# Bargaining Power and Value Sharing in Distribution Networks: a Cooperative Game Theory Approach

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**Abstract**

Costs and benefits associated with a distribution network (e.g., gas pipelines, water irrigation) are shared by agents connected by the network. Therefore, economic incentives to join or to expand a network depend on how the network surplus is being distributed, which in turn depends on a variety of factors: position of each agent (e.g., a country) in a specific network, its reliability in the cooperation scheme (e.g., geo-political stability), existence of market distortions and availability of outside options (e.g., alternative energy sources). This study is aimed at presenting a game theory methodology that can be applied to real world cases, having the potential to shed light on several political economy issues.

The methodology is presented and illustrated with application to a fictious network structure. The method is based on a two-stage process: first, a network optimization model is used to generate payoff values under different coalitions and network structures; a second model is subsequently employed to identify cooperative game solutions. Any change in the network structure entails both a variation in the overall welfare level and in the distribution of surplus among agents, as it affects their relative bargaining power. Therefore, expected costs and benefits, at the aggregate as well as at the individual level, can be compared to assess the economic viability of any investment in network infrastructure. A number of model variants and extensions are also considered: changing demand, exogenous instability factors, market distortions, externalities and outside options.

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# 1 Introduction

# 2 Methodology

We consider a network, made of arcs and nodes. An arc connects two nodes $i$ and $j$, but not all pairs of nodes are generally connected. In addition to nodes we consider supply and demand points, both of which are connected to a node in the network by an access link (directed). Each demand point is associated with a demand curve, expressing the required demand quantity volume as a (negatively sloped) function of the market delivery price or cost.

All arcs and all access links are associated with an increasing and convex function cost function $C_{ij}$ of the flow $f_{ij}$. For access supply links, this could be interpreted as production cost. For demand links, this would express a final market distribution cost. For intermediate arcs, the function refers to transportation costs.

A discrete and finite number of agents operate in the network. Agents can cooperate in coalitions $\Gamma$, where an agent can joint at most one coalition. Each coalition $\Gamma$ has access rights to a number of arcs, links and nodes of the networks. $A(\Gamma)$ is the set of all nodes connected by arcs controlled by the coalition $\Gamma$.

For each coalition and its associated network, we consider a network market equilibrium (NME). A network market equilibrium is found when flows in the arcs and links are determined such that:

1. Demand access links flows equal demand levels, defined by demand functions computed at marginal delivery costs (demand is served);
2. Total costs (production, transportation, distribution) are minimized;
3. Total incoming flows in each (transit) node equal total outgoing flows (flow balance constraint).

A NME is the solution of the following mathematical optimization problem:

$$\max_{f_{ij},(i,j)\in A(\Gamma)} W_\Gamma = \sum_{d\in D} \int_0^{f_{id}} P_d(f)df - \sum_{(i,j)\in A(\Gamma)} C_{ij}(f_{ij}) \qquad (1)$$

s.t.:

$$\sum_{s\in S} f_{sj} + \sum_{i\in N} f_{ij} - \sum_{i\in N} f_{ji} - \sum_{d\in D} f_{jd} = 0 \qquad \forall j \in N \qquad (2)$$

where:

$D$ is the set of demand points;

$S$ is the set of supply points;

$N$ is the set of transit nodes;

$A(\Gamma)$ is the set of admissible pairs of nodes/points connected by arcs/links, for which the coalition $\Gamma$ possesses access rights;

$f_{ij}$ is the flow from node/point $i$ to node/point $j$;

$P_d(f)$ is the inverse demand curve at point $d$;

$C_{ij}$ is the cost function of the arc/link connecting $i$ to $j$.

Solving a NME problem identifies the total net welfare $W$ obtainable from a certain network. This total welfare is virtually distributed among all parties involved in the network. For example, think of nodes, or points of supply/demand, as countries. Each country contributes to the functioning and possibly to the construction of the network infrastructure, receiving benefits is terms of consumer surplus, tax revenue or profits.

Clearly, there is no obvious way to determine how the overall pie of total welfare would be split. Therefore, to discuss the implications of surplus allocation, we make use of cooperative game theory. A cooperative game equilibrium is a normative concept applied to the distribution of benefits or costs in a group. Among the various equilibrium concepts proposed in the literature, we use here the Shapley value[1], because of its simplicity of computation and easiness of interpretation: the Shapley value assign to each agent a payoff which is proportional to her "contribution" in all possible forming coalitions, that is the difference between the overall surplus obtained by a cooperative coalition with and without the agent. The Shapley values of a game can be readily interpreted as an allocation of benefits (or costs) which reflects the relative bargaining power of each party.

In order to compute a Shapley value distribution for a network with the characteristics described above, one needs to compute the welfare associated with all possible coalitions and individual agents. Each agent is here associated with a node, therefore computing the maximum welfare for a coalition amounts to solving a NME where all arcs connect pairs of nodes belonging to the coalition. In other words, we consider sub-networks obtained from the big network by removing those links where at least one of the two extremes brings to an agent not in the coalition. The smaller the coalition, the smaller the network, the lower the welfare that can be obtained. Furthermore, many coalitions may actually get zero welfare. This is true for all individuals (one member coalitions), for coalitions including only demand or only supply agents, or where demand and supply agents are disconnected.

In a set of $N$ agents, there are $2^N$ possible coalitions, including the grand coalition (all agents inside) and the empty one. To compute the Shapley value, or any other distribution concept of cooperative game theory (e.g., the nucleolus),

[1] Alternative equilibrium concepts, like the nucleolus, may not change the qualitative results of our analysis. The nucleolus is the central point of the core, the set of all allocations in which no individual and no coalition gets less than what it can get without collaboration from other agents. The nucleolus is sometimes hard to compute and it is most appropriate in contexts focusing on coalition formation, rather than (as in the present case) of bargaining power and surplus sharing.

the first step is solving for the NME and obtaining the welfare level associated with any sub-coalition, possibly excluding those ones which have obviously a zero welfare. This can be done with optimization software like GAMS or AIMMS, or mathematical packages like Mathematica or, if the network is not too complex, using a solver embedded into spreadsheets like Microsoft Excel or LibreOffice Calc. Once surpluses for all possible combinations of agents in the set have been obtained, the Shapley value can be computed using an algorithm, for example the one proposed by Carter (1993)[3], based on Mathematica.

Different network structures imply, of course, different distributions of welfare. Hubert and Ikonnikova (2011)[2], Hubert and Cobanli (2012)[1] adopt the methodology described above to assess the distribution of surplus in gas distribution networks for Eastern Europe and the Middle East. The existing pipeline infrastructure is taken as a benchmark, to be contrasted with alternative network structures in which new links are added or the capacity of some existing links is expanded. These alternative scenarios are based on investment projects under discussion or realization.

Assessing how welfare changes and how it is differently distributed when a network is modified allows evaluating the individual incentives to undertake the proposed investment. Any network enlargement necessarily increases the overall welfare, which can be measured in monetary terms, but this could not be sufficient to cover the costs. Furthermore, some investment may not need the involvement of all parties. Think, for example, the addition of a new link, whose realization requires the involvement of only the agents located at its two sides. As a change in the network topology influences the bargaining power and the distribution of welfare, it may well be the case that a certain investment may not be globally justified, yet be locally viable.

# 3 An Illustrative Example

To illustrate the meaning of surplus allocation in a network, let us consider a fictious network structure as depicted in Figure 1. There are five agents (i.e., countries), each one associated with one node: A, B, C, D, E. All costs associated with arcs and links are constant, possibly up to a capacity limit. A and E are suppliers. E has a higher production cost (15 instead of 10) and it is, furthermore, affected by an upper supply capacity limit of 50 (this is indicated by a number in parentheses, otherwise there are no capacity constraints). All intermediate links have a unit transport cost of 5, except for the link connecting B to D, which has a cost of 8 but a maximum capacity of only 10. B, C and D are demand points, each one associated with a simple linear demand curve of the type $Q = \Psi$-p, where $\Psi$ is the maximum price in the market and also a measure of the market size. There are no distribution costs.

Flows in the network of Figure 1 can be allocated by solving the mathematical program (1-2). Consider, for example, the demand market D ($Q = 95 - p$). For D, the least cost supplier is E (15+5=20). However E cannot provide more than 50 units, which is less what would be required at a price of 20 (95-20=75).

Figure 1: A reference network structure

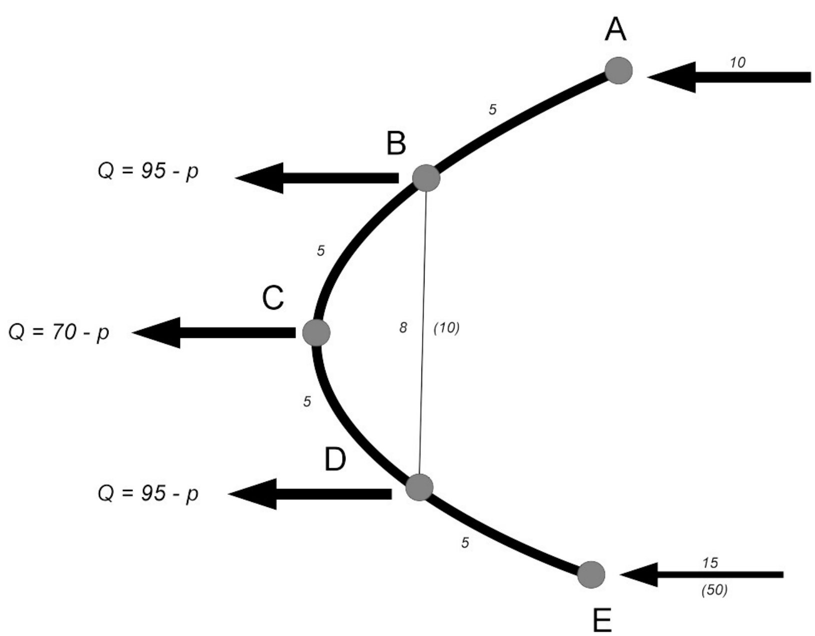
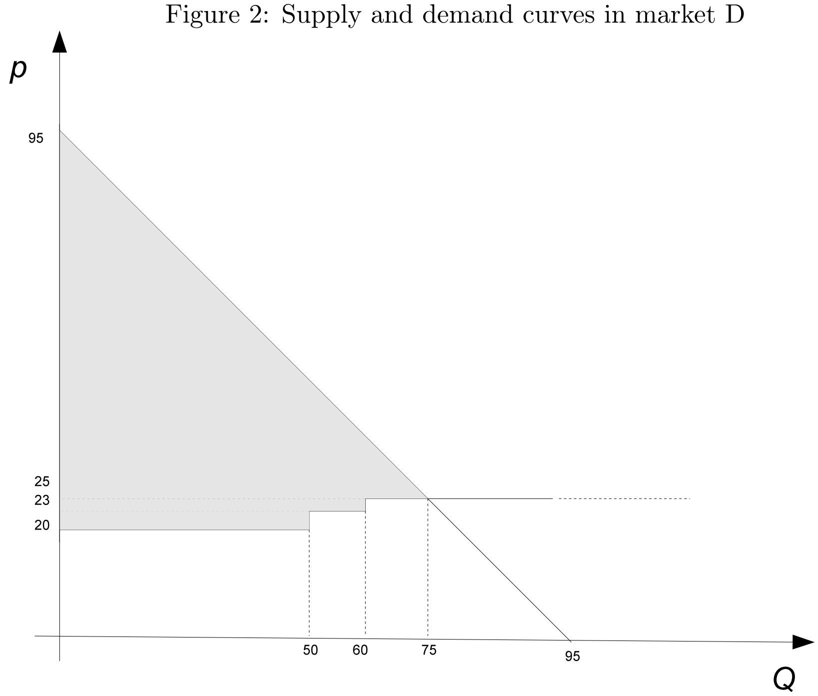


Figure 2: Supply and demand curves in market D




The second least cost alternative is A through the A-B-D path (10+5+8=23), which is also capacity constrained. Finally, it is possible to supply D trough the path A-B-C-D at a cost 10+5+5+5=25. At a price 25, 70 units are demanded and delivered at D: 50 from E, 10 from B, 10 from C. The total surplus generated in market D is 2720, corresponding to the area below the inverse demand curve but above the stepwise supply curve in Figure 2. This surplus adds to the one generated in B (3200) and C (1250) to the total welfare W (7170).

How is this total welfare going to be virtually distributed among the five agents? This depends on the relative bargaining power. Consider, for example, supplier E. The bargaining power of E has to do with what the other agents can get without E. For example, a coalition {A,B,C,D} could run a network without the D-E link. Market D, in this case, would live without its most convenient supply source, which would reduce welfare in D by 250 [(25-20)*50], lowering total welfare from 7170 to 6920.

Welfare for other sub-coalitions can be computed in a similar way, allowing to compute a Shapley distribution for the cooperative game on the network. Shapley values for this base case are reported in Table 1, in the column "Base".

Values under the heading "Ext." refer, instead, to an alternative case, where the original structure of the network as in Figure 1 has been modified, by removing the capacity limit in the link B-D. This enhancement increases the overall welfare, from 7170 to 7192, as it lowers the market price (from 25 to 23) in the

Table 1: Shapley value surplus distributions

| agents \ cases | **Base** | **Ext.** | **Diff.** |
|---|---|---|---|
| **A** | 2031.67 | 2099.58 | +67.92 |
| **B** | 2346.67 | 2429.92 | +83.25 |
| **C** | 655 | 550 | -105 |
| **D** | 1405 | 1488.25 | +83.25 |
| **E** | 731.67 | 624.25 | -107.42 |
| ***Total*** | ***7170*** | ***7192*** | ***+22*** |

D market. Furthermore, it changes the surplus distribution, actually harming the agents C and E.

C is made worse off because it would be by-passed whenever D is served from B (or B from D). Consequently, any threat from C of not joining a coalition would be weakened, thereby reducing its bargaining power. Analogously, the threat from E of not serving D would reduce welfare in that market by an amount [(23-20)*50=150] smaller than it was before (250), because D can now revert to a fairly efficient alternative supply source.

It is interesting to notice how the variation in surplus affects the incentives to undertake the investment. At the aggregate level, the investment in capacity expansion would be desirable if its cost would be lower than 22, that is, the total welfare gain. However, it may be the case that the expansion of capacity in B-D only requires cooperation between agents B and D, possibly with the contribution of A. In this case, if the investment costs more than 22 but less than 166.5 (83.25+83.25), it would be undertaken, despite the fact that it would not be socially desirable. In other words, there would be a *negative externality* generated by the expansion of capacity in the link B-D.

# 4 Extensions

## 4.1 Changing Demand

Consider a case where demand in the smallest market C increases from $Q = 70 - p$ to $Q = 80 - p$. This obviously raises the overall welfare obtainable in the network, from 7170 to 7720. It also changes, however, the bargaining power of all parties, at it is shown in Table 2, comparing the base case with the one with expanded demand in C.

Much of the welfare gain accrues to C. However, it also goes to the nodes which are involved in the supply of C, in proportion to their contribution. As C is typically supplied through the route A-B-C, A and B are also getting significant gains.

Table 2: Shapley value surplus distributions

| agents \ cases | **Base** | **Exp.** | **Diff.** |
|---|---|---|---|
| **A** | 2031.67 | 2208.33 | 176.67 |
| **B** | 2346.67 | 2500.42 | 153.75 |
| **C** | 655 | 850.42 | 195.42 |
| **D** | 1405 | 1417.08 | 12.08 |
| **E** | 731.67 | 743.75 | 12.08 |
| ***Total*** | ***7170*** | ***7720*** | ***550*** |

Table 3: Shapley distributions with instability in C

| agents \ cases | **Base** | **Ext.** | **Diff.** |
|---|---|---|---|
| **A** | 1982 | 2057.92 | +75.92 |
| **B** | 2297 | 2390.75 | +93.75 |
| **C** | 602 | 507.5 | -94.5 |
| **D** | 1393.67 | 1487.42 | +93.75 |
| **E** | 765.33 | 623.42 | -141.92 |
| ***Total*** | ***7040*** | ***7067*** | ***+27*** |

## 4.2 Exogenous Instability

Suppose that one agent in the set, say agent C, is affected by some exogenous factors undermining her "reliability" as a partner in any coalition. For example, C could refer to a geo-politically unstable country. We assume that there is some probability that the C partner is not available and cooperating. A simple way to capture this exogenous instability is to compute the expected payoffs for all potential coalitions, considering that the coalition could shrink to a smaller one, excluding C. For example, with a 10% probability, the expected payoff of the grand coalition {A,B,C,D,E} would be 0.9*P(ABCDE)+0.1*P(ABDE), where P() is the payoff computed from the NME as in the previous section.

Using this methodology to modify the payoffs for all sub-coalitions including C, new Shapley value distributions can be computed. Table 3 presents the new values, corresponding to the ones in Table 1, under exogenous instability for C.

We see that total expected welfare is lower and, not surprisingly, C is the member which is losing proportionally more (602 instead of 655). Agent E gains from the instability in C (765.33 instead of 731.67), because she has more bargaining power now. Indeed, if the path serving market D through C would be disrupted, D could get no more than 60, because of capacity constraints. The price would then go up to 35, making the potential threat by E of not serving D very serious.

If capacity in the link B-D is enlarged, total welfare would increase by 27, which is a bigger increment than before. The value for E falls to 623.42, because to serve the D market it is not necessary to pass through C if capacity is unbounded in B-D. More importantly, gains for B and D together now sum up to 93.75*2=187.5, which is significantly more than the value without instability in C (166.5), whereas the global gain only increases from 22 to 27. We can therefore deduce that: (a) instability in the node C increases the likelihood that capacity in the arc B-D is enlarged, (b) it is more likely that negative externalities are generated and the network is inefficiently expanded.

## 4.3 Outside Options

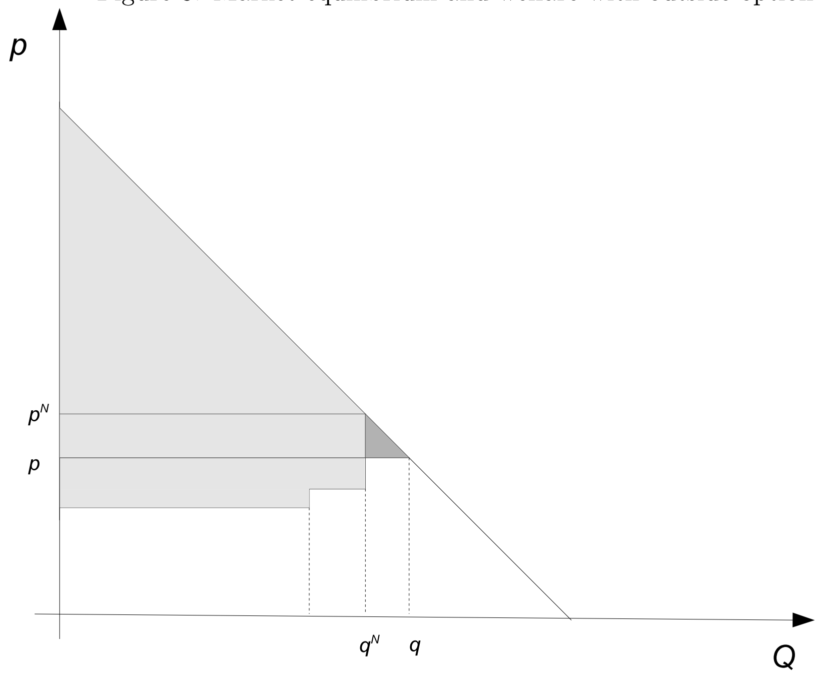
Very often, markets have access to alternative energy sources outside the network. For example, instead of relying (only) on gas or oil, distributed trough pipelines, a country can get energy from renewable sources (e.g. solar). These "outside options" typically have two key characteristics: (1) they are more costly than conventional, network-based goods under normal market conditions; (2) their exploitation does not require cooperation with other agents. In this case, even if an agent may not find it convenient to utilize the outside option when a cooperative network is in place, the mere availability of the outside option affects her bargaining power and the distribution of surplus.

To illustrate the point, consider a market like D in the numerical example discussed above. In the base case, D obtains a good through the network at the price 25. Suppose that D could have produced, autonomously, the same good at a constant cost of 30. Clearly, domestic production would not be economically viable under these conditions. However, to compute the Shapley value we did consider the welfare obtainable by all possible sub-coalitions. The coalition including only D would have got zero surplus in the initial case, but the possibility of autonomous domestic production brings the potential welfare to 2112.5 (price 30, consumed quantity 65). The sub-coalition {A,B,D,E} delivered 60 to market D, bringing the price at 35, which is higher than 30. When the outside option is available, the equilibrium price would instead be 30, and D would be served by both imports through the network and domestic production. This case is depicted in Figure 3.

The price $\bar{p}$ is the constant marginal cost of domestic production, which constitutes an upper bound on the equilibrium price. The network delivers quantity $q^N$ which, without domestic supply, would have brought about a price of $p^N$. Now the price is kept at $\bar{p}$, the quantity consumed is $q$, where $q^N$ is delivered by the network and $q - q^N$ is internally produced. The availability of the outside option implies an higher consumer surplus. The gain corresponds to the dark grey shaded area in Figure 3.

To compute the network market equilibrium when outside options, expressed as "backstop technologies" at cost $\bar{p}$, are available, the optimization program (1) has to be modified in the following way:

Figure 3: Market equilibrium and welfare with outside option




$$\max_{f_{ij},(i,j)\in A(\Gamma)} W_\Gamma = \\ \sum_{d\in D}\left[\int_0^{f_{id}} P_d(f)df + \int_{f_{id}}^{\overline{P}_d(f)^{-1}} (P_d(f) - \overline{P}_d)df\right] - \\ - \sum_{(i,j)\in A(\Gamma)} C_{ij}(f_{ij}) \tag{3}$$

where $\overline{P}_d$ is the exogenous price of domestic production in market $d$ (possibly very high if no outside option is available), and $\overline{P}_d(f)^{-1}$ is the quantity consumed at this price $\overline{P}_d$ .

Results for the case of a backstop technology at cost 30 in market D are shown in Table 4, under the column “Option”.

We see that total network value is unaffected by the presence of an outside option in D, as it is not economically efficient to use the alternative technology if the network would be run cooperatively by all agents. However, the option significantly improves the bargaining power of agent D, as she makes a much bigger contribution to welfare in all possible coalitions (including the singleton D). In fact, with the possibility of autonomous domestic production, agent D gets the higher share of total surplus, at the expenses of all other agents.

Table 4: Shapley distributions with outside option in node D

| agents \ cases | **Base** | **Option** | **Diff.** |
|---|---|---|---|
| **A** | 2031.67 | 1656.25 | -375.42 |
| **B** | 2346.67 | 2135.83 | -210.84 |
| **C** | 655 | 520.21 | -134.79 |
| **D** | 1405 | 2532.71 | +1127.21 |
| **E** | 731.67 | 325 | -406.67 |
| ***Total*** | ***7170*** | ***7170*** | ***0*** |

## 4.4 Market Distortions

The analysis conducted so far assumes that all the potential surplus generated within the network is appropriated by the parties involved in the different coalitions. This hypothesis is consistent with the existence of perfectly competitive markets for network goods or, alternatively, with the presence of a monopolistic supplier, which can perfectly price discriminate among her customers. This second explanation may be defended on the ground that many international networks for oil and gas are based on block pricing schemes, that is, contracts specifying quantity volumes and total prices beforehand.

However, the model described above can be easily modified to accommodate for the existence of distortions in specific markets, like oligopolies or taxes. A common characteristic of market distortions is that the quantity volume exchanged is lower than in the optimum or, equivalently, that market prices are higher than what they would be. Exogenous reductions in consumption volumes can be easily introduced by setting appropriate values for the capacity parameters $k_{jd}$ in distribution links (or, equivalently, by changing the cost functions $C_{jd}$ in the more general formulation). This amounts to assume the existence of import quotas, possibly justified in terms of energy policy[2].

Alternatively, market distortions can imply taxes or profit mark-ups on top of competitive prices. These may also be easily introduced in the model by making the capacity parameters $k_{jd}$ endogenous, dependent on market prices or delivered quantities.

## 4.5 Exogenous Surplus Factors

Cooperation benefits (or costs) may go beyond the network where cooperation takes place, involving multiple policy dimensions. For example, suppose that nodes in the illustrative example of Figure 1 are countries, and that countries B and D were engaged in a past conflict. A political "peace dividend", associated with cooperation between two former enemies, may then play a role in the distribution of surplus and in the justification of investments in the network infrastructure.

[2]For example, a government in a country may want to have a portfolio of energy sources, thereby restricting access to the least cost ones.

Table 5: Shapley distributions with exogenous surplus factors

| agents \ cases | **Base** | **Ex.S.** | **Diff.** |
|---|---|---|---|
| **A** | 2031.67 | 2032.5 | +0.83 |
| **B** | 2346.67 | 2350.83 | +4.17 |
| **C** | 655 | 655 | 0 |
| **D** | 1405 | 1409.17 | +4.17 |
| **E** | 731.67 | 732.5 | +0.83 |
| ***Total*** | ***7170*** | ***7180*** | ***+10*** |

This case could be considered in the example above by raising the payoff of all coalitions including both B and D (e.g, by adding 10 to the surpluses obtained in the NME). The Shapley values computed after such modification are displayed in Table 5, where they are compared with the base case.

As could be expected, much of the exogenous extra gain (+10) accrues, symmetrically, to B and D. However, part of it also goes to A and E. Why this should be so? In order to grab the additional surplus, B and D must be part of the game, but the network must also be functioning, delivering the goods produced in A and E. If there are no suppliers in a coalition, the coalition would get zero surplus in any case, even if both B and D are into it.

# 5 Conclusion

# References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