A model of firm exit under inefficiency and uncertainty*

Simone Pieralli†, Silke Hüttel‡, Martin Odening§

Abstract

This paper examines the impact of technical efficiency on the optimal exit timing of firms in a stochastic dynamic framework. While an extensive literature deals with exit behavior under output price uncertainty and efficiency of firms separately, the interplay of these two aspects is rarely studied. Starting from a standard real options approach we incorporate technical efficiency via a production function and derive an optimal price trigger at which firms irreversibly exit a market. The profit function in the optimization problem inherits properties from the production function via a dual Legendre transform. We consider two types of production technologies which differ in the way efficiency interacts with the primal technology. Assuming separability of efficiency on the primal technology we show that higher efficiency and higher returns to scale make the firm more reluctant to exit irreversibly the market. We then extend this model to a case where efficiency is not separable from other inputs and we derive explicit results for a Cobb-Douglas production function. Somehow surprisingly we find that higher efficiency does not always increase the reluctance to exit if firms exhibit low returns to scale.

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†Corresponding author: simone.pieralli@agrar.hu-berlin.de. Research Associate at the Department of Agricultural Economics, Humboldt-Universität Berlin, Unter den Linden 6, D-10115 Berlin, Germany and Ph.D. Candidate at University of Maryland-College Park, USA.

‡Junior Professor of Department of Agricultural Economics, Humboldt-Universität Berlin

§Professor of Department of Agricultural Economics, Humboldt-Universität Berlin

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1 Introduction

Decisions to quit from production or even to exit a market are among the most far-reaching managerial decisions. Firms' exit decisions are dynamic by nature and have to be made in a uncertain economic environment. Moreover, it is costly to reverse them. That is, liquidation values for specific production assets in place are considerably lower than investment outlays required to start production. In view of this irreversibility firms consider exit decisions carefully and usually firms do not re-enter a market once production has been shut down. The analysis of exit decisions is challenging but important from a sectoral perspective. Structural change in an industry is, in essence, the outcome of aggregated entry and exit decisions of firms. Thus understanding exit behavior of firms is essential to predict the velocity of structural change and adjustment processes, which in turn affect the competition and the competitiveness of an industry.

Given the relevance of exit decisions it is not surprising that many attempts have been made to explain why and when firms quit and which economic factors influence this decision and particularly its timing (see, for example, Musshoff et al. 2012, and the literature cited therein). Two strands of literature are of particular interest for understanding firms' exit decisions. First, given the costly reversibility of exit decisions made under uncertain future expectations, the real options approach provides a convenient model framework to analyze firms' exit decisions. Second, it is undisputed that the firms' relative performance is a crucial determinant of long-run survival. Thus, efficiency analysis is a base for analyzing firms' decisions to continue or to quit production. These two fields have received extensive attention, but only separately. A joint treatment of these two aspects is the topic of this paper.

Real options theory is used to analyze irreversible decisions under uncertainty by exploiting the analogy between financial options and (dis)investments (Dixit and Pindyck 1994). It asserts that deferring an exit decision may increase a firm's profit even if the expected present value of cash flows falls below the liquidation value. Quitting from production deletes the option to benefit from increasing returns in the future and this loss of flexibility must be covered by the liquidation value of the firm. Thus, compared with traditional stopping rules lower cash flows are tolerated before it becomes optimal to exit (e.g. Dixit 1989, and Odening, Musshoff, and Balmann 2005). This finding has been used to rationalize sluggish disinvestment and exit behavior of firms. Real options theory allows derivation of hypotheses on the impact of economic variables such as sunk cost, volatility, and flexibility on the timing of firm exit. For example O'Brien and Folta (2009) consider the impact of uncertainty and sunk costs on exit behavior, confirming that uncertainty dissuades from exiting but only when sunk costs are sizable.

Efficiency of firms has not been considered as a determinant of real option values and exit triggers so far. Heterogeneity of exit triggers among firms is the result of different exposure to price risk or differences in the expected profit flow. Clearly, inefficient firms will face lower expected stochastic returns compared to efficient competitors and this difference will be translated into the optimal exit time. In other words, technical inefficiency is typically implied by the specification of the stochastic process of the firm's future profits. Thus technical efficiency does not enter the real option model as an explicit parameter; it is rather merged with other model parameters.

The second relevant strand of literature, that is complementary to the real options perspective, emphasizes the impact of efficiency on firm exits. Goddard et al. (1993) argue that more efficient firms show superior performance and are more viable in a competitive environment. They earn higher profits and increase their market shares at the expense of less efficient firms thereby increasing industry concentration. This view is often labeled as "efficient structure hypothesis" in the literature and can be traced back to Demsetz (1973). An implication of this hypothesis is that efficient and inefficient firms cannot coexist in the long run. The hypothesis that technical inefficiency increases the probability of firm exit has been empirically tested. Among others, Tsionas and Papadogonas (2006), Kumbhakar, Tsionas, and Sipiläinen (2009), and Wheelock and Wilson (2000) find a positive correlation between inefficiency and exit. It remains open whether inefficiency causes firms to leave the market. At the same time it can be observed that inefficient firms persist in the market, at least in the short run (Emvalomatis, Stefanou, and Lansink 2011). This finding suggests to distinguish between short run and long run efficiency. This distinction is emphasized by the concept of dynamic efficiency (e.g. Silva and Stefanou 2007, and Rungsuriyawiboon and Stefanou 2007). Dynamic efficiency measurement acknowledges difference in the adjustment of variable and quasi-fixed inputs in production. Changes in the level of quasi-fixed factors entail additional costs attached to adjusting the capital stock in the long run, for instance through temporarily foregone outputs. Such costs may prevent firms from immediately realizing otherwise optimal investments or disinvestments[1]. Dynamic efficiency models usually assume static expectations of revenues and costs. Thus they fail to take into account the value of waiting, which in turn underlies the real options models[2].

Against this background, the purpose of this paper is to bridge the two aforementioned strands of literature. In particular, exit under output price uncertainty is considered while allowing for technical inefficiency. We start from a standard real options model and use a generic production function with an efficiency term. We derive the properties inherited to the instantaneous profit function from the original production function by using a dual Legendre transformation. This allows deriving flexibly the substitution properties of the production function among multiple inputs in a general setting. We do not impose a priori specific functional forms of the production function which possibly cause inflexibilities among production inputs. Depending on how efficiency is assumed to interact with the technology (in a separable or non-separable manner), uncertainty impacts firms' reluctance to exit the market differently. Efficiency increases the reluctance to exit the market in the separable case. In contrast, in the non-separable case, the efficiency parameter interacts directly with the returns to scale parameter and it results in a non-monotonic impact on the optimal exit trigger prices. Very inefficient firms are shown more reluctant to exit the market than higher efficient ones. The paper closest to ours is Lambarraa, Stefanou, and Gil (2009) who study the

[1]The notion of adjustment costs is shared with the real option models where these costs are modeled explicitly.

[2]An exception is Hüttel, Narayana, and Odening (2011) who incorporate cost uncertainty into the dynamic efficiency model of Rungsuriyawiboon and Stefanou (2007).

inefficiency of Spanish olive farmers using a real options approach. They consider the effect of inefficiency with a Cobb-Douglas technology and its persistence on investment decisions. Nonetheless, they do not show directly in the technology the impact of inefficiency on farmers' exit decision under uncertainty. In contrast, our model allows to rationalize the co-existence of differently efficient firms in the market through the interaction of uncertain output price and real options effects.

In the next section we present the model, without making functional assumptions on the way inputs interact to produce output. Structure on the production technology, with specific functional forms examples, helps exemplifying in a simple framework the theory exposed. We then consider exemplarily a Cobb-Douglas production function and derive explicit exit conditions for a separable efficiency term. Numerical simulations are carried out to illustrate our theoretical results. The third section extends the model to a non-separable case. The last section concludes.

2 A model for firm exit decisions under uncertainty and inefficiency

2.1 A general framework

Our model departs from the standard real options approach suggested by Dixit (1989). In contrast to Dixit (1989) we do not consider entry and exit simultaneously but focus on the optimal timing of the exit decision. That means we assume an existing firm with potential infinite life, that is already active in a market. It buys inputs $\mathbf{x} \in \mathbb{R}^P_+$ at non-stochastic cost $\mathbf{w} \in \mathbb{R}^P_{++}$ to produce output y that can be sold at stochastic price $p \in \mathbb{R}_{++}$. We are interested in a critical threshold for the stochastic price that triggers the firm's market exit. Output price is assumed to follow a Geometric Brownian motion process:

$$\frac{dp}{p} = \alpha dt + \sigma dz \tag{1}$$

where α is the drift rate of the stochastic process, σ is its volatility, and dz is the increment of a Wiener process. At each instant, the firm faces the choice of whether to continue production or to leave the market. In the case of continuing, the firm earns a profit flow $\pi(p, \mathbf{w})$ where $\pi : \mathbb{R}_{+}^{1+P} \rightarrow \mathbb{R}_{+}$. Exit is instead irreversible and firms get a positive liquidation value L upon exit. The decision problem of the firm constitutes an optimal stopping problem that can be solved by stochatic dynamic programming techniques. The solution procedure involves two steps: first we have to determine the value of the active firm $V(p, t)$ as a function of the stochastic profit flow and thus the price. Actually, $V(p, t)$ contains the whole sequence of operating options. This value implies an optimally adjusted level of variable inputs. Second, we have to calculate the option to exit, i.e. to quit from production in exchange for the liquidation value L. The optimal exit triggers are derived as part of the solution.

The value of the firm at a certain time period t is equal to the sum of the operating profit over a short interval time $(t, t + dt)$ and the continuation value after time $t + dt$:

$$V(p, t) = \pi(p, \mathbf{w})dt + E(V(p + dp)e^{-\rho dt}) \tag{2}$$

where ρ is an exogenously specified discount rate.

Applying Ito's lemma yields the following second order differential equation between the value of the firm and the profit flow[3]:

$$V'(p)\alpha p + \frac{1}{2}V''(p)\sigma^2 p^2 - \rho V(p) + \pi(p, \mathbf{w}) = 0 \tag{3}$$

In order to link efficiency and exit decision making, there is need to model the production technology explicitly. We first derive the general form of the stochastic profit flow. Unless for simple functional forms, an explicit solution for the profit function is difficult to attain. To circumvent this problem we use the dual Legendre transform and derive the structural properties of the profit function implicitly (Fuss and McFadden 1978, Jorgenson and Lau

[3]Since we consider an infinite time horizon problem, time is not a decision variable and will be omitted hereafter.

1974). Using these findings, we present the impact of efficiency on optimal exit behavior if efficiency is modeled in a separable manner from the production function (subsection 2.2). We provide an illustration of the results for the Cobb-Douglas case (subsection 2.3). We then extend the model to incorporate a non-separable efficiency term in the Cobb-Douglas case (section 3).

2.2 Separable efficiency

We assume the existence of a firm with production function $f : y = f(\mathbf{x})$, with the same characteristics as in Fuss and McFadden (1978)[4], which transforms a vector of inputs $\mathbf{x}$ into a scalar output y, where $f : \mathbb{R}_+^P \rightarrow \mathbb{R}_+$. At this stage no further assumptions are imposed on the precise functional form of f, which adds flexibility to our analysis.

The short run profit of a firm is defined as revenue less costs:

$$\pi(p, \mathbf{w}) = sup_{\mathbf{x}} \left(pf(\mathbf{x}) - (\mathbf{w}'\mathbf{x}) \mid \mathbf{x} \in \mathbb{R}_+^P \right) \tag{4}$$

Here, we follow the convention in Fuss and McFadden (1978) and obtain a normalized profit function $\pi^* : \mathbb{R}_+^P \rightarrow \mathbb{R}_+$ as:

$$\pi^*(\mathbf{w}^*) = sup_{\mathbf{x}} \Big(f(\mathbf{x}) - (\mathbf{w}^{*\prime}\mathbf{x}) \mid \mathbf{x} \in \mathbb{R}_+^P \Big) \tag{5}$$

where input prices are normalized by the output price p: $\mathbf{w}^* = \mathbf{w}/p$. This normalized profit function results from the dual Legendre transformation (see Fuss and McFadden (1978) for more details). In a further step, the function in (5) is used to derive a profit function to be implicitly included in the non-homogeneous part of equation (3) in the basic set-up of the model.

Efficiency is introduced in the primal production function through a separable short-term

[4] In particular, f is a finite, non-negative, real-valued, continuous, smooth, twice-continuously differentiable, monotonic, concave and bounded function; inaction is possible.

production efficiency parameter; as a result, the dual normalized profit function is then also separable in efficiency. This is achieved through multiplying the production function $f(\mathbf{x})$ by a scalar efficiency parameter $a \in (0, 1]$, where maximum efficiency is when $a = 1$. The resulting normalized profit function encompasses efficiency:

$$\pi_a^*(\mathbf{w}^*, a) = sup_{\mathbf{x}} \left(af(\mathbf{x}) - (\mathbf{w}^{*'}\mathbf{x}) \mid \mathbf{x} \in \mathbb{R}_+^P \right) \tag{6}$$

where $\pi_a^* : \mathbb{R}_+^{1+P} \to \mathbb{R}_+$ and where $\mathbf{x}$ depend implicitly on output and input prices, and efficiency level.

One class of production functions very used in empirical estimates of production technologies is the class of homogeneous production functions. While other sets of assumptions are possible (Fuss and McFadden 1978), we exemplify our approach with such a set of functions. We assume the function f to be homogeneous of degree k with respect to the inputs with $k < 1$. Accordingly, the normalized profit function will be homogeneous of degree $-k/(1-k)$ in the normalized input prices $\mathbf{w}^*$. In order to derive the effect of efficiency, we collect out the efficiency terms and define a non-decreasing function $h : \mathbb{R}_+ \to \mathbb{R}_+$. The normalized profit function can then be expressed as follows:

$$\pi_a^*(\mathbf{w}^*, a) = h(a) g^{-k/(1-k)}(\mathbf{w}^*) \tag{7}$$

where $g^{-k/(1-k)} : \mathbb{R}_+^P \to \mathbb{R}_+$ is a homogeneous function g of degree $-k/(1-k)$. The non-normalized profit function including efficiency (π_a), by the assumptions on the production function, is separable between output and input prices:

$$\pi_a(p, \mathbf{w}, a) = p\pi_a^*(\mathbf{w}^*, a) \tag{8}$$

In order to express this function in a similar manner as in (7), that is in a multiplicatively separable form, we further define two separate functions $h_1(p) : \mathbb{R}_+ \to \mathbb{R}_+$, and $g^*(\mathbf{w}) : \mathbb{R}_+^P \to \mathbb{R}_+$. The non-normalized profit function expressed in multiplicatively separable terms of $h(a), h_1(p)$ and $g^*(\mathbf{w})$ is given by:

$$\pi_a(p, \mathbf{w}, a) = g^*(\mathbf{w})h(a)h_1(p) \tag{9}$$

Since the non-normalized profit function is obtained from the normalized profit function by multiplying it by p, the homogeneity properties of g^* with respect to $\mathbf{w}$ (non-normalized input prices) are the same as the ones of g with respect to $\mathbf{w}^*$. That is, both are homogeneous of degree $-k/(1-k)$. The following Lemma[5] derives then the degree of homogeneity of the profit function in output price.

Lemma A profit function of the type $\pi_a(p, \mathbf{w}, a)$ homogeneous of degree $-k/(1-k)$ in input prices $\mathbf{w}$ will be homogeneous of degree $1/(1-k)$ in output price p.

Given the Lemma, $h_1(p)$ is a homogeneous function of degree $1/(1-k)$. In a further step we summarize $g^*(\mathbf{w})$ and $h(a)$ in a multiplicative factor $X = g^*(\mathbf{w})h(a)$. The profit function (9) is rewritten in terms of X and reduces to a multiplication of two terms:

$$\pi_a(h_1(p), \mathbf{w}, a) = Xh_1(p) \tag{10}$$

In order to be more general we rewrite $h_1(p)$ as a function of a positive constant λ such that $h_1(p) = h_1(\lambda p) \mid_{\lambda=1}$. The profit function that captures the efficiency of the firm is expressed in a compact multiplicative form:

$$\pi_a(h_1(p), \mathbf{w}, a) = Xh_1(\lambda p) \mid_{\lambda=1} \tag{11}$$

In the next step we incorporate the profit function (11) that accounts for a separable efficiency term into the optimality conditions of an active firm (equation 3).

[5] The proof, similar to the one in Lau's chapter in Fuss and McFadden (1978) and Kumbhakar (2001), can be found in the Appendix.

The value of an *active* firm in terms of the enhanced profit function is

$$V'(p)\alpha p + \frac{1}{2}V''(p)\sigma^2 p^2 - \rho V(p) + \pi_a(h_1(\lambda p), \mathbf{w}, a) = 0 \tag{12}$$

Following Dixit (1989) the solution of the non-homogeneous second order differential equation (12) is given by:

$$V(p) = B_1 p^{\beta_1} + B_2 p^{\beta_2} + V_p(p) \text{ if } p > p_* \tag{13}$$

where p_* is the price level that triggers an irreversible exit from the market. That is, if the market price p falls below the trigger price the firm will optimally leave the market. The decision maker waits until the net worth of the firm is lower than the liquidation value (L) to get out of the market. This will be made explicit in the following steps.

Trying a solution of the type $X_1 h_1(\lambda p)$ we get:

$$V_p(p) = \frac{X\lambda^{1/(1-k)} h_1(p)}{\delta'} \tag{14}$$

where $\delta' = \rho - \frac{\alpha}{\lambda(1-k)} - \frac{k\sigma^2}{2\lambda^2(1-k)^2}$ is a risk adjusted discount rate. β_1 and β_2 are, respectively, the positive and negative roots of the quadratic equation associated with the second order differential equation as shown in (12). Further assuming that h_1 is homogeneous of degree γ, the fundamental quadratic equation is in this case equivalent to:

$$Q(\gamma) = \frac{\alpha\gamma}{\lambda} + \frac{\gamma(\gamma-1)\sigma^2}{2\lambda^2} - \rho = 0 \tag{15}$$

The positive root of the fundamental quadratic is

$$\beta_1 = \frac{1}{2} - \frac{\alpha\lambda}{\sigma^2} + \left\{ \left[\frac{\alpha\lambda}{\sigma^2} - \frac{1}{2} \right]^2 + 2\frac{\rho\lambda^2}{\sigma^2} \right\}^{1/2} \tag{16}$$

The negative root is

$$\beta_2 = \frac{1}{2} - \frac{\alpha\lambda}{\sigma^2} - \left\{ \left[\frac{\alpha\lambda}{\sigma^2} - \frac{1}{2} \right]^2 + 2\frac{\rho\lambda^2}{\sigma^2} \right\}^{1/2} \tag{17}$$

Ruling out bubble solutions $V(p)$ becomes:

$$V(p) = \frac{X\lambda^{1/(1-k)}h_1(p)}{\delta'} \tag{18}$$

where δ' is the negative of the fundamental quadratic (15), evaluated at $1/(1-k)$. Because we need to assume that $\delta' > 0$, the negative of the fundamental quadratic evaluated at $1/(1-k)$ needs to be positive. That is, $1/(1-k)$ is between the two roots of the fundamental quadratic. Since we focus on the exit strategy, we require $1/(1-k) > \beta_2$ which amounts to a restriction on the degree of homogeneity of the production function: $(\beta_2 - 1)/\beta_2 > k$.

The value of the option (F) for an active firm is equal to the sum of the values of the exit option and the value of the active firm $V(p)$ in (18):

$$F(p) = A_1 p^{\beta_1} + A_2 p^{\beta_2} + \frac{X\lambda^{1/(1-k)}h_1(p)}{\delta'} \text{if } p > p_* \tag{19}$$

The value of the option to exit is given by the first two terms in the right-hand side of the above equation $A_1 p^{\beta_1} + A_2 p^{\beta_2}$. The value of the exit option will be zero if prices are high enough since there is no incentive for the firm to leave the market. The constant A_1, associated with the positive root β_1, should be zero implying

$$F(p) = A_2 p^{\beta_2} + \frac{X\lambda^{1/(1-k)}h_1(p)}{\delta'} \text{ if } p > p_* \tag{20}$$

To solve for the trigger price level p_* and the constant A_2 we invoke the value matching condition:

$$F(p_*) = L \tag{21}$$

and the smooth pasting condition:

$$F'(p_*) = 0 \tag{22}$$

This yields:

$$A_2 = -\frac{\lambda^{\frac{k}{1-k}} X \partial h_1(p_*)/\partial p_* p_*}{\delta' \beta_2 p_*^{\beta_2}} \tag{23}$$

Inserting (23) into (20), then into (21), and rearranging, gives an implicit definition for the trigger price:

$$g^*(\mathbf{w})h(a)h_1(\lambda p_*) = \delta' L \left(\frac{\beta_2 \lambda}{\beta_2 \lambda - \frac{1}{1-k}} \right) \tag{24}$$

The optimality condition (24) states that the instantaneous profit on the left-hand side must equal the appropriately discounted liquidation value ($\delta' L$), times a multiple $\left(\frac{\beta_2 \lambda}{\beta_2 \lambda - \frac{1}{1-k}} \right)$, which is lower than unity. Equation (24) shows that the exit price trigger decreases in a. That is, more efficient firms have a comparatively lower exit trigger compared to less efficient ones. Thus, the reluctance to irreversibly leave the market increases for more efficient firms.

The degree of homogeneity of the production function (k) has an impact on the level of exit trigger prices $h_1(\lambda p_*)$. In particular, an increase in k in (24) decreases both the multiplier of liquidation value L, and δ', implying a higher reluctance of firms who have a higher degree of homogeneity in inputs. However, the effect of k on the level of exit trigger prices can be different depending on the specification of the production function.

2.3 Simulations

To quantify the aforementioned effects we construct an illustrative example. A simple type of production technology that responds to the properties considered here is the Cobb-Douglas. For exemplification purposes we use a production function[6] with one input and one output:

$$f^{CD}(x) = x^{\theta} \tag{25}$$

where $f^{CD} : \mathbb{R}_+ \to \mathbb{R}_+$. Observed output y is lower than or equal to maximal producible output:

$$y = f^{CD} h(e^{-\phi}) = f^{CD} e^{-\phi} = x^{\theta} e^{-\phi} \tag{26}$$

[6] In particular, we omit the dependence on returns to scale parameter θ because it is a given parameter that shapes the production function.

where $\phi \in [0, \infty)$ is an inefficiency parameter, so that $a = e^{-\phi}$ can be considered an efficiency term which is separable from input x and output y.[7] The Cobb-Douglas technology results in a separable profit function. Since efficiency enters in a multiplicatively separable way in the production function, the efficiency term is multiplicatively separable also in the dual profit function:

$$\pi_\phi(p, w, \phi) = e^{-\frac{\phi}{1-\theta}}(1-\theta)\left(\frac{\theta}{w}\right)^{\frac{\theta}{1-\theta}} p^{\frac{1}{1-\theta}} \tag{27}$$

where $\pi_\phi : \mathbb{R}^3_+ \to \mathbb{R}_+$. Second order conditions, in order for a derivative method to identify a maximum for profit, impose that $\theta \leq 1$, which in turn implies non-increasing returns to scale on the production function[8].

Noticing that, for the Cobb-Douglas case, $\lambda = 1$, degree of homogeneity k is equal to θ, $\gamma = \frac{1}{1-\theta}$, $h_1(\lambda p) = p^\gamma$, and $g^*(\mathbf{w})h(e^{-\phi}) = e^{-\frac{\phi}{1-\theta}}(1-\theta)\left(\frac{\theta}{w}\right)^{\frac{\theta}{1-\theta}}$, we obtain the following price trigger as a special case of (24):

$$e^{-\frac{\phi}{1-\theta}}(1-\theta)\left(\frac{\theta}{w}\right)^{\frac{\theta}{1-\theta}} p_*^{\frac{1}{1-\theta}} = \delta' \mid_{\lambda=1} L\left(\frac{\beta_2}{\beta_2 - \frac{1}{1-\theta}}\right) \tag{28}$$

Apparently, the efficiency term affects only the net worth, i.e., efficiency acts simply as a shifter of trigger prices under the assumption of multiplicative separability. For $\phi = 0$, equation (28) reduces to the standard real options exit price trigger with variable output (Dixit and Pindyck 1994). More inefficient firms are less reluctant to exit the market. In adding more flexibility to their standard model, Dixit (1989) recognized that exit would be at lower levels than usually predicted under investment under uncertainty with the introduction of variable scale. The inclusion of efficiency tends to counteract this by shifting up exit trigger price levels for more inefficient firms.

Equation (28) can be solved for p_*. The outcome is illustrated in figures 1 to 3 for different

[7]In this context, an extension to multiple inputs is possible under the assumption that the efficiency parameter enters as a shifter of the whole production function, contracting efficient output across inputs equally.

[8]For a well-defined solution we require $\theta < 1$.

parameter constellations, which will be introduced below. The aim of these simulations is to illustrate, for a population of simulated firms, the interaction between efficiency, uncertainty (volatility of output price), and exit trigger prices.

In order to simulate the triggers, we randomly draw 15000 times and discard the first 5000 a pseudo-random univariate normal deviate. This is used to simulate the returns to scale parameter of a population of production units[9]. Their returns to scale parameters are normally distributed with mean 0.5 and standard deviation 0.07, so that most of simulated units (99.74%) have a returns to scale parameter between 0.29 and 0.71.

In each figure we have in the upper plot $\phi = 0.7$, which corresponds to a lower level of efficiency of approximately 50%. In the middle plot we have $\phi = 0.4$, which represents a medium level of efficiency of almost 67%. Finally in the lower plot we have $\phi = 0.1$, which corresponds to a higher level of efficiency, i.e. 90%. Drift rate of uncertain output price is null in these simulations. An overview of the other simulation parameters used is given in table 1.

Table 1: Simulation parameters for separable efficiency Cobb-Douglas case

	Parameters
Figure 1	L=1, w=0.01, $\sigma=\{0.02, 0.09\}$
Figure 2	L=$\{1, 4\}$, w=0.01, $\sigma= 0.02$
Figure 3	L=1, w=$\{0.01, 0.015\}$, $\sigma= 0.02$

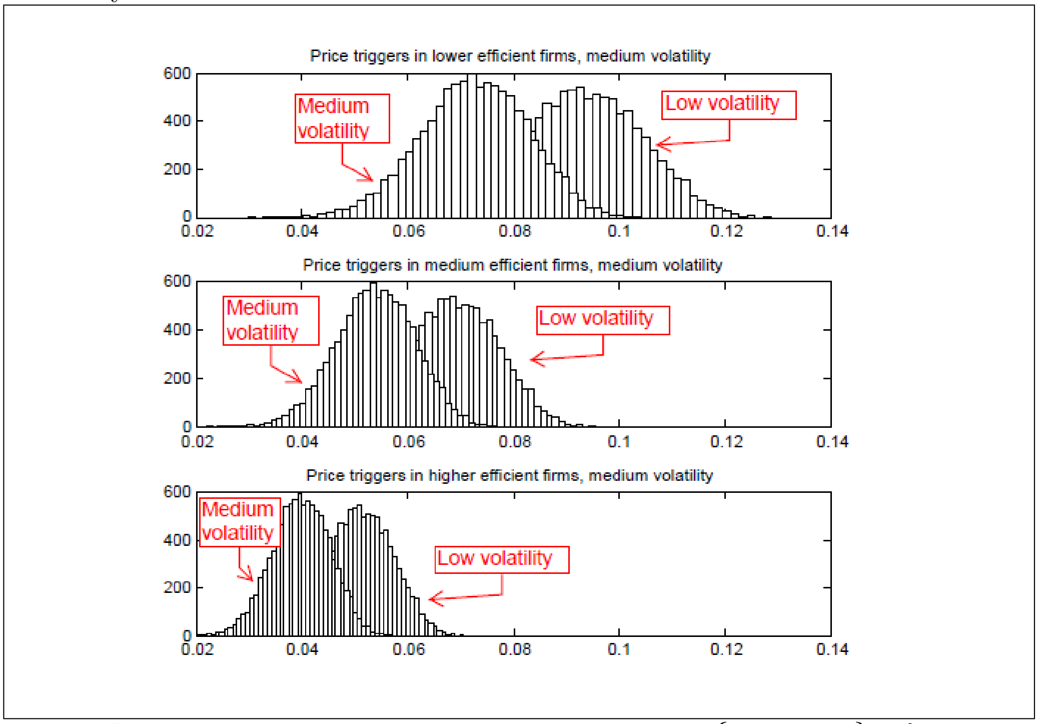
In figure 1 we consider a production unit that has liquidation value normalized to 1 and input cost fixed at 0.01[10]. In all plots in this figure we vary the level of volatility from a low level of $\sigma = 0.02$ to a medium level of $\sigma = 0.09$. The resulting distributions from varying the level of volatility, for each level of efficiency, are plotted as two overlapping histograms: the histograms on the left, for each plot, correspond to higher volatility of prices, the histograms

[9]Note that we do not want to derive in this instance an industry-wide equilibrium but just want to see how single heterogeneous firms react to price risk.

[10]An increasing marginal product, when increasing returns to scale θ, for input level greater than 1, calls for decreasing trigger prices if input cost w is fixed. If instead the marginal product decreases when increasing the returns to scale parameter θ, then trigger prices will increase. This can happen if input level is lower than 1. In these simulations we only use levels of input cost that ensures that input levels in correspondence of the trigger prices are greater than 1.

on the right correspond to lower volatility of prices.

Figure 1: Exit trigger prices for separable efficiency in a Cobb-Douglas production function with simulated returns to scale for different levels of volatility of output price and different efficiency levels



Note: Parameters: $L = 1$, $w = 0.01$, $\sigma = \{0.09, 0.02\}$, $\theta \sim N(0.5, 0.0049)$. Volatility $\sigma = 0.09$ (left histogram) and $\sigma = 0.02$ (right histogram). Efficiency: $e^{-\phi} = 0.497$ (upper panel), $e^{-\phi} = 0.67$ (middle panel), and $e^{-\phi} = 0.9$ (lower panel). Sample size: 10000. [Note one observation had to be excluded to avoid negative δ'].

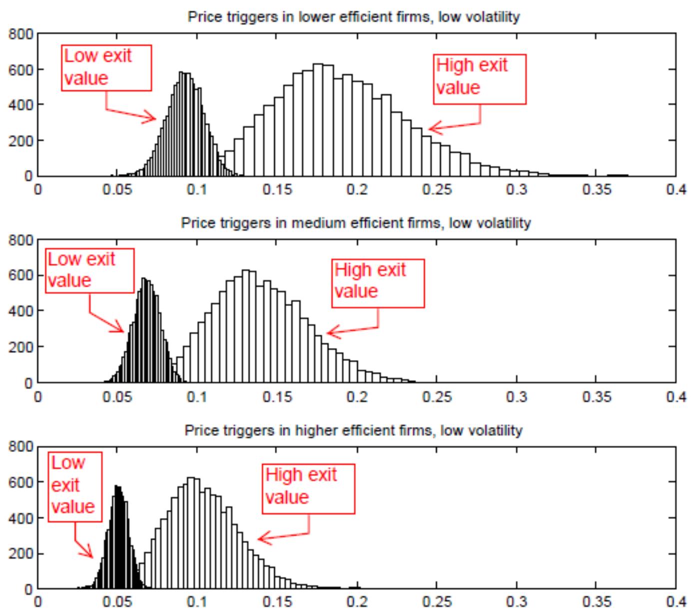
For a given efficiency level firms are more reluctant to exit the higher the volatility is, which is a standard result of real options theory. It is interesting to note, however, that the impact of efficiency depends on the level of price volatility. The higher the price risk, the lower is the impact of efficiency on the optimal price trigger. Looking at the left histograms, we see a lower shift to the left when increasing efficiency. That is, in a more volatile market one can expect to find a higher heterogeneity of firms with respect to their efficiency, at least in the short run[11]. The graph further shows that different firms can coexist in the market at a given exit trigger price for different efficiency levels. If only firms more efficient than a

[11] Note this conclusion abstracts from liquidity aspects that may force firms to quit production.

certain threshold were to be in the market for a given output price, then distributions of exit trigger prices, in correspondence of different levels of efficiency, for the same volatility level, shall be non-overlapping.

In figure 2 we consider what happens when we vary the liquidation value of the production unit from 1 to 4 while keeping input cost fixed at 0.01. To isolate the effect of varying liquidation value we keep the level of volatility of output price at $\sigma = 0.02$. At each level of efficiency, we obtain two distributions of trigger prices for each level of liquidation value: the histograms on the left, for each plot, correspond to lower liquidation value, the histograms on the right correspond to higher liquidation value. Higher liquidation values reduce the reluctance to leave the market.

Figure 2: Exit trigger prices for separable efficiency in a Cobb-Douglas production function with simulated returns to scale for different levels of liquidation value and different efficiency levels

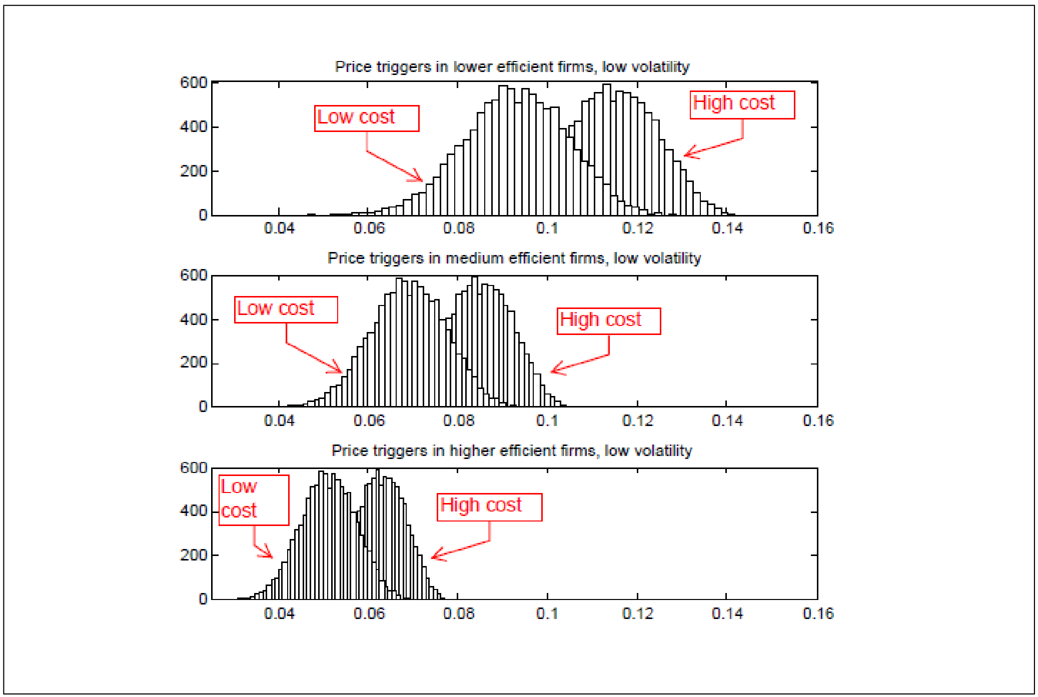


Note: Parameters: $L = \{1, 4\}$, $w = 0.01$, Volatility $\sigma = 0.02$, $\theta \sim N(0.5, 0.0049)$. $L = 1$ (left histogram) and $L = 4$ (right histogram). Efficiency: $e^{-\phi} = 0.497$ (upper panel), $e^{-\phi} = 0.67$ (middle panel), and $e^{-\phi} = 0.9$ (lower panel). Sample size: 10000.

In addition to shifting to the right the distribution of exit trigger prices, higher liquidation value results in more heterogeneous reactions by the firms. In particular, the reaction of less efficient firms is more heterogeneous than reactions by more efficient firms. The exit trigger prices for more efficient firms are more concentrated.

In figure 3 we consider the impact of different unit costs on the exit trigger prices. We vary unit costs from 0.01 to 0.015 while keeping liquidation value of the production unit fixed at 1. Also in this case we keep the level of volatility of output price at $\sigma = 0.02$.

Figure 3: Exit trigger prices for separable efficiency in a Cobb-Douglas production function with simulated returns to scale for different levels of unit costs and different efficiency levels



Note: Parameters: $L = 1$, $w = \{0.01, 0.015\}$, Volatility $\sigma = 0.02$, $\theta \sim N(0.5, 0.0049)$. $w = 0.01$ (left histogram) and $w = 0.04$ (right histogram). Efficiency: $e^{-\phi} = 0.497$ (upper panel), $e^{-\phi} = 0.67$ (middle panel), and $e^{-\phi} = 0.9$ (lower panel). Sample size: 10000.

For each plot, the histograms on the left correspond to lower unit cost, the histograms on the right correspond to higher unit cost. Higher unit cost reduces reluctance to exit the market irreversibly. Efficiency influences substantially the firms that should be present in the market for different levels of prices. In particular, it reduces the dispersion of the trigger prices at which firms exit. More efficient firms would tend to exit at more similar exit trigger price levels. Less efficient firms exit first, more efficient ones stay longer in the market.

Summarizing, for all scenarios considered here, we find a monotonic decrease of the exit trigger prices at varying degree, when increasing efficiency. This implies higher inertia for more efficient units.

3 Model extension: non-separable efficiency

In this section we show that the relation between efficiency and exit in the separable case cannot be generalized with the same strength if efficiency is included in a non-separable manner in the production technology. A single multiplicative efficiency term restricts efficiency to act only as a shifter with respect to all production factors (Orea and Álvarez 2006). It also implies a unitary elasticity of output with respect to the efficiency term.

A natural extension is to include a non-multiplicative efficiency term. A non-multiplicative efficiency implies efficiency cannot be separated from the inputs and output in determining the level of trigger prices. By assuming a specific functional form for the non-separable efficiency and for the production function, we show, in a very simple framework, that under decreasing returns to scale it is possible that less efficient firms are more reluctant to exit the market than more efficient ones.

3.1 Non-separable efficiency and Cobb-Douglas technology

In this subsection we use again a Cobb-Douglas technology with one input to derive the optimal exit trigger prices. However, we now assume that the production function is directly transformed by efficiency. We rely on a Box-Cox transformation (Box and Cox 1964), but other transformations would also be possible. The observed output transformed by efficiency is:

$$y = \frac{(f^{CD}(x))^{\xi} - 1}{\xi} = \frac{(x^{\theta})^{\xi} - 1}{\xi} \tag{29}$$

where $\xi \in (0,1)$ is considered in this case as an efficiency parameter. The profit function then takes the form:

$$\pi_\xi(p,w,\xi) = p^{-\frac{1}{\xi\theta-1}} w^{\frac{\xi\theta}{\xi\theta-1}} \left(\left(\frac{1}{\xi}\right)\left(\frac{1}{\theta}\right)^{\frac{\xi\theta}{\xi\theta-1}} - \left(\frac{1}{\theta}\right)^{\frac{1}{\xi\theta-1}} \right) - \frac{p}{\xi} \tag{30}$$

where $\pi_\xi : \mathbb{R}^3_+ \to \mathbb{R}_+$.

Second order conditions for a maximum impose that for a well-defined solution $\xi\theta < 1$. Different combinations of returns to scale and efficiency can satisfy this[12]. To simplify notation we rewrite (30) as:

$$\pi_\xi(p,w,\xi) = Kp^\eta - Dp \tag{31}$$

where $\eta = -\frac{1}{\theta\xi-1}$, $K = \frac{w^{1-\eta}\theta^\eta}{\eta-1}$, and $D = \frac{\eta\theta}{\eta-1}$ [13]. With this profit function the value of the active firm should satisfy the following second order differential equation, analogous to (12):

$$V'(p)\alpha p + \frac{1}{2}V''(p)\sigma^2 p^2 - \rho V(p) + Kp^\eta - Dp = 0 \tag{32}$$

Carrying out the same steps as in the previous section we attain the following expression for the optimal exit trigger price:

$$\frac{Kp_*^\eta}{\eta'} = L\frac{\beta_2}{\beta_2-\eta} + \frac{(1+\alpha-\rho)Dp_*}{\eta'}\frac{\beta_2-1}{\beta_2-\eta} \tag{33}$$

where $\eta' = \rho - \alpha\eta - 1/2\sigma^2\eta(\eta-1)$ is a risk adjusted discount rate. The left-hand side of equation (33) is the usual value of the net worth of the project appropriately discounted. The first term of the right-hand side is the fraction of the exit value L that can be recovered upon exit from the market. This is very similar to the separable case apart from the influence of efficiency term ξ on η.

[12] In particular either $\theta < 1/\xi$ or $\xi < 1/\theta$

[13] The second term $-Dp$ is mainly present because of the normalization present in the Box-Cox transformation used to provide continuity at 0. Without this, the first term most important for our results is still present.

Additionally, there is another term[14]. This is $\frac{(1+\alpha-\rho)Dp_*}{\eta'}$, which rearranged is $\frac{(1+\alpha-\rho)p_*\eta\theta}{\eta'(\eta-1)}$ and, finally, $\frac{(1+\alpha-\rho)p_*}{\xi\eta'}$. This shows how efficiency and the price level interact directly, appropriately discounted and multiplied by $\frac{\beta_2-1}{\beta_2-\eta}$. If $\xi\theta < 1$ then η is greater than 1 and $\frac{\beta_2-1}{\beta_2-\eta}$ is lower than unity. Substituting back into K and D and rearranging terms, we obtain:

$$\frac{1}{\eta'}\left(\frac{p_*^\eta w^{1-\eta}\theta^\eta}{\eta-1} - \frac{(1+\alpha-\rho)p_*\eta\theta}{\eta-1}\frac{\beta_2-1}{\beta_2-\eta}\right) - L\frac{\beta_2}{\beta_2-\eta} = 0 \tag{34}$$

Applying chain rule allows us to see how efficiency affects (34). The derivative of η with respect to ξ is positive $\frac{\theta}{(1-\xi\theta)^2}$. Accordingly, the sign of the derivative of equation (34) with respect to ξ[15] will be the same as the sign of the derivative of (34) with respect to η, given that $\eta > 1$. The latter derivative is:

$$\frac{1}{\eta'}\left(\frac{p_*^\eta w^{1-\eta}\theta^\eta}{\eta-1}(\ln p_* + \ln w + \ln\theta - \frac{1}{\eta-1}) - \frac{(1+\alpha-\rho)p_*(\beta_2-1)}{(\beta_2-\eta)(\eta-1)}(\frac{-\theta}{\eta-1} + \frac{\theta\eta}{\beta_2-\eta})\right) - L\frac{\beta_2}{(\beta_2-\eta)^2} = 0 \tag{35}$$

The sign of the terms of the derivative are all positive except $\ln p_* + \ln w + \ln\theta - \frac{1}{\eta-1}$. The latter derivative can be negative. In particular, for low values of returns to scale parameter θ, lower efficiency can be linked to higher reluctance to exit. Very low efficient firms, under decreasing returns, can be more reluctant to exit than higher efficient ones. On the other hand, for higher values of θ, reluctance increases with efficiency.

3.2 Simulations

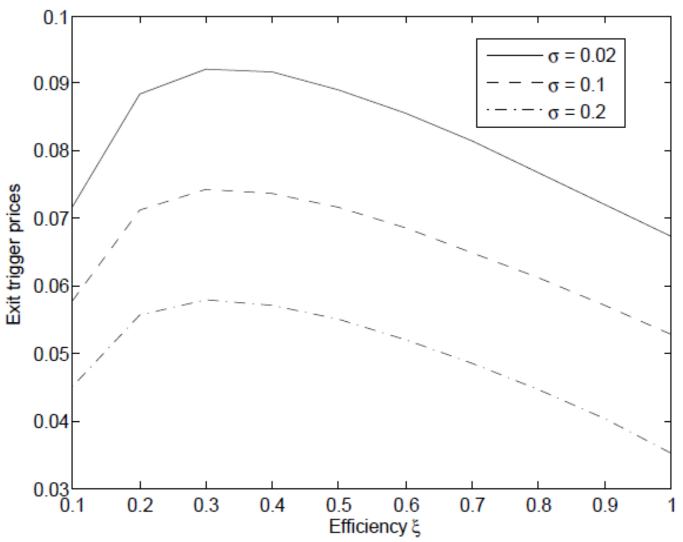
To illustrate the difference with the separable case in the previous section we propose simulations that show the relationship between uncertainty, trigger prices, and a non-separable efficiency term. We illustrate the result by means of numerical simulations using similar

[14]Again it is to be stressed that, if we were to adopt other forms of non-separable efficiency terms, this result might not be present.

[15]We note here that the same is also the sign of the derivative with respect to θ.

parameter settings as in the separable case. Figure 4 depicts the impact of efficiency under different levels of price risk (parameters: L=1, w=0.01, θ= 0.5, σ= {0.02, 0.2}). We find here a non-monotonic relationship for very low values of efficiency. This happens when very low levels of efficiency interact with low levels of returns to scale making the degree of homogeneity of the function very low.

Figure 4: Exit trigger prices for non-separable efficiency in a Cobb-Douglas production function with different levels of efficiency and for different levels of output price volatility for a medium returns to scale function



Note: Parameters: $w = 0.01$, $\theta = 0.5$, and varying volatility level of output price from $\sigma = 0.02$ to $\sigma = 0.2$.

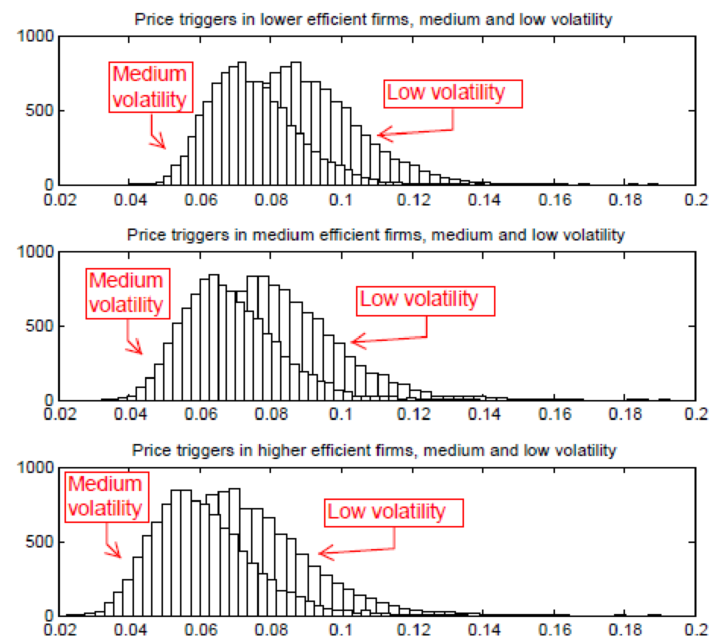
In the following we simulate a population of firms where the returns to scale parameter is simulated as in the separable case. To make the comparison easier, for different levels of efficiency, in each figure, we have, approximately as in the separable case, in the upper plot $\xi = 0.5$, in the middle plot $\xi = 0.7$, and in the lower plot $\xi = 0.9$. An overview of the simulations parameters used is given in Table 2.

Table 2: Simulation parameters for non-separable efficiency Cobb-Douglas case

	Parameters
Figure 5	L=1, w=0.01, σ= {0.02, 0.09}
Figure 6	L={1, 4}, w=0.01, σ= 0.02
Figure 7	L=1, w={0.01, 0.015}, σ= 0.02

Figure 5 shows again two series of overlapping histograms: the histograms on the left correspond, in each plot, to higher volatility of prices $\sigma = 0.09$, the histograms on the right correspond to lower volatility of prices $\sigma = 0.02$. We see, in aggregate, just a slight decrease of exit trigger prices, when increasing volatility, for given level of efficiency. In this non-separable case, even more markedly, efficiency does not seem to discriminate between firms that should be in and out of the market.

Figure 5: Exit trigger prices for non-separable efficiency in a Cobb-Douglas production function with simulated returns to scale for different levels of volatility of output price and different efficiency levels

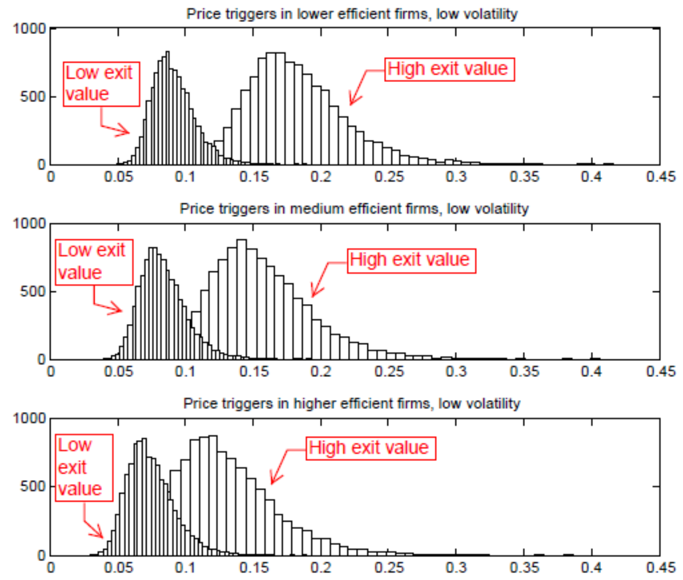


Note: Parameters: $L = 1$, $w = 0.01$, $\sigma = \{0.09, 0.02\}$, $\theta \sim N(0.5, 0.0049)$. Volatility $\sigma = 0.09$ (left histogram) and $\sigma = 0.02$ (right histogram). Efficiency: $\xi = 0.5$ (upper panel), $\xi = 0.7$ (middle panel), and $\xi = 0.9$ (lower panel). Sample size: 10000.

Greater shifts toward left appear for a higher change in value of price risk, signaling higher reluctance to exit in presence of higher price risk.

In figure 6 we consider varying the liquidation value, as in the separable case, between 1 and 4. For each level of efficiency, the histograms on the left correspond, for each plot, to lower liquidation value, the histograms on the right correspond to higher liquidation value. Higher liquidation value reduces reluctance to exit the market irreversibly. As in the separable case, exit trigger prices, at increases in liquidation values, are more diversified.

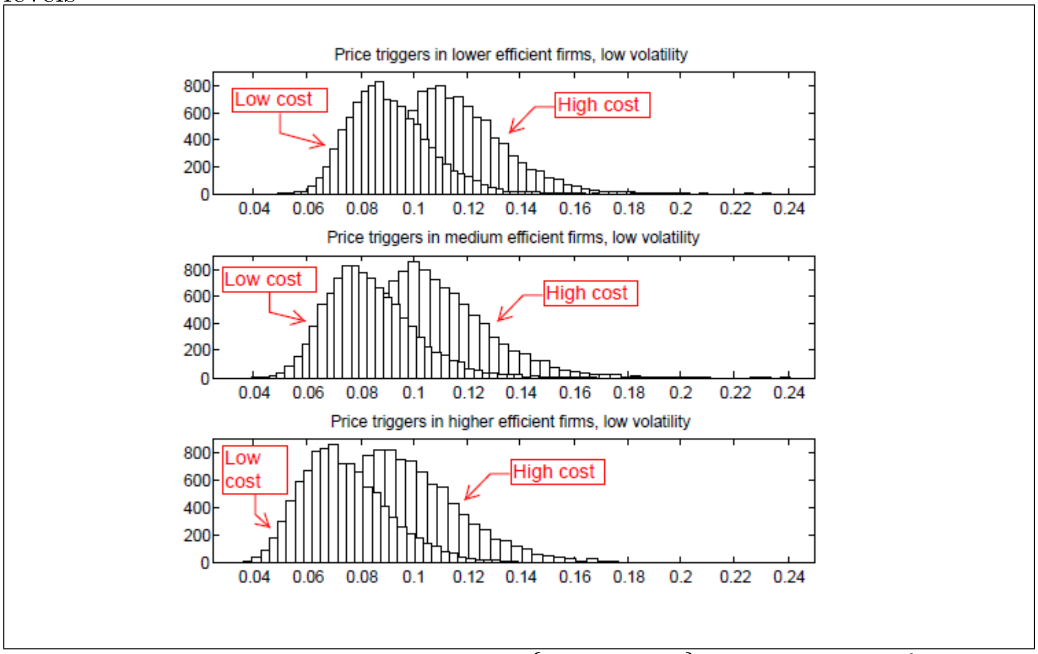
Figure 6: Exit trigger prices for non-separable efficiency in a Cobb-Douglas production function with simulated returns to scale for different levels of liquidation value and different efficiency levels



Note: Parameters: $L = \{1, 4\}$, $w = 0.01$, $\sigma = 0.02$, $\theta \sim N(0.5, 0.0049)$. Liquidation value $L = 1$ (left histogram) and $L = 4$ (right histogram). Efficiency: $\xi = 0.5$ (upper panel), $\xi = 0.7$ (middle panel), and $\xi = 0.9$ (lower panel). Sample size: 10000.

In figure 7 we consider the effect of varying unit costs from 0.01 to 0.015 on the exit trigger prices. For each plot, as before, the histograms on the left correspond to lower unit cost, the histograms on the right correspond to higher unit cost. Higher unit cost reduces reluctance to exit the market irreversibly. While we notice a clear effect on trigger prices due to higher unit costs among firms with similar efficiency levels, we only notice a slight shift towards lower trigger prices when increasing efficiency. The efficiency effect seems less important when considered in a non-separable framework.

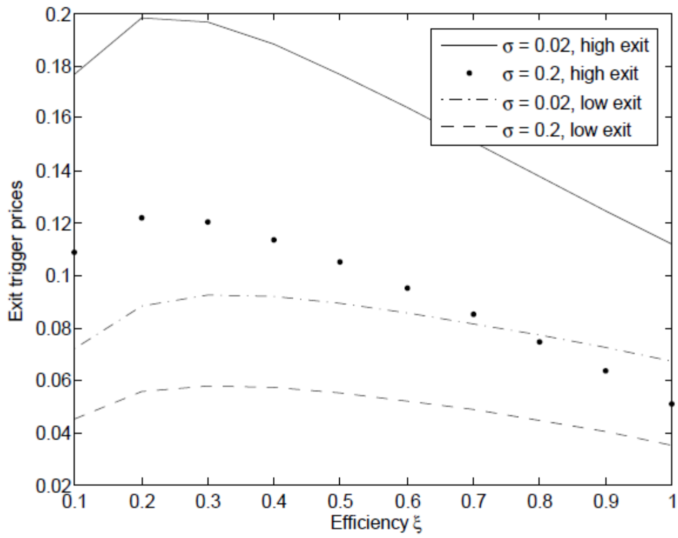
Figure 7: Exit trigger prices for non-separable efficiency in a Cobb-Douglas production function with simulated returns to scale for different levels of unit costs and different efficiency levels



Note: Parameters: $L = 1$, $w = \{0.01, 0.015\}$, $\sigma = 0.02$, $\theta \sim N(0.5, 0.0049)$. Input price $w = 0.01$ (left histogram) and $w = 0.04$ (right histogram). Efficiency: $\xi = 0.5$ (upper panel), $\xi = 0.7$ (middle panel), and $\xi = 0.9$ (lower panel). Sample size: 10000.

We illustrate more specifically in figure 8 the interaction of volatility level with the effect of increasing liquidation value. In particular, we note that, proportionally, reduction in reluctance (i.e. an increase in exit trigger prices), due to changes in liquidation value, is much more pronounced for lower volatility levels than for higher ones. Higher volatility reduces the importance of the change in liquidation value.

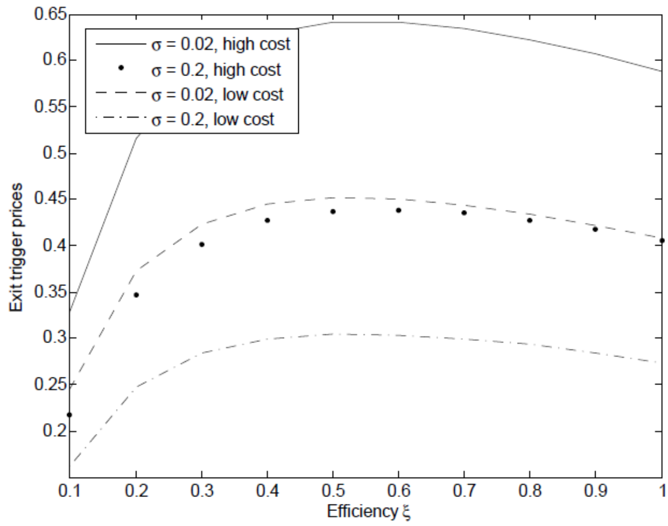
Figure 8: Exit trigger prices for non-separable efficiency in a Cobb-Douglas production function for high and low liquidation (exit) values, across different efficiency levels and for different levels of output price volatility



Note: Parameters: $w = 0.01$, $\theta = 0.5$, and liquidation value varying from $L = 1$ to $L = 4$.

Similarly, in figure 9 we show that the increase in exit trigger prices due to an increase in unit costs, when increasing costs of the same amount, is shown to be proportionally lower for higher volatility levels.

Figure 9: Exit trigger prices for non-separable efficiency in a Cobb-Douglas production function for high and low unitary costs, across different efficiency levels and for different levels of output price volatility



Note: Parameters: $L = 1$, $\theta = 0.5$, and input cost varying from $w = 0.01$ to $w = 0.015$.

4 Conclusions

In this paper we develop a model to include production efficiency in the evaluation of exit behavior of firms when subject to stochastic output price that follows a Geometric Brownian motion process. We do so by modeling directly the technological structure of a production firm and deriving implicitly, without assuming a functional form, a dual profit function through a Legendre transformation.

To exemplify this methodology, a general class of results for homogeneous production functions is derived with an efficiency term separable from the rest of the production factors, but without assuming a specific functional form of the production function. More efficient firms are more reluctant to exit the market believing in their potential to get profitable again if prices raise. Firms with higher degree of homogeneity in inputs are generally more reluctant to exit irreversibly the market, even though this result depends on the chosen class of production functions.

These results are supported by analytical results derived for the exemplifying case of a Cobb-Douglas technology under separability of efficiency. Higher efficiency increases reluctance to exit from the market. Differently efficient production units with different returns to scale can coexist at a level of output price under the conditions developed in this study. Volatility affects differently exit trigger prices, depending on the level of efficiency.

We then derive the case when the efficiency term is non-separable in a Cobb-Douglas technology. Higher efficiency does not always increase reluctance to exit when very low returns to scale are present.

We develop a set of numerical simulations for the specific Cobb-Douglas case to support the theoretical findings about the behavior of exit trigger prices under different efficiency levels and returns to scale. As expected, in the separable case lower efficiency increases the exit trigger prices making higher the incentive to exit from the market. In the non-separable case, instead, efficiency, because it interacts directly with the returns to scale parameter, has a less strong, and not necessarily monotonic, relationship with exit trigger prices. It seems that efficiency affects less the exit trigger prices under non-separability, because, in our model, efficiency and returns to scale are, in a sense, substitutable.

Finally, some classical results of theory of investment under uncertainty are common to all our numerical simulations. In particular, while volatility increases reluctance to exit the market, higher unitary costs and higher liquidation values decrease the reluctance to exit irreversibly the market.

It is important to stress that our framework proposes a general methodology. Derived results are just an example of the possible assumptions on the primal technology that could result in different dual profit functions. Nonetheless, our example is general enough to show how efficiency can be included in a structural manner into the technology, without assuming a specific production functional form, to derive firm exit behavior.

The question whether efficiency works as a shifter on the exit trigger or whether it is non-monotonically related to the exit trigger, and, if so, how it interrelates with price uncertainty needs to be answered using firm data. For instance, one could use a two-stage procedure.

In a first stage the firm specific efficiency could be measured using standard approaches like a stochastic frontier analysis or data envelopment analysis. In a second stage the predicted technical efficiency could then enter a binary choice model (stay or exit) or a hazard rate model as explanatory variable. Alternatively, we could estimate jointly firm specific efficiency, and the likelihood of staying or exiting the market. Such econometric models allow for testing interaction effects between efficiency and uncertainty but also for non-monotonic relations between efficiency and exit probability.

5 Appendix

Proof of ***Lemma*** A profit function of the type $\pi_a(p, \mathbf{w}, a)$ homogeneous of degree $-k/(1-k)$ in input prices will be homogeneous of degree $1/(1-k)$ in output price p.

Proof The proof is similar to the one in Lau's chapter in Fuss and McFadden (1978) and Kumbhakar (2001). Because the profit function has to be positively linearly homogeneous with respect to all output and input prices, by Euler's theorem:

$$\sum_{i=1}^{P} \frac{\partial \pi_a}{\partial w_i} w_i + \frac{\partial \pi_a}{\partial p} p = \pi_a \tag{36}$$

But we also know that the profit function is homogeneous of degree $-k/(1-k)$ in input prices so that by Euler's theorem:

$$\sum_{i=1}^{P} \frac{\partial \pi_a}{\partial w_i} w_i = -k/(1-k)\pi_a \tag{37}$$

If we substitute in the previous equation:

$$-k/(1-k)\pi_a + \frac{\partial \pi_a}{\partial p} p = \pi_a \tag{38}$$

Rearranging, we are able to show that the profit function has to be homogeneous of degree $1/(1-k)$ in p by Euler's theorem:

$$\frac{\partial \pi_a}{\partial p} p = 1/(1-k)\pi_a \ \text{ Q.E.D.} \tag{39}$$

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