.000	

b	c	total
	5.001	5.001
l	5.000	
l (t+h+f)	5.000	
total	5.000	

welfare

a	b
6.722	6.722

Table 8: Effects of the Increase in Factor Endowment of Countries A and B (Basic Model)

number of firms (n)

a	b
15.420	15.420

number of firms (an)

	a	b
a	10.650	27.650
b	27.650	10.650

trade flow (pdot)

	a	b
a	70.228	70.234
b	70.234	70.228

trade flow (qdot)

	a	b
a	-16.542	-16.542
b	-16.542	-16.542

trade flow (phidot)

	a	b
a	0.997	-2.343
b	-2.343	0.997

trade flow (total quantity)

	a	b	exp
a	-8.565	9.089	-4.338
b	9.089	-8.565	-4.338
imp	-4.338	-4.338	

trade flow (total value)

	a	b	exp
a	55.647	85.707	63.498
b	85.707	55.647	63.498
imp	63.498	63.498	

total fixed cost (h)

a	b
15.420	15.420

total fixed cost (f)

	a	b
a	10.650	27.650
b	27.650	10.650

total transportation cost

	a	b
a		9.089
b	9.089	

input-output table (quantity)

a	m	s	c
m	-4.335	10.832	0.266
s	-4.341	10.831	14.533
s (t+h+f)	14.544		
l	-4.333	10.833	
k	-4.335	10.832	

b	m	s	c
m	-4.335	10.832	0.266
s	-4.341	10.831	14.533
s (t+h+f)	14.544		
l	-4.333	10.833	
k	-4.335	10.832	

input-output table (price)

a	m	s	c
m	62.286	62.286	62.286
s	42.070	42.070	42.070
s (t+h+f)	42.070		
l	0.000	0.000	
k	169.910	169.910	

b	m	s	c
m	62.286	62.286	62.286
s	42.070	42.070	42.070
s (t+h+f)	42.070		
l	0.000	0.000	
k	169.910	169.910	

input-output table (value)

a	m	s	c	total
m	55.251	79.864	62.717	63.968
s	35.903	57.458	62.717	54.459
s (t+h+f)	62.733			57.458
l	-4.333	10.833		61.075
k	158.210	199.146		
total	63.968	57.458	61.075	

b	m	s	c	total
m	55.251	79.864	62.717	63.968
s	35.903	57.458	62.717	54.459
s (t+h+f)	62.733			57.458
l	-4.333	10.833		61.075
k	158.210	199.146		
total	63.968	57.458	61.075	

welfare

a	b
2.971	2.971

Table 9: Effects of the Increase in Labor Endowment of Countries A and B (Extended Model)

number of firms (n)

a	b
-10.120	-10.120

number of firms (an)

	a	b
a	-5.933	-20.900
b	-20.900	-5.933

trade flow (pdot)

	a	b
a	-34.109	-34.104
b	-34.104	-34.109

trade flow (qdot)

	a	b
a	20.542	20.542
b	20.542	20.542

trade flow (phidot)

	a	b
a	-1.062	3.050
b	3.050	-1.062

trade flow (total quantity)

	a	b	exp
a	14.606	-7.474	9.319
b	-7.474	14.606	9.319
imp	9.319	9.319	

trade flow (total value)

	a	b	exp
a	-24.485	-39.028	-28.283
b	-39.028	-24.485	-28.283
imp	-28.283	-28.283	

total fixed cost (h)

a	b
-10.120	-10.120

total fixed cost (f)

	a	b
a	-5.933	-20.900
b	-20.900	-5.933

total transportation cost

	a	b
a		-7.474
b	-7.474	

input-output table (quantity)

a	m	s	c
m	9.320	-5.845	5.660
s	9.366	-5.829	-9.159
s (t+h+f)	-9.756		
l	9.333	-5.833	
k	9.335	-5.832	

b	m	s	c
m	9.320	-5.845	5.660
s	9.366	-5.829	-9.159
s (t+h+f)	-9.756		
l	9.333	-5.833	
k	9.335	-5.832	

input-output table (price)

a	m	s	c
m	-31.710	-31.710	-31.710
s	-20.570	-20.570	-20.570
s (t+h+f)	-20.570		
l	0.000	0.000	
k	-80.530	-80.530	

b	m	s	c
m	-31.710	-31.710	-31.710
s	-20.570	-20.570	-20.570
s (t+h+f)	-20.570		
l	0.000	0.000	
k	-80.530	-80.530	

input-output table (value)

a	m	s	c	total
m	-25.345	-35.701	-27.845	-28.602
s	-13.131	-25.200	-27.845	-23.427
s (t+h+f)	-28.319			-25.200
l	9.333	-5.833		-27.090
k	-78.712	-81.665		
total	-28.602	-25.200	-27.090	

b	m	s	c	total
m	-25.345	-35.701	-27.845	-28.602
s	-13.131	-25.200	-27.845	-23.427
s (t+h+f)	-28.319			-25.200
l	9.333	-5.833		-27.090
k	-78.712	-81.665		
total	-28.602	-25.200	-27.090	

welfare

a	b
2.524	2.524

Table 10: Effects of the Increase in Capital Endowment of Countries A and B (Extended Model)

number of firms

a	b
1	1

number of active firms

	a	b
a	0.6	0.4
b	0.4	0.6

trade flow (cif)

	a	b
a	185	35
b	35	185

trade flow (fob)

	a	b
a	185	30
b	30	185

Pareto shape parameter

4.25

CES elasticity

4

Table 11: Given Information for Calibrating the Basic Model

number of firms

a	b
1	1

number of active firms

	a	b
a	0.6	0.4
b	0.4	0.6

trade flow (cif)

	a	b
a	90	35
b	35	90

trade flow (fob)

	a	b
a	90	30
b	30	90

input-output table (value)

a	m	s	c
m	25	20	80
s	45	30	20
l	25	40	
k	25	10	

b	m	s	c
m	25	20	80
s	45	30	20
l	25	40	
k	25	10	

Pareto shape parameter

4.25

CES elasticity

4

Table 12: Given Information for Calibrating the Extended Model