Puzzling Out Feldstein-Horioka: an Extensive Analysis using Time Varying Parameter Models[1]

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Abstract. This paper carries out a time varying parameter model based analysis of the well-known Feldstein-Horioka puzzle on a wide dataset consisting of 126 countries and 51 years of data. A lot of research has been done on single countries or small country-groups, but we try to aggregate our by-country estimates and analyze them at once, searching for country-group patterns. We confirm the finding that the worldwide savings-retention coefficient has declined significantly, with the most pronounced period of the 90's and early 2000's, and along with this, the standard deviation of by-country savings-retention coefficients ha also declined. For now we research the coefficients' distribution in different country-income categories but there seems to be no explanatory force of country-income.

JEL (Journal of Economic Literature) codes: C32, F32

Keywords: investment and saving, Feldstein-Horioka puzzle, time varying parameters, Kalman filter

1 Introduction and theoretical background

The puzzle of Feldstein and Horioka (1980, henceforth FH) stems from the early eighties: the authors identified the correlation between a country's savings-to-GDP and investment-to-GDP ratio as a measure of international capital mobility. By using a cross-section analysis they concluded that the long-awaited capital mobility was not significantly present. The puzzle rose from the fact that the seminal article inspired a vast amount of literature with different research methods and data samples, but without a real wide consensus.

There are a vast number of studies researching the Feldstein-Horioka puzzle with time-series methods, and still a large group that deal with time-varying parameter models. There are two existing approaches to the problem, and both originate from regressing a country's investment-to-GDP ratio on the savings-to-GDP ratio on a longitudinal (time series) sample. If we assume the non-stationarity of the two series, this can be regarded as a cointegrating equation.

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[1] This is a work-in-progress, therefore a very preliminary draft version of the paper, compiled for the Ecomod2012 conference. At the conference we will probably show much more results.

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One possible (and often used) solution is to make the cointegrating vector itself time varying. This means allowing at least the savings retention coefficient (and optionally the constant term) to change over time. This approach is quite simple, easy to implement, and the results are more likely to be interpretable. However, there is a theoretical problem: a cointegrating link between variables is by definition a long term one, whereas this model allows changing the relationship in the short term. Moreover, it does not estimate any error correction, so the theoretical anticipation may be clearer if we drop the “cointegration” term and just assume a simple time-varying correlation.

The other method – which is clearly less often used – eliminates the previously mentioned theoretical drawback and estimates a cointegrating relationship which is constant in time. The adjustment coefficients in the error correction equations are the time varying parameters, therefore not the cointegration but the intensity of the adjustment is assumed to change over time. The error correction equations optimally use dynamics which means more parameters to estimate and a loss of observations (due to lags used) thus it’s harder to reach interpretable results and it works better on a higher – say quarterly – data frequency.

A lot of literature deals with one specific country or a smaller group like the G7. Our aim with this paper is to “go further” and estimate the time varying savings retention coefficient / cointegrating adjustment coefficient with both methods for a wide set of countries and present the aggregated results.

After this introductory chapter we show the relevant literature, Chapter 3 unfolds the models described and shows the data used, Chapter 4 presents the results and Chapter 5 concludes.

2 A brief literature review

Economic theory predicts that under perfect capital mobility capital will tend to flow into the country where the highest yield is achievable thus domestic savings and domestic investments should remain uncorrelated. However, in the original Feldstein-Horioka study the authors found that the correlation between domestic savings and domestic investment during the period 1960-74 was high for the sample of 16 OECD countries. They estimated the following well-known equation ever since:

$$\left(\frac{I}{Y}\right)_i = \beta_0 + \beta_1 \left(\frac{S}{Y}\right)_i + \varepsilon_i \qquad \text{(FH)}$$

Here I means gross domestic investment, S stands for gross domestic savings and Y denotes GDP (later we’ll use i and s for GDP fractions). In other words they regressed the average of the investment rates on the average of savings rates using a cross-sectional sample and they also estimated β_1 for sub-periods, using five-year-average rates. A β_1 near 1 would mean that domestic savings and domestic investment are highly correlated. They found that β_1 was about 0.9 for every sample period, which they interpreted as a lack of capital mobility.

Their findings have been widely discussed. Lots of studies used simple OLS estimates for various groups of countries and periods. Many papers (e.g. Turner (1986), Feldstein and Bachetta (1989), Bayoumi (1990), Sinn (1992)) confirmed the original findings at least partially. Several empirical researches, e.g. Blanchard and Giavazzi (2002) showed that the coefficient in question is much lower than estimated by FH, and it's especially lower for developing countries as emphasized by e.g. Frankel et al. (1987), Coakley et al. (1999) or Kasuga (2004).

Researchers used a wide set of econometric techniques to confirm or to refuse the original argument. Numerous studies used time series methods to examine the long-run and short-run connection between the saving and investment rates. Several authors tried to find cointegration, e.g. Jansen (1996), De Vita (2002). VAR models have been constructed e.g. by Bodman (1995). To rule out spurious regressions some studies attempted to investigate if the direction of causality runs from saving to investment or inversely. See for example Leachman (1990) or Boon (2000).

Various theoretical macroeconomic models have been built to explain the puzzle. Several studies stressed output shocks and the current account sustainability, e.g. Roubini (1988), Baxter and Crucini (1993) separated basic savings and true savings, Barro, Mankiw and Sala-i Martin (1995) introduced a growth model with human capital, Obstfeld and Rogoff (2000) created a model with budget constraints and transport costs. The main critiques of these works incorporate the omitted variables, the econometric methods, and the data used.

Early papers related to the puzzle used periods averages in terms of saving- and investment rates but that method seemed to be ineligible. Time-series studies used annual rates, but the estimated coefficients were constant over time which led to unsophisticated results. Using time-varying parameter approach (TVP) is the most effective way to capture the changes in capital mobility over time.

Ho (2000) applied a two-state, first-order Markov regime-switching model for Taiwan using quarterly data for the 1979-95 interval and concluded that capital mobility is more likely to have been high in the studied period. Other three East-Asian countries in addition to Taiwan (Korea, Hong Kong, Singapore) have been also studied by Sun (2004) using a TVP model. He claimed that due to the financial liberalization, during the 1980-97 period capital was very mobile, moreover the mobility was rising.

Papapetrou (2006) used recursive OLS, rolling OLS, Kalman filter and Markov switching regime modeling technique to capture major policy regime changes. Using quarterly data for Greece for the period 1980-2003 she found that the estimated time varying coefficients declined over time indicating that due to the financial liberalization the capital mobility increased.

Telatar et al. (2007) carried out an empirical research using a Markov switching model with heteroscedastic disturbances and they found that the effect of policy regime changes can be observed. They used annual data for 9 European countries for the period 1970-2002. They concluded that two groups of countries can be perceived in the sample. In six of nine countries (Belgium, Denmark, Finland, France, Italy and Sweden) a significant fall has occurred in the saving retention coefficient around 1994, after the creation of the European Monetary Union, which was a step to the direction of the European capital market integration. Capital mobility increased considerably as the FH coefficient declined. However in the remaining three countries (Germany, Netherlands, UK) they didn't find such a substantial switching point.

Hatemi-J and Hacker (2007) analyzed the capital mobility in Sweden between 1993 and 2004. They used Kalman filter and found that the correlation is comparatively low and has declined before 1995 when Sweden joined the EU. Surprisingly they observed that capital mobility hasn't increased after joining the Union.

Gomes et al. (2008) investigated the investment-saving connection for the second half of the 20th century in three South-American countries (Argentina, Brazil, Chile). They asserted that the coefficient showed relatively high volatility over time, thus the time varying parameter approach is much better than the simple OLS estimation.

Evans et al. (2008) examined a very long period for 8 countries. They found that the saving retention coefficient showed remarkable volatility from 1850 to 1990. Their results were mixed as for example in the USA capital mobility seemed to have been low throughout the whole period in contrast with Canada. Changes in the value of the coefficient did not show any unanimous pattern.

Arisoy and Ucak (2010) studied the G7 countries trying to catch the changes in the relationship of saving and investment rates. Using Kalman-filter they did not confirm the earlier results as they did not find significant evidence that capital mobility has declined over time. The authors found that the coefficient remained relatively stable during the period from 1960 to 2007 except for Italy where a significant drop was observed.

We can see that the TVP literature of the Feldstein-Horioka puzzle is relatively bold but lacks any research to use the TVP methods for a large enough or the entire set of countries. Our paper wishes to fill this gap.

3 Methodology

We now present our two models used. In both cases we build a state space model by taking the main equations as observation equations while we assume that the time varying parameters are random walks – these make up the state equations. The models are estimated by the Kalman filter (and smoother) via maximum likelihood, but we can also use recursive or rolling window least squares by taking only the signal equations[4]. All methods share the feature that not only the time varying parameters themselves, but their standard errors can also be estimated.

The time varying cointegrating equation (TVCE) model postulates that both the savings retention coefficient and a constant term follow random walks independent of each other, while we observe the cointegrating regression:

$$\begin{aligned}\beta_{0,t} &= \beta_{0,t-1} + \omega_{0,t}\\ \beta_{1,t} &= \beta_{1,t-1} + \omega_{1,t}\end{aligned} \qquad \text{(TVCE ST)}$$

$$i_t = \beta_{0,t} + \beta_{1,t}s_t + \varepsilon_t \qquad \text{(TVCE OBS)}$$

Here i_t represents the country's investment-to-GDP ratio and s_t the savings-to-GDP ratio. We assume independence between the two $\omega_{i,t}$, $i = 0{,}1$ innovations while both are uncorrelated with the ε_t residuals by definition. Of course in this model we are interested in the $\beta_{1,t}$ savings retention coefficient sequence and its standard error, this indicator directly shows the level of co-movement between investment and saving. When thinking of cointegration, it can be interpreted as a long-term equilibrium rate, and if we drop the integratedness assumption, it shows a correlation coefficient. The value of 1 can be thought as the analyzed country's capital markets are fully closed while the value of 0 shows a "perfect" openness. Negative values are somewhat harder to interpret.

The time varying error correction term (TVECT) model assumes a conventional cointegrating equation, along with an error correction model with time varying adjustment coefficients. The ECM consists of two equations: one for the investment adjustment, and one for the savings:

[4] We set the sample size for rolling window OLS to 10 years which seems to be quite short but this way we manage to get a 41 year long estimate sequence on a 50 year long data sample. Similarly, the minimum sample size for recursive OLS is also 10.

$$i_t - \beta_0 + \beta_1 s_t + u_t \qquad \text{(TVECT CE)}$$

$$\begin{aligned} \alpha_{i,t} &= \alpha_{i,t-1} + \omega_{i,t} \\ \alpha_{s,t} &= \alpha_{s,t-1} + \omega_{s,t} \end{aligned} \qquad \text{(TVECT ST)}$$

$$\begin{aligned} \Delta i_t &= \sum_{l=1}^{k} \phi_{i,i}^{(1)} \Delta i_{t-l} + \sum_{l=1}^{k} \phi_{i,s}^{(1)} \Delta s_{t-l} + \alpha_{i,t} u_{t-1} + \varepsilon_{i,t} \\ \Delta s_t &= \sum_{l=1}^{k} \phi_{s,i}^{(1)} \Delta i_{t-l} + \sum_{l=1}^{k} \phi_{s,s}^{(1)} \Delta s_{t-l} + \alpha_{s,t} u_{t-1} + \varepsilon_{s,t} \end{aligned} \qquad \text{(TVECT OBS)}$$

Here we are primarily interested in the sequence of the adjustment coefficient of investment $\alpha_{i,t}$ which should be negative to show real adjustment to the cointegrating relationship. Similarly to before, we assume no correlation within the ε's group and within the ω's group, and non-zero cross-correlations are set to zero by construction. The estimation of such a model is twofold: first the cointegration equation is estimated – we use Johansen's method (in EViews), but we could also use simple OLS[5]. The u_t residuals are then saved, and in the second step, the state space model / recursive OLS / rolling window OLS can be estimated as usual (this part is carried out in GAUSS). The lag length k is set to 1 for yearly data and up to 4 for quarterly data so that we can control for autoregressive effects in the equations.

As we argued before, the TVCE model can be interpreted simply as a time changing correlation model where we drop the assumption of cointegration entirely. The second model however is a fully written cointegration and error correction representation where the nonstationarity of the i_t and s_t processes plays a crucial role. We know that both series are bounded by construction; what's more, macro says that in the long run i_t will be equal to s_t. Both of these arguments point to the fact that the frequency and sample length used is very important: if we used an extremely long sample, the integratedness of the two series couldn't hold any more. We will conduct unit root tests but the literature suggests that on sample lengths comparable to ours the series are not stationary.

After we have our estimated beta (TVCE) and alpha (TVECT) sequences, we need to put the results in a more concise form: we have to aggregate the series on the cross section dimension. When doing so, we simply leave out the countries where there is no data but we are aware of the fact that this may cause a selection bias. We try to avoid it by identifying any extreme values whose nonexistence in the early sample could cause false averages.

There are two ways we are dealing with aggregation. The first is more complex but makes use of the supplied standard errors: we calculate both the mean and standard error of the average

[5] Using OLS to estimate a cointegration equation is called the Engle-Granger method and is known for its drawbacks, e.g. it's not invariant to normalization (that is, which variable is on the left hand side).

beta or alpha at every observation for any desired set of countries. The standard error calculation requires a correlation matrix of the coefficient sequences: here more assumptions can be done: the correlation values can either be chosen arbitrarily (say, all be zeros, or ones) or they can be calculated from the estimated sequences. We choose the latter method and calculate correlations on pairwise samples but in practice this does not matter too much.

When using the Kalman filter there would be another, more theoretically sound method to estimate cross correlations of the beta (and alpha) sequences: put all countries' equations in one stacked state space model and thus we could estimate the full covariance matrix of the state variables. The practical problem with such an approach is that it produces a quadratically increasing number of parameters to be estimated and it simply does not work even for a moderate number of countries.

The second way of sequence aggregation is simply analyzing the empirical distribution of betas (alphas) at a given point in time for a given set of countries. This does not need averaging (which also raises the question what weights we should use) and helps us much more to deal with the variable length data series: it shows any extreme data points and distribution skews which should be taken with care.

In this version of the paper we use only a 51 year long (1960-2010) panel of 126 countries taken from the World Bank Database where the main sources are the OECD National Accounts and the World Bank national accounts. Investment rate is gross capital formation (% of GDP) and saving rate is gross domestic savings (% of GDP). We didn't take into account countries with relatively too low GDP or permanently negative saving rate. Several time series are missing data typically in the first 10-20 years and the last 2-5 years or so, the longest sample for which we have all data is 1990-2005. We are in the process of compiling another, quarterly database comprising of data for about 25 developed countries. These series will have much more data points, ranging from 40 (10 years) to about 200 (50 years) and thus more useful for especially the TVECT estimation.

4 Preliminary results

To get a glance of the overall results, let's use our aggregating method which also deals with standard error bands on the full world – the entire set of countries. Figures 1a-b show the averaged time-varying point estimates with confidence intervals using the recursive least squares, rolling window least squares, Kalman filter and smoother, for the TVCE and TVECT models respectively. The TVCE estimates show a clear descending path throughout the sample from levels of about 0.50-0.60 to 0.30-0.40, and the confidence limits also support the significance of the decline, especially the more trusted Kalman algorithms. Considering that the OLS methods place too much weight on the older observations we can suggest that much of this descent happened in the 60's (with less emphasis) and much more importantly in the 90's and early 2000's, in other periods the estimates show more or less constant behavior. Keep in mind that in 1990-2005 we have observations for all countries; this statement is therefore is very clear.

As far as the TVECT model is concerned, all the estimates are on the negative side (which is in line with the assertion of cointegration adjustment) and we can see a very mild convergence towards zero: the smoothed estimates appear almost constant; the two OLS estimates probably show an outlier effect in the 70's and only the Kalman filter produces a very slight change. It seems therefore from these two results that to presume a constant cointegration equation throughout our 50 year period is probably unacceptable. We will continue using the TVECT model with the quarterly data set.

Our second result set so far is the further analysis of the Kalman filter TVCE results by time and by country groupings based on income. The World Bank supplies us with the distribution of all countries into four groups based on income: low, lower middle, upper middle and high. This grouping depends – of course – on time, and it is probably done using newest (or relatively new) data. However, there is a very strong correlation among the income rankings of countries in time and so we didn't count with this changing effect. We show the distributions of the filtered betas grouped by income at different points in time on figures 2a-f (different graphs for every time point) and 3a-d (different graphs for every income category). From the results we can infer the following:

- The decline of the savings-retention coefficient in time is again very clear. On the 1960, 1970 and 2010 graphs there are a lot less data points (due to the short series) but the process can be well seen from 1980 to 2000.
- The initially present extreme values are diminishing in time, therefore the standard deviation of beta estimates declines in time, and this is true for all income categories. The Kalman filter has a tendency though that its first estimates are ill-valued and this may cause at least partially this observation. However, again from 1980 to 2000 the tendency goes on which affirms our finding.
- Not much difference can be observed between country groups at any point in time. Income therefore does not seem to explain the differences in the savings-retention coefficient.
- The high income countries' beta distribution tends to have two modes throughout the entire sample. We need to identify which countries belong to which mode; this could perhaps explain the distribution.

5 Conclusion and near future research

To sum up the results detailed in the previous chapter: the TVCE induced savings retention coefficient declined, and its standard deviation also declined in time. This means that countries whose capital markets were more closed in the past opened up more throughout the sample. These results suit well our intuitions; therefore the research so far is a success. However, there is surely a lot more to do; in the last words we supply a list of research for this paper:

- Supply the distribution of p-values of the t-tests for the i_t and s_t series.
- TVCE: take other country groups: LDC, EU, EMU, OECD vs. non-OECD, etc.
- TVCE: take the smoothed results and analyze for extreme values at sample ends.
- TVECT: compile and use the quarterly database, look at α_s as well.
- Compute the common component of the betas / alphas.

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Appendix: Figures

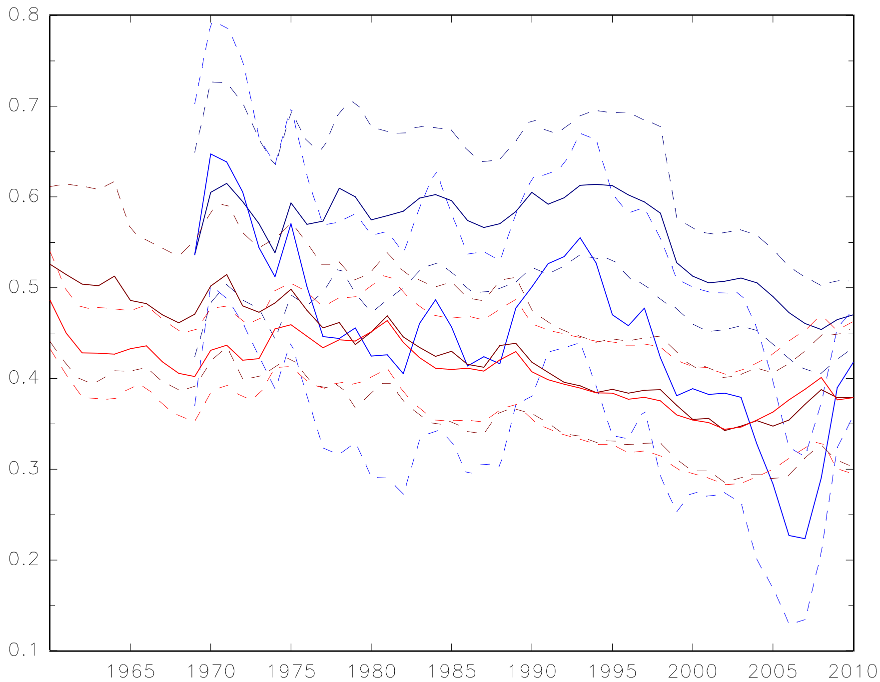


Figure 1a –TVCE $\beta_{1,t}$ estimates and error bands for the world by averaging the by-country estimates
Dark blue = Recursive OLS, Blue = Rolling window OLS, Dark red = Kalman filter, Red = Kalman smoother

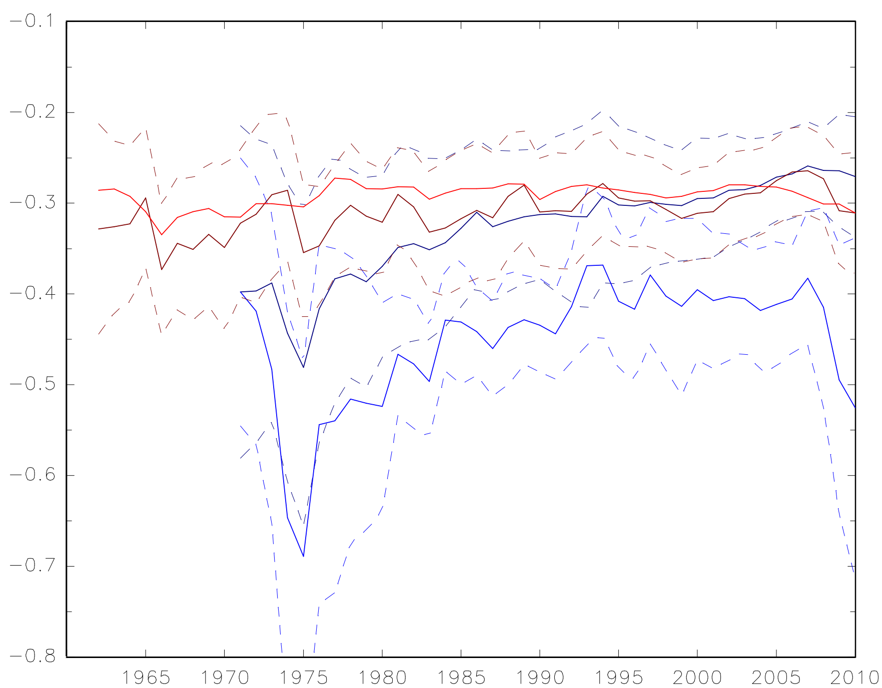
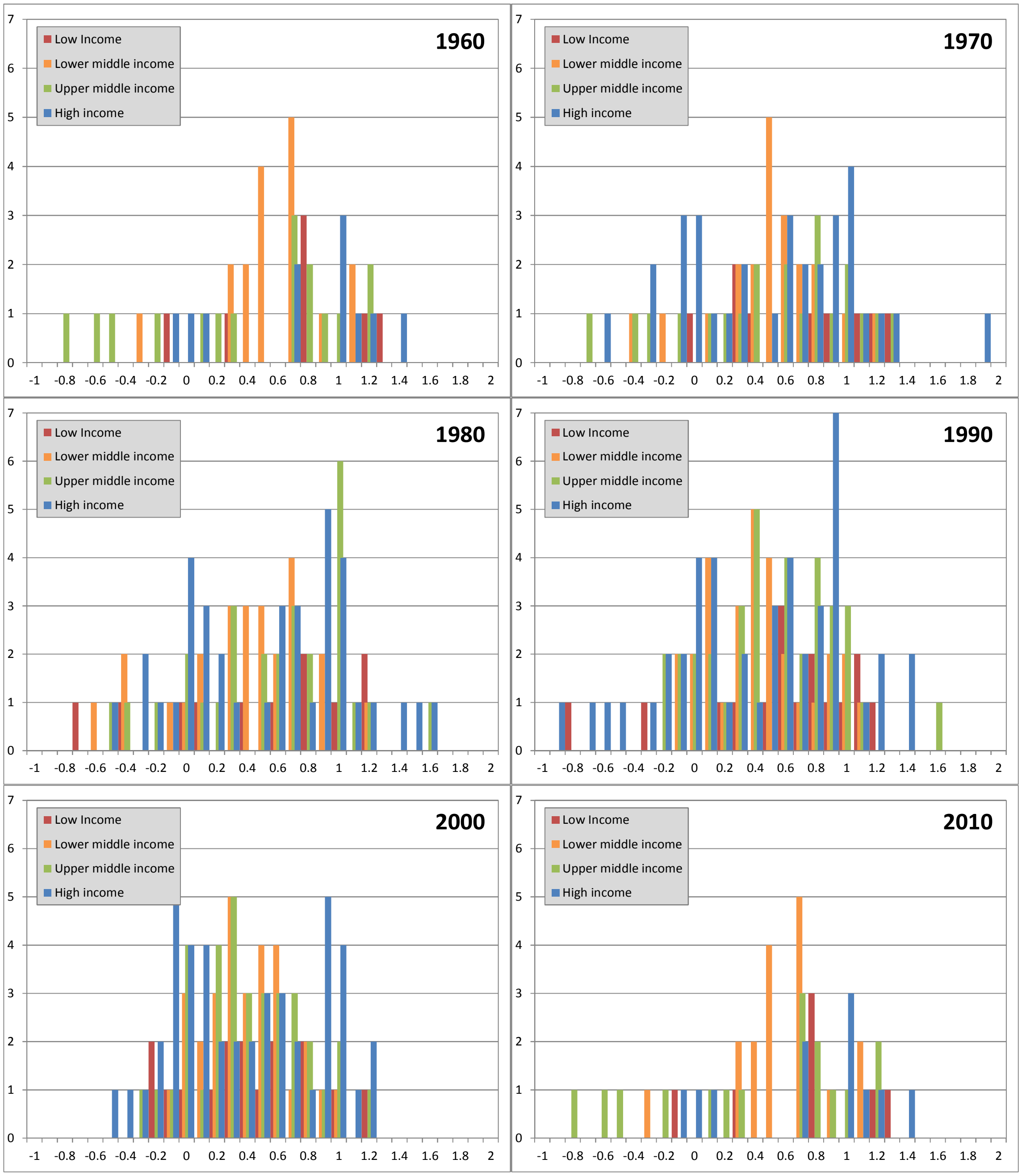
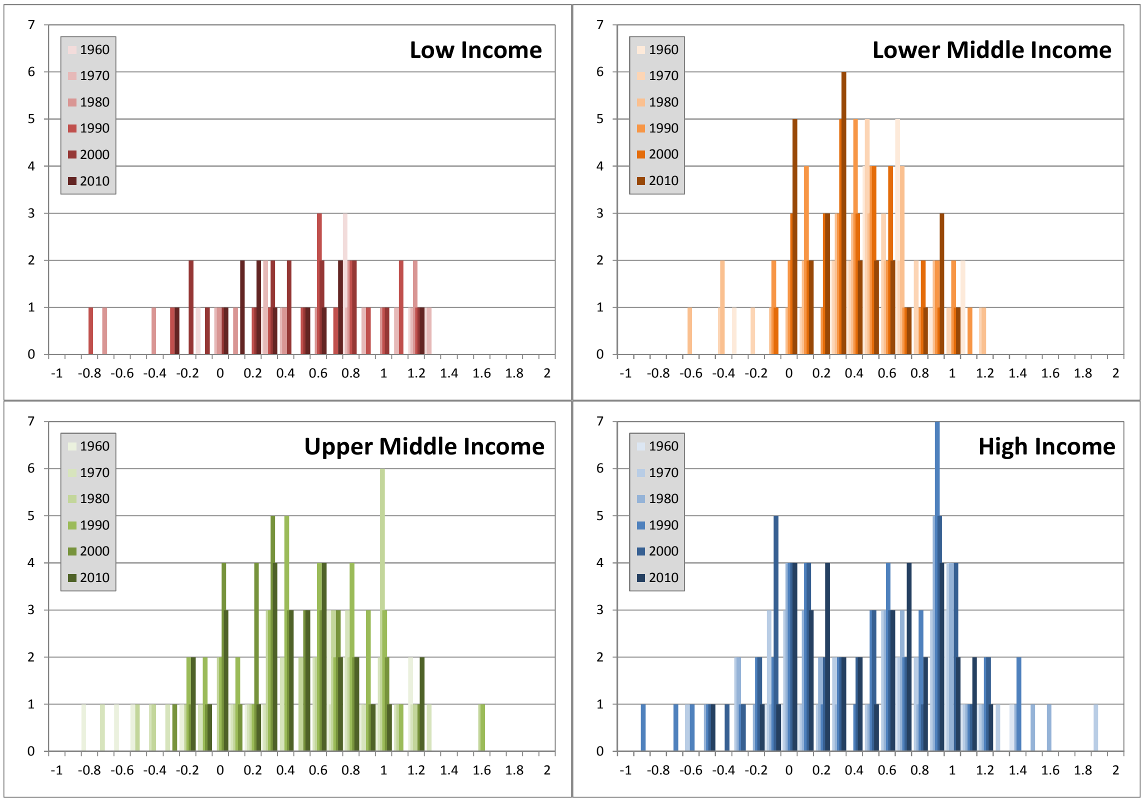


Figure 1b –TVECT $\alpha_{i,t}$ estimates and error bands for the world by averaging the by-country estimates
Dark blue = Recursive OLS, Blue = Rolling window OLS, Dark red = Kalman filter, Red = Kalman smoother



Figures 2a-f – Distributions of the Kalman Filter TVCE $\beta_{1,t}$ estimates by countries of different income

Separate graphs by points in time (1960-2010, 10 year increments)



Figures 3a-d – Distributions of the Kalman Filter TVCE $\beta_{1,t}$ estimates by time

Separate graphs by countries of different income (low, lower middle, upper middle, high)