ESTIMATION OF VECTOR ERROR CORRECTION MODEL WITH GARCH ERRORS: MONTE CARLO SIMULATION AND APPLICATION

Koichi Maekawa[*]
Hiroshima University of Economics
kc-mae@hue.ac.jp

Kusdhianto Setiawan[#]
Faculty of Economics and Business
Universitas Gadjah Mada
s.kusdhianto@ugm.ac.id

ABSTRACT

The standard vector error correction (VEC) model assumes the iid normal distribution of disturbance term in the model. This paper extends this assumption to include GARCH process. We call this model as VEC-GARCH model. However as the number of parameters in a VEC-GARCH model is large, the maximum likelihood (ML) method is computationally demanding. To overcome these computational difficulties, this paper searches for alternative estimation methods and compares them by Monte Carlo simulation. As a result a feasible generalized least square (FGLS) estimator shows comparable performance to ML estimator. Furthermore an empirical study is presented to see the applicability of the FGLS.

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This manuscript consists of two parts. On Part I we developed the estimation method and on Part II we applied the new method for testing international CAPM.

(*) the author of part I of this manuscript

(#) the co-author of part I and author of part II of this manuscript, presenter of the paper in the conference

PART I

ESTIMATION METHOD DEVELOPMENT

1. INTRODUCTION

Vector Error correction (VEC) model is often used in econometric analysis and estimated by maximum likelihood (ML) method under the normality assumption. ML estimator is known as the most efficient estimator under the *iid* normality assumption. However there are disadvantages such that the normality assumption is often violated in real date, especially in financial time series, and that ML estimation is computationally demanding for a large model. Furthermore in our experience of empirical study error terns in VEC model often show a GARCH phenomenon, which violates *iid* assumption. To overcome these disadvantages and to reduce computational burden of ML estimator it may be worthwhile to reconsider the feasible generalized least square (FGLS) estimator instead of ML estimator (MLE) because FGLS method is relatively free from the distributional assumptions and ease computational burden.

The purpose of this paper is to examine the finite sample properties of FGLS estimator in VEC-GARCH model by Monte Carlo simulation and by real data analysis of the international financial time series. The paper is organized as follows: Section 2 briefly surveys the multivariate GARCH (MGARCH hereafter) model; Section 3 describes VEC representation of the vector autoregressive (VAR) model; Section 4 presents a VEC-GARCH model and shows that this model can be estimated by FGLS within the framework of the seemingly unrelated regression (SUR) model; Section 5 examines the performance of FGLS by Monte Carlo simulation; Section 6 presents an empirical application of VEC-GARCH model and shows the applicability of FGLS; finally Section 7 gives some concluding remarks.

2. MULTIVARIATE GARCH

Multivariate GARCH model has been developed and applied in financial econometrics and numerous literature were published. The recent development in this area were surveyed by Bauwens, L., S. Laurent and J. V. K. Rombouts (2006) and T. Teräsvirta (2009) . Before investigating MGARCH model in this paper we briefly introduce MGARCH model focusing on relevant MGARCH models in our study.

2.1. vech-GARCH model

The univariate GARCH model has been generalized to *N*-variable multivariate GARCH models in many ways. The most straightforward generalization is the following *vech*-GARCH model by Bollerslev, Engle, and Woodridge (1988):

$$r_t = H_t^{1/2}\eta_t \ \ with\, E(r_t) = 0,\ \ E(\eta_t\eta_t') = I \tag{1}$$

where $r_t = (r_{1t}, \cdots, r_{it}, \cdots, r_{Nt})'$, and r_t is assumed to follows a multivariate normal distribution $N(0, H_t)$. An element of the variance covariance matrix H_t is denoted by h_{ijt} :$H_t = \left[h_{ijt}\right]$. In the most general *vech*-GARCH model $vech(H_t)$ is given by

$$vech(H_t) = c + \sum_{j=1}^{q} A_j\, vech\left(r_{t-j}r_{t-j}'\right) + \sum_{j=1}^{p} B_j\, vech\left(H_{t-j}\right) \tag{2}$$

where $vech(\cdot)$ is an operator that vectorizes a symmetric matrix. We briefly illustrate the 2-variable case (*N*=2) for simplicity. For *N*=2 and *p*=*q*=1 $vech(H_t)$ is written as follows:

$$vech\begin{pmatrix} h_{11,t} & h_{12,t} \\ h_{21,t} & h_{22,t} \end{pmatrix} = \left(h_{11,t}, h_{21,t}, h_{22,t}\ \right)',$$

and c is an (*N*(*N*+1)/2)×1=3×1 vector, and A_j and B_j are *N*(*N*+1)/2×*N*(*N*+1)/2 =3×3 parameter matrices. Then $vech(H_t) \coloneqq h_t$ is written as

$$h_t = \begin{bmatrix} h_{11,t} \\ h_{21,t} \\ h_{22,t} \end{bmatrix}$$

$$= \begin{bmatrix} c_{01} \\ c_{02} \\ c_{03} \end{bmatrix} + \begin{bmatrix} a_{11} & a_{12} & a_{13} \\ a_{21} & a_{22} & a_{23} \\ a_{31} & a_{32} & a_{33} \end{bmatrix} \begin{bmatrix} r_{1,t-1}^2 \\ r_{1,t-1}r_{2,t-1} \\ r_{2,t-1}^2 \end{bmatrix} + \begin{bmatrix} b_{11} & b_{12} & b_{13} \\ b_{21} & b_{22} & b_{23} \\ b_{31} & b_{32} & b_{33} \end{bmatrix} \begin{bmatrix} h_{11,t-1} \\ h_{12,t-1} \\ h_{22,t-1} \end{bmatrix} \quad (3)$$

or

$$\begin{aligned} h_{11,t} &= c_{01} + a_{11}r_{1,t-1}^2 + a_{12}r_{1,t-1}r_{2,t-1} + a_{13}r_{2,t-1}^2 \\ &\quad + b_{11}h_{11,t-1} + b_{12}h_{12,t-1} + b_{13}h_{22,t-1} \\ h_{21,t} &= c_{02} + a_{21}r_{1,t-1}^2 + a_{22}r_{1,t-1}r_{2,t-1} + a_{23}r_{2,t-1}^2 \\ &\quad + b_{21}h_{11,t-1} + b_{22}h_{12,t-1} + b_{23}h_{22,t-1} \\ h_{22,t} &= c_{03} + a_{31}r_{1,t-1}^2 + a_{32}r_{1,t-1}r_{2,t-1} + a_{33}r_{2,t-1}^2 \\ &\quad + b_{31}h_{11,t-1} + b_{32}h_{21,t-1} + b_{33}h_{22,t-1} \end{aligned}$$

This representation is very general and flexible but there is a disadvantage that only a sufficient condition for the positive definiteness of the matrix H_t is known. Furthermore the number of parameters is $(p+q)(N(N+1)/2)^2 + N(N+1)/2$ which is large unless N is small. For example, if $p = q = 1$ and $N = 2$, the number of parameters is 21, if N=3 it is 78. This may cause computational difficulties.

2.2. Diagonal vech model

To reduce such disadvantages mentioned above Bollerslev, Engle, and Wooldridge (1988) proposed diagonal *vech* model in which the coefficient matrices A_j and B_j are assumed diagonal. In this case the number of parameters is reduced to $(p+q+1)N(N+1)/2)$. For example, if $p = q = 1$ and $N = 2$ then the number is 9, and if N=3 it is 8. Furthermore, in this case the necessary and sufficient conditions for the positive definiteness of H_t are obtained by Bollerslev, Engle, and Nelson (1994). The variance equation (3) is simplified as follows:

$$h_t = \begin{bmatrix} h_{11,t} \\ h_{21,t} \\ h_{22,t} \end{bmatrix} = \begin{bmatrix} c_{01} \\ c_{02} \\ c_{03} \end{bmatrix} + \begin{bmatrix} a_{11} & 0 & 0 \\ 0 & a_{22} & 0 \\ 0 & 0 & a_{33} \end{bmatrix} \begin{bmatrix} r_{1,t-1}^2 \\ r_{1,t-1}r_{2,t-1} \\ r_{2,t-1}^2 \end{bmatrix} + \begin{bmatrix} b_{11} & 0 & 0 \\ 0 & b_{22} & 0 \\ 0 & 0 & b_{33} \end{bmatrix} \begin{bmatrix} h_{11,t-1} \\ h_{21t-1} \\ h_{22,t-1} \end{bmatrix}$$

or

$$\begin{aligned} h_{11,t} &= c_{01} + a_{11}r_{1,t-1}^2 + b_{11}h_{11,t-1} \\ h_{21,t} &= c_{02} + a_{22}r_{1,t-1}r_{2,t-1} + b_{22}h_{21,t-1} \\ h_{22,t} &= c_{02} + a_{22}r_{2,t-1}^2 + b_{33}h_{22,t-1} \end{aligned}$$

2.3. BEKK model

To ensure the positive definiteness of H_t Engle and Kroner (1995) proposed a following model called as Baba-Engle-Kraft-Kroner (BEKK) model.

$$\mathrm{H_t} = \mathrm{CC'} + \sum_{j=1}^{q}\sum_{k=1}^{K} \mathrm{A'_{kj}}\, \mathrm{r_{t-j}r'_{t-j}A_{kj}} + \sum_{j=1}^{q}\sum_{k=1}^{K} \mathrm{B'_{kj}H_{t-j}B_{kj}} \tag{4}$$

where A_{kj}, B_{kj}, C are $N{\times}N$ coefficient matrices, C is a lower triangular matrix. Although this decomposition of the constant term can ensure the positive definiteness of H_t, which is the advantage of this model, the number of parameters is quite large. Because of this, estimation of this model is often infeasible for a large model. When K=1 this model is written as

$$H_t = CC' + A'r_{t-1}r'_{t-1}A + B'H_{t-1}B \tag{5}$$

In this case the number of parameters is $np = (\mathrm{p}+\mathrm{q})\mathrm{N}^2 + \mathrm{N}(\mathrm{N}+1)/2$. If $p = q = 1$ and N=2, then $np = 11$, and $np = 24$ for N=3. If $N \geq 4$ it may not be feasible to estimate this model. To reduce number of parameters it is common and popular to assume that the coefficient matrices A, B are diagonal. This model is called Diagonal BEKK model (Engle and Kroner (1995)). In this model $np = (p+q)N + N(N+1)/2$. If $p = q = 1$ and N=2, then $n\mathrm{p} = 7$, and $np = 12$ for N=3. For small size Diagonal BEKK model the calculation is feasible. However, even in Diagonal BEKK model, np will be large when N is not small. For example, np=35 when $p = q = 1$ and N=5.

We illustrate several versions of (5) for a simple case N=2 and K=1:

Unrestricted BEKK. In this case the variance covariance matrix $\mathrm{H_t} = \begin{bmatrix} \mathrm{h_{11,t}} & \mathrm{h_{12,t}} \\ \mathrm{h_{21,t}} & \mathrm{h_{22,t}} \end{bmatrix}$ is expressed as

$$\mathrm{H_t} = \begin{bmatrix} \mathrm{c_{11}} & 0 \\ \mathrm{c_{21}} & \mathrm{c_{22}} \end{bmatrix}\begin{bmatrix} \mathrm{c_{11}} & \mathrm{c_{21}} \\ 0 & \mathrm{c_{22}} \end{bmatrix} + \begin{bmatrix} \mathrm{a_{11}} & \mathrm{a_{12}} \\ \mathrm{a_{21}} & \mathrm{a_{22}} \end{bmatrix}' \begin{bmatrix} \mathrm{r^2_{1,t-1}} & \mathrm{r_{1,t-1}r_{2,t-1}} \\ \mathrm{r_{2,t-1}r_{1,t-1}} & \mathrm{r^2_{2,t-1}} \end{bmatrix}\begin{bmatrix} \mathrm{a_{11}} & \mathrm{a_{12}} \\ \mathrm{a_{21}} & \mathrm{a_{22}} \end{bmatrix} + \begin{bmatrix} b_{11} & b_{12} \\ b_{21} & b_{22} \end{bmatrix}' H_{t-1} \begin{bmatrix} b_{11} & b_{12} \\ b_{21} & b_{22} \end{bmatrix}$$

or

$$
\begin{aligned}
h_{11,t} &= c_{11}^2 + a_{11}^2 r_{1,t-1}^2 + 2a_{11}a_{21}r_{1,t-1}r_{2,t-1} + a_{21}^2 r_{2,t-1}^2 + b_{11}^2 h_{11,t-1} \\
&\quad +2b_{11}b_{21}h_{12,t-1} + b_{21}^2 h_{22,t-1} \\
h_{12,t} &= c_{11}c_{21} + a_{11}a_{12}r_1^2 + \left(a_{21}a_{12} + a_{11}a_{22}\right)r_1 r_2 + a_{21}a_{12}r_2^2 + b_{11}b_{12}h_{11,t-1} \\
&\quad +\left(b_{21}b_{12} + b_{11}b_{22}\right)h_{12,t-1} + b_{21}b_{12}h_{22,t-1} \\
h_{22,t} &= c_{12}^2 + c_{22}^2 + a_{12}^2 r_1^2 + 2a_{12}a_{22}r_1 r_2 + a_{22}^2 r_2^2 + b_{12}^2 h_{11,t-1} \\
&\quad +2b_{12}b_{22}h_{12,t-1} + b_{22}^2 r_2^2
\end{aligned}
$$

where H_t is positive definite by construction.

Diagonal BEKK is expressed as

$$
\begin{aligned}
\mathrm{H_t} = &\begin{bmatrix} \mathrm{c_{11}} & 0 \\ 0 & \mathrm{c_{22}} \end{bmatrix}\begin{bmatrix} \mathrm{c_{11}} & 0 \\ 0 & \mathrm{c_{22}} \end{bmatrix} + \begin{bmatrix} \mathrm{a_{11}} & 0 \\ 0 & \mathrm{a_{22}} \end{bmatrix}' \begin{bmatrix} \mathrm{r_{1,t-1}^2} & \mathrm{r_{1,t-1}r_{2,t-1}} \\ \mathrm{r_{2,t-1}r_{1,t-1}} & \mathrm{r_{2,t-1}^2} \end{bmatrix}\begin{bmatrix} \mathrm{a_{11}} & 0 \\ 0 & \mathrm{a_{22}} \end{bmatrix} \\
&+ \begin{bmatrix} b_{11} & 0 \\ 0 & b_{22} \end{bmatrix}' H_{t-1} \begin{bmatrix} b_{11} & 0 \\ 0 & b_{22} \end{bmatrix}
\end{aligned}
$$

or

$$
\begin{aligned}
h_{11,t} &= c_{11}^2 + a_{11}^2 r_{1,t-1}^2 + b_{11}^2 h_{11,t-1} \\
h_{12,t} &= a_{11}a_{22}r_{1,t-1}r_{2,t-1} + b_{11}b_{22}h_{12,t-1} \\
h_{22,t} &= c_{22}^2 + a_{22}^2 r_{2,t-1}^2 + b_{22}^2 h_{2,t-1}
\end{aligned}
$$

where $h_{ij,t}$ in these variance covariance equations only depend on their own lagged values $h_{ij,t-1}$.

Engle and Kroner (1995) shows that the diagonal *vech* and the diagonal BEKK are equivalent as follows: By stacking the diagonal elements of A and B of the diagonal vech model, i.e.,

$$\alpha = (a_{11}, a_{22}, a_{33})', \qquad \beta = (b_{11}, b_{22}, b_{33})'$$

and write

$$\Sigma_\mathrm{t} = \mathrm{M} + \alpha\alpha' \odot \mathrm{r_{t-1}} \mathrm{r'_{t-1}} + \beta\beta' \odot \Sigma_{\mathrm{t-1}}$$

then it is easy to see that $vech(\Sigma_\mathrm{t})$ is identical to the diagonal *vech*.

There are many other types of multivariate GARCH model. They are surveyed, for example, in Bauwens, L., S. Laurent and J. V. K. Rombouts (2006) and Silvennoinen and Terasvirta (2009).

Bollerslev, Engle, and Wooldridge (1988) introduced a restricted version of the general multivariate *vec* model of GARCH with following representation:

$$\mathrm{H_t} = \Omega + \mathrm{A} \odot \mathrm{r_{t-1}r'_{t-1}} + \mathrm{B} \otimes \mathrm{H_{t-1}}$$

where the operator $\odot$ is the Hadamard product and $\otimes$ is Kronecker Product. To ensure the positive semi-definiteness (PSD) there are several ways for specifying coefficient matrices. One example is to specify Ω, A, and B as products of Cholesky factorized triangular matrices. Such parameterization will be used in the latter section in this paper.

2.4. Log-likelihood function of vech-GARCH

If the distribution of errors η_t is a multivariate normal, then the log-likelihood function of (1) is given by

$$\sum_{t=1}^{T} l_t(\theta) = c - \frac{1}{2}\sum_{t=1}^{T} ln|H_t| - \frac{1}{2}\sum_{t=1}^{T} r_t' H_t^{-1} r_t \quad (6)$$

In calculating MLE we have to invert H_t at every time *t*. This is computationally tedious when *T* and *N* are not small. Furthermore H_t is often noninvertible.

3. VEC REPRESENTATION OF VAR MODEL

We consider *M*-variate and *k*-th order vector autoregressive time series $Y_t = [y_{1,t} \dots y_{i,t} \dots y_{M,t}]'$

$$\mathrm{Y_t} = \Pi_1 \mathrm{Y_{t-1}} + \cdots + \Pi_k \mathrm{Y_{t-k}} + \epsilon_t \quad (7)$$

This model is called Vector Autoregressive (VAR) Model. The subscript *t* denotes time: $t = 1,2,\cdots,n$. The errors ϵ_t are assumed to follows *iid* *M*-dimensional multivariate normal distribution $N(0, \Sigma)$. Note that Σ does not depend on time *t*. Later in this paper we consider the time dependent case, i.e., Σ_t. Now by introducing a $M \times M$ matrix Π defined by

$$\Pi = I_p - \Pi_1 - \cdots - \Pi_k$$

We can rewrite (7) as

$$\Delta Y_t = C^0 + \Pi Y_{t-1} + \Phi \Delta Y_{t-1} + \varepsilon_t \quad (8)$$

where,

Y_{t-1} $= \left[y_{1,t-1} \dots y_{i,t-1} \dots y_{M,t-1}\right]'$: a vector of first order lagged of Y_t.

ΔY_t $= \left[\Delta y_{1,t}\ \Delta y_{2,t} \dots \Delta y_{i,t} \dots \Delta y_{M,t}\right]'$: a vector of first difference of Y_t at time *t*.

C^0 $= [c_1^0\ c_2^0 \dots c_i^0 \dots c_M^0]'$: a vector of constant terms.

ε_t $= [\varepsilon_1\ \varepsilon_2 \dots \varepsilon_M]'$: a vector of disturbance errors which is assumed *iid M*-dimensional multivariate normal distribution $N(0, \Sigma)$.

In what follows we consider a case in which all elements in Y_t are I(1). In this case as the left hand side variables $\Delta \mathrm{Y_t}$ are sationary I(0) the right hand side of (8) should be also stationary. To ensure the stationarity of the right hand side of (8), the rank of the coefficient matrix Π is less than *M* or rank(Π)<*M*. The reason is as follows: if rank(Π)=*M* then there exists Π^{-1} and the equation (8) can be solved for I(1) variable Y_{t-1} as a linear combination of stationary variables $\Delta \mathrm{Y_t}$ and $\Delta \mathrm{Y_{t-1}}$. This is a contradiction. This is because why rank(Π)<M. Under this rank condition Π can be decomposed as follows:

$$\Pi = AB$$

where

A $= [a_1\ a_2 \dots a_i \dots a_M]'$: vector of coefficients in cointegrating equation (loading matrix that contains adjustment parameters) and,

B $= [b_1\ b_2 \dots\ b_i \dots b_M]$: a vector of cointegrating coefficient,

Φ $= \begin{bmatrix} \varphi_{11} & \cdots & \varphi_{1M} \\ \vdots & \ddots & \vdots \\ \varphi_{M1} & \cdots & \varphi_{MM} \end{bmatrix}$: a *M* by *M* matrix,

where BY_{t-1} is assured to be stationary (Granger's representation theorem). The stationarity of BY_{t-1} means that a linear combination of elements in Y_{t-1} is stationary, in such elements are called as co-integrated and B is called as co-integration vector. The coefficient matrix *A* is called as loading vector because *A* conveys cointegrated variables to the system.

4. VECTOR ERROR CORRECTION WITH GARCH ERRORS (VEC-GARCH MODEL)

4.1. VECM with BEKK errors

So far we have considered the standard Vector Error Correction Model (VECM), where a set of time series is nonstationary at level, but stationary at their first differences and $\varepsilon_t \sim iid\ N(0, \Sigma)$. Matrix Π represents the long run relationship between the variables in Equation (8) and Johansen (1988) proposed a maximum likelihood estimation of (8) for the case of the rank of matrix $\Pi = r$, where $0 < r < M$.

In what follows, we relaxed the assumption of homoscedasticity of the errors. Instead, we assume that ε_t has zero mean and time dependent variance-covariance matrix of H_t that has the BEKK GARCH structure as given by (6):

$$H_t = CC' + A' r_{t-1} r_{t-1}' A + BH_{t-1}B'.$$

4.2. SUR representation

VEC model with GARCH errors can be represented by Seemingly Unrelated Regression (SUR) model as follows. SUR representation of VEC model seems to be worthwhile to consider. For simplicity we consider three-equation VEC model such as:

$$\Delta Y_t = \Pi Y_{t-1} + \Phi \Delta Y_{t-1} + \varepsilon_t$$

or

$$\begin{bmatrix} \Delta Y_{1,t} \\ \Delta Y_{2,t} \\ \Delta Y_{3,t} \end{bmatrix} = \begin{bmatrix} \pi_{11} & \pi_{12} & \pi_{13} \\ \pi_{21} & \pi_{22} & \pi_{23} \\ \pi_{31} & \pi_{32} & \pi_{33} \end{bmatrix} \begin{bmatrix} Y_{1,t-1} \\ Y_{2,t-1} \\ Y_{3,t-1} \end{bmatrix} + \begin{bmatrix} \phi_{11} & \phi_{12} & \phi_{13} \\ \phi_{21} & \phi_{22} & \phi_{23} \\ \phi_{31} & \phi_{32} & \phi_{33} \end{bmatrix} \begin{bmatrix} \Delta Y_{1,t-1} \\ \Delta Y_{2,t-1} \\ \Delta Y_{3,t-1} \end{bmatrix} + \begin{bmatrix} \varepsilon_{1,t-1} \\ \varepsilon_{2,t-1} \\ \varepsilon_{3,t-1} \end{bmatrix},$$

for *t*=1, 2, …, *n*.

Alternatively this system can be written as

$$\begin{aligned} \Delta Y_{1,\cdot} &= Y_{-1}\Pi_1' + \Delta Y_{-1}\Phi_1' + \varepsilon_1 \\ \Delta Y_{2,\cdot} &= Y_{-1}\Pi_2' + \Delta Y_{-1}\Phi_2' + \varepsilon_2 \\ \Delta Y_{3,\cdot} &= Y_{-1}\Pi_3' + \Delta Y_{-1}\Phi_3' + \varepsilon_3 \end{aligned} \quad (9)$$

where Π_i and Φ_i are the *i*th row of Π and Φ respectively, i.e.,

$$\mathit{\Pi} = \begin{bmatrix} \pi_{11} & \pi_{12} & \pi_{13} \\ \pi_{21} & \pi_{22} & \pi_{23} \\ \pi_{31} & \pi_{32} & \pi_{33} \end{bmatrix} = \begin{bmatrix} \mathit{\Pi}_1 \\ \mathit{\Pi}_2 \\ \mathit{\Pi}_3 \end{bmatrix},\ \mathit{\Phi} = \begin{bmatrix} \phi_{11} & \square_{12} & \phi_{13} \\ \phi_{21} & \phi_{22} & \phi_{23} \\ \phi_{31} & \phi_{32} & \phi_{33} \end{bmatrix} = \begin{bmatrix} \mathit{\Phi}_1 \\ \mathit{\Phi}_2 \\ \mathit{\Phi}_3 \end{bmatrix},$$

$$Y_{-1} = \begin{pmatrix} y_{11} & y_{21} & y_{31} \\ \vdots & \vdots & \vdots \\ y_{1,n-1} & y_{2,n-1} & y_{3,n-1} \end{pmatrix}, \qquad \Delta Y_{-1} = \begin{pmatrix} \Delta y_{11} & \Delta y_{21} & \Delta y_{31} \\ \vdots & \vdots & \vdots \\ \Delta y_{1,n-1} & \Delta y_{2,n-1} & \Delta y_{3,n-1} \end{pmatrix},$$

$\Delta Y_{i,\cdot} = [\Delta Y_{i,2}, \Delta Y_{i,3}, \dots, \Delta Y_{i,t}, \dots, \Delta Y_{i,n}]'$ and,
$\varepsilon_i = [\varepsilon_{i,2}, \varepsilon_{i,3}, \dots, \varepsilon_{i,t}, \dots, \varepsilon_{i,n}]'$.

Defining new matrices X and β by

$$\mathrm{X} = [\mathrm{Y}_{-1}, \Delta\mathrm{Y}_{-1}] \text{ and } \beta' = [\beta_1', \beta_2', \beta_3'],$$

the 3-equation VEC model (8) can be written as SUR model as follows:

$$\begin{bmatrix} \Delta\mathrm{Y}_1 \\ \Delta\mathrm{Y}_2 \\ \Delta\mathrm{Y}_3 \end{bmatrix} = \begin{bmatrix} \mathrm{X} & 0 & 0 \\ 0 & \mathrm{X} & 0 \\ 0 & 0 & \mathrm{X} \end{bmatrix} \beta + \begin{bmatrix} \varepsilon_1 \\ \varepsilon_2 \\ \varepsilon_3 \end{bmatrix}.$$

We assume that $\mathrm{E}(\varepsilon) = 0$, $\mathrm{E}(\varepsilon_{\mathrm{is}}, \varepsilon_{\mathrm{it}}) = 0$ for $\mathrm{s} \neq \mathrm{t}$, and the variance and covariance $\mathrm{E}(\varepsilon_{\mathrm{it}}^2) = \mathrm{h_{iit}}$ and $\mathrm{E}\big(\varepsilon_{\mathrm{it}}, \varepsilon_{\mathrm{jt}}\big) = \mathrm{h_{ijt}}$ follow MGARCH(1,1). Let us define $\Omega = \mathrm{E}(\varepsilon\varepsilon')$, or in the complete form:

$$\Omega = \begin{bmatrix} \Omega_{11} & \Omega_{12} & \Omega_{13} \\ \Omega_{21} & \Omega_{22} & \Omega_{23} \\ \Omega_{31} & \Omega_{32} & \Omega_{33} \end{bmatrix}$$

where, Ω_{ij} is a n×n diagonal matrix where its main diagonal elements are elements of *n*-vector of $h_{ij,t}$ and zeros on the off diagonal elements and, $\Omega_{ij} = \Omega_{ji}$, i.e.,

$$\Omega_{ij} = \begin{bmatrix} h_{ij,1} & & & & \\ & \ddots & & 0 & \\ & & h_{ij,t} & & \\ & 0 & & \ddots & \\ & & & & h_{ij,n} \end{bmatrix}$$

Thus we have

$$
\Omega = \left[\begin{array}{c|c|c}
\begin{matrix} h_{11,1} & & & & \\ & \ddots & & 0 & \\ & & h_{11,t} & & \\ & 0 & & \ddots & \\ & & & & h_{11,n} \end{matrix} &
\begin{matrix} h_{12,1} & & & & \\ & \ddots & & 0 & \\ & & h_{12,t} & & \\ & 0 & & \ddots & \\ & & & & h_{12,n} \end{matrix} &
\begin{matrix} h_{13,1} & & & & \\ & \ddots & & 0 & \\ & & h_{13,t} & & \\ & 0 & & \ddots & \\ & & & & h_{13,n} \end{matrix} \\ \hline
\begin{matrix} h_{21,1} & & & & \\ & \ddots & & 0 & \\ & & h_{21,t} & & \\ & 0 & & \ddots & \\ & & & & h_{21,n} \end{matrix} &
\begin{matrix} h_{22,1} & & & & \\ & \ddots & & 0 & \\ & & h_{22,t} & & \\ & 0 & & \ddots & \\ & & & & h_{22,n} \end{matrix} &
\begin{matrix} h_{23,1} & & & & \\ & \ddots & & 0 & \\ & & h_{23,t} & & \\ & 0 & & \ddots & \\ & & & & h_{23,n} \end{matrix} \\ \hline
\begin{matrix} h_{31,1} & & & & \\ & \ddots & & 0 & \\ & & h_{31,t} & & \\ & 0 & & \ddots & \\ & & & & h_{31,n} \end{matrix} &
\begin{matrix} h_{32,1} & & & & \\ & \ddots & & 0 & \\ & & h_{32,t} & & \\ & 0 & & \ddots & \\ & & & & h_{32,n} \end{matrix} &
\begin{matrix} h_{33,1} & & & & \\ & \ddots & & 0 & \\ & & h_{33,t} & & \\ & 0 & & \ddots & \\ & & & & h_{33,n} \end{matrix}
\end{array}\right]
$$

where $h_{ij,t}$ follow multivariate MGARCH(1,1) process.
After obtaining an estimate $\widehat{\Omega}$, we have FGLS,

$$\widehat{\beta} = [X'\widehat{\Omega}^{-1}X]^{-1}X'\widehat{\Omega}^{-1}\Delta Y.$$

Note that inverting a large and sparse matrix Ω often causes computational problems such as memory size, computer time, and inaccurate numerical results. To avoid those problems we propose the following algorithm: After estimating MGARCH process we construct a relatively small matrix $\widehat{H}_t$ and its inverse $\widehat{H}_t^{-1}$ at each time *t* such that,

$$\widehat{H}_t = \begin{bmatrix} \hat{h}_{11,t} & \hat{h}_{12,t} & \hat{h}_{13,t} \\ \hat{h}_{21,t} & \hat{h}_{22,t} & \hat{h}_{23,t} \\ \hat{h}_{31,t} & \hat{h}_{32,t} & \hat{h}_{33,t} \end{bmatrix}, \text{and } \widehat{H}_t^{-1} = \begin{bmatrix} \hat{h}_t^{11} & \hat{h}_t^{12} & \hat{h}_t^{13} \\ \hat{h}_t^{21} & \hat{h}_t^{22} & \hat{h}_t^{23} \\ \hat{h}_t^{31} & \hat{h}_t^{32} & \hat{h}_t^{33} \end{bmatrix} \tag{10}$$

where $\widehat{H}_t$ and $\hat{h}_{ij,t}$ are estimated variance covariance of MGARCH.

Replacing $\hat{h}_{ij,t}$ with $\hat{h}_t^{ij}$ in $\widehat{\Omega}$ we have easily obtain $\widehat{\Omega}^{-1}$ without inverting a large matrix Ω.

5. MONTE CARLO SIMULATION

5.1 Data generating Process (DGP)

Monte Carlo simulation is carried out by generating artificial data of three series. The data generating process (DGP) is repeated for 1000 times. We run the simulation for the number of observations *n* : 100, 300 and 500. For removing the initial value effect, we generate 2*n* observations for each series and remove the first half of the generated data in each simulation run. The true model for generating the data is specified as follows:

$$Y_t = PY_{t-1} + QY_{t-2} + U_t \tag{11}$$

or in stacked model it can be restated as,

$$\begin{bmatrix} y_{1t} \\ y_{2t} \\ y_{3t} \end{bmatrix} = \begin{bmatrix} p_{11} & p_{12} & p_{13} \\ p_{21} & p_{22} & p_{23} \\ p_{31} & p_{32} & p_{33} \end{bmatrix} \begin{bmatrix} y_{1t-1} \\ y_{2t-1} \\ y_{3t-1} \end{bmatrix} + \begin{bmatrix} q_{11} & q_{12} & q_{13} \\ q_{21} & q_{22} & q_{23} \\ q_{31} & q_{32} & q_{33} \end{bmatrix} \begin{bmatrix} y_{1t-2} \\ y_{2t-2} \\ y_{3t-2} \end{bmatrix} + \begin{bmatrix} u_{1t} \\ u_{2t} \\ u_{3t} \end{bmatrix}$$

where U_t follow GARCH process, $U_t \sim N(0, H_t)$ and H_t follows the diagonal BEKK:

$$H_\mathrm{t} = \mathrm{M}^* + \alpha^* \odot \varepsilon'_{\mathrm{t}-1}\varepsilon_{\mathrm{t}-1} + \beta^* \odot \mathrm{H}_{\mathrm{t}-1}$$

with

$$\alpha^* = \begin{bmatrix} \alpha_{11}^2 & \alpha_{11}\alpha_{22} & \alpha_{11}\alpha_{33} \\ \alpha_{11}\alpha_{22} & \alpha_{22}^2 & \alpha_{22}\alpha_{33} \\ \alpha_{11}\alpha_{33} & \alpha_{22}\alpha_{33} & \alpha_{33}^2 \end{bmatrix} = \begin{bmatrix} 0.090 & 0.180 & 0.060 \\ 0.180 & 0.360 & 0.120 \\ 0.060 & 0.120 & 0.040 \end{bmatrix}$$

$$\beta^* = \begin{bmatrix} \beta_{11}^2 & \beta_{11}\beta_{22} & \beta_{11}\beta_{33} \\ \beta_{11}\beta_{22} & \beta_{22}^2 & \beta_{22}\beta_{33} \\ \beta_{11}\beta_{33} & \beta_{22}\beta_{33} & \beta_{33}^2 \end{bmatrix} = \begin{bmatrix} 0.090 & 0.150 & 0.120 \\ 0.150 & 0.250 & 0.200 \\ 0.120 & 0.200 & 0.160 \end{bmatrix}$$

$$\mathrm{M}^* = \begin{bmatrix} m_{11}^2 & 0 & 0 \\ 0 & m_{22}^2 & 0 \\ 0 & 0 & m_{33}^2 \end{bmatrix} = \begin{bmatrix} 0.025 & 0 & 0 \\ 0 & 0.090 & 0 \\ 0 & 0 & 0.049 \end{bmatrix}$$

α^*, β^* are transformed matrices of $\alpha'\alpha$ and $\beta'\beta$ where where α and β are [0.3,0.6,0.2], [0.3,0.5,0.4] respectively. M* is a transformed matrix of M'M where M is a diagonal matrix with its diagonal elements are [0.5,0.3,0.7]. Equivalently, the variance-covariance equations are as follow:

$$h_{11\mathrm{t}} = 0.025 + 0.09u_{1t}^2 + 0.09h_{11,t-1}$$
$$h_{21\mathrm{t}} = 0.18u_{1t}u_{2t} + 0.15h_{21,t-1}$$

$$
\begin{aligned}
h_{31\mathrm{t}} &= 0.06u_{3t}u_{1t} + 0.12h_{31,t-1} \\
h_{22\mathrm{t}} &= 0.09 + 0.36u_{2t}^2 + 0.25h_{22,t-1} \\
h_{32\mathrm{t}} &= 0.12u_{3t}u_{2t} + 0.2h_{32,t-1} \\
h_{33\mathrm{t}} &= 0.049 + 0.04u_{3t}^2 + 0.16h_{33,t-1}
\end{aligned}
$$

Equation (11) can be rewritten as Vector Error Correction Model (VECM):

$$\Delta Y_t = \Pi Y_{t-1} + \phi \Delta Y_{t-1} + U_t \tag{12}$$

where $\Pi = P + Q - I$ and $\phi = -Q$. The true values of P and Q are set as follow:

$$P = \begin{bmatrix} 0.7 & 0 & 0 \\ 0 & 0.3 & 0 \\ 0.5 & 0 & 0.4 \end{bmatrix} \text{ and } Q = \begin{bmatrix} 0.3 & 0 & 0 \\ 0 & 0.7 & 0 \\ 0.5 & 1 & 0.1 \end{bmatrix}$$

thus $\Pi = \begin{bmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 1 & 1 & -0.5 \end{bmatrix}$ which can be decomposed into loading vector $[0 \quad 0 \quad 1]'$ and cointegrating vector $[1 \quad 1 \quad -0.5]$.

Before we generate Y_t, we have to generate U_t ~N(0, H_t) as follows.

Step 1. Generate $\varepsilon_t = \begin{bmatrix} \varepsilon_{1t} \\ \varepsilon_{2t} \\ \varepsilon_{3t} \end{bmatrix} \sim N(0, I)$

Step 2. Generate H_t using Diagonal BEKK model from ε_t

Step 3. Transform ε_t to U_t by applying Cholesky Decomposition: $U_t = L_t \varepsilon_t$, where L_t is lower triangular matrix obtained from decomposing $H_t = L_t {L_t}'$.

By construction, the positive definiteness (PD) of H_t is assured.

5.2. Estimation Strategy

Under the above DGP we carried out Monte Carlo simulation for the following five cases:

Case 1 (OLS): We estimate parameters equation by equation in equation (9) by OLS without considering GARCH error structure and obtain the followings:

$$
\begin{aligned}
\Delta Y_{1,\cdot} &= Y_{-1}\widehat{\Pi}_1' + \Delta Y_{-1}\widehat{\Phi}_1' + \hat{u}_1 \\
\Delta Y_{2,\cdot} &= Y_{-1}\widehat{\Pi}_2' + \Delta Y_{-1}\widehat{\Phi}_2' + \hat{u}_2 \\
\Delta Y_{3,\cdot} &= Y_{-1}\widehat{\Pi}_3' + \Delta Y_{-1}\widehat{\Phi}_3' + \hat{u}_3
\end{aligned}
$$

Case 2 (VECM): We estimate parameters in equation (12) by VECM system equation without considering GARCH error structure and obtain the followings:

$$\Delta Y_t = \widehat{\Pi} Y_{t-1} + \widehat{\Phi} \Delta Y_{t-1} + \widehat{U}_t$$

Case 3 (FGLS-OLS-GARCH/FOLSH): First we calculate OLS residuals $\hat{u}_i$ for each equations without considering GARCH error structure as in Case 1. Next, we use $\hat{u}_i$ for obtaining variance covariance matrix $\widehat{H}_t$ and $\widehat{H}_t^{-1}$ in the diagonal BEKK model. Having $\widehat{H}_t$ and $\widehat{H}_t^{-1}$ in hand we can construct $\widehat{\Omega}$ and $\widehat{\Omega}^{-1}$ to have feasible generalized least square (FGLS) estimator.

Case 4 (FGLS-VECM-GARCH/FVECH): We use VECM system equations as in Case 2 for estimating $\widehat{\Omega}$. First we obtain each residual $\tilde{u}_i$ from VECM in Case 2. Next, we use $\tilde{u}_i$ for obtaining variance covariance matrix $\widehat{H}_t$ and $\widehat{H}_t^{-1}$ in the diagonal BEKK model. Having $\widehat{H}_t$ and $\widehat{H}_t^{-1}$ in hand we can construct $\widehat{\Omega}$ and $\widehat{\Omega}^{-1}$ to have feasible generalized least squre (FGLS) estimator.

Case 5 (MLE): We estimate all parameters in the mean equation (12) and the diagonal BEKK variance equation (5) by MLE and obtain the estimated system as follows:

Mean equation: $\Delta \mathrm{Y}_{\mathrm{t}} = \widehat{\Pi} \mathrm{Y}_{\mathrm{t}-1} + \widehat{\Phi} \Delta \mathrm{Y}_{\mathrm{t}-1} + \widehat{U}_{\mathrm{t}}$

Variance equation: $\widehat{H}_t = \hat{C}\hat{C}' + \hat{A}'\widehat{U}_{t-1}\widehat{U}_{t-1}'\hat{A} + \hat{B}'\widehat{H}_{t-1}\hat{B}$

or equivalently the variance-covariance equations are as follow:

$$\begin{aligned}
\hat{h}_{11\mathrm{t}} &= \widehat{\mathrm{m}}_{11}^2 + \hat{a}_{11}^2 \hat{u}_{1\mathrm{t}-1}^2 + \hat{b}_{11}^2 \hat{h}_{11,t-1} \\
\hat{h}_{21\mathrm{t}} &= \hat{a}_{22}\hat{a}_{11}\hat{u}_{2t}\hat{u}_{1t-1} + \hat{b}_{22}\hat{b}_{11}\hat{h}_{21,t-1} \\
\hat{h}_{31\mathrm{t}} &= \hat{a}_{33}\hat{a}_{11}\hat{u}_{3t}\hat{u}_{1t-1} + \hat{b}_{33}\hat{b}_{11}\hat{h}_{31,t-1} \\
\hat{h}_{22\mathrm{t}} &= \widehat{\mathrm{m}}_{22}^2 + \hat{a}_{22}^2 \hat{u}_{2\mathrm{t}-1}^2 + \hat{b}_{22}^2 \hat{h}_{22,t-1} \\
\hat{h}_{32\mathrm{t}} &= \hat{a}_{33}\hat{a}_{22}\hat{u}_{3t}\hat{u}_{2t-1} + \hat{b}_{33}\hat{b}_{22}\hat{h}_{32,t-1} \\
\hat{h}_{33\mathrm{t}} &= \widehat{\mathrm{m}}_{33}^2 + \hat{a}_{33}^2 \hat{u}_{3\mathrm{t}-1}^2 + \hat{b}_{33}^2 \hat{h}_{33,t-1}
\end{aligned}$$

In estimating the parameters we maximize log likelihood function as specified in Equation (6). We run the simulation in Eviews program (version 7.2). For Case 5, in order to starting the iteration, the initial values of VECM parameters (the mean equation) were set based on single OLS equations as in Case 1. Meanwhile, the initial values for MGARCH parameters in the variance equations were set based on univariate GARCH.

5.2. Simulation Results

The main estimation methods under investigation in this paper are FGLS-based estimator (FOLSH and FVECH) and Maximum Likelihood Estimator (MLE). These strategies are taking into account the presence of MGARCH error structure. Presumably, the strategies are expected to outperform the other strategies that neglect the MGARCH error structure (OLS and VECM). Summary of simulation results is presented in Table 1. From the table, we observed that estimation methods FOLSH, FVECH, and MLE seem to outperform the other methods (OLS and VECM); the mean of the estimated parameter from 1000 times simulation run tends to be closer to its true value in most cases.

Table 1: Parameter Estimates from Monte Carlo Simulation

n=100											
		OLS		VECM		FOLSH		FVECH		MLE	
Parameters	True Value	Mean	Std.Dev.	Mean	Std.Dev.	Mean	Std.Dev.	Mean	Std.Dev.	Mean	Std.Dev.
$\hat{\pi}_{11}$	0.000	-0.048	0.082	-0.011	0.074	-0.043	0.082	-0.042	0.081	-0.038	0.078
$\hat{\pi}_{12}$	0.000	-0.010	0.083	-0.011	0.074	-0.008	0.083	-0.008	0.084	-0.007	0.079
$\hat{\pi}_{13}$	0.000	0.005	0.040	0.005	0.037	0.003	0.040	0.003	0.040	0.003	0.037
$\hat{\varphi}_{11}$	-0.300	-0.272	0.127	-0.279	0.127	-0.275	0.129	-0.275	0.131	-0.282	0.122
$\hat{\varphi}_{12}$	0.000	-0.001	0.082	0.018	0.132	-0.002	0.085	-0.001	0.084	-0.002	0.079
$\hat{\varphi}_{13}$	0.000	0.001	0.049	-0.520	0.149	0.001	0.052	0.001	0.050	0.001	0.048
$\hat{\pi}_{21}$	0.000	-0.017	0.099	-0.019	0.090	-0.009	0.092	-0.009	0.094	-0.010	0.076
$\hat{\pi}_{22}$	0.000	-0.072	0.109	-0.020	0.091	-0.051	0.098	-0.047	0.102	-0.040	0.084
$\hat{\pi}_{23}$	0.000	0.009	0.047	0.010	0.045	0.004	0.043	0.004	0.049	0.005	0.036
$\hat{\varphi}_{21}$	0.000	0.016	0.133	0.000	0.080	0.010	0.127	0.010	0.132	0.009	0.103
$\hat{\varphi}_{22}$	-0.700	-0.647	0.101	-0.668	0.100	-0.656	0.095	-0.660	0.095	-0.670	0.087
$\hat{\varphi}_{23}$	0.000	-0.003	0.057	-1.018	0.105	-0.004	0.052	-0.003	0.053	-0.001	0.045
$\hat{\pi}_{31}$	1.000	1.026	0.101	1.025	0.102	1.027	0.109	1.026	0.109	1.026	0.109
$\hat{\pi}_{32}$	1.000	1.025	0.101	1.026	0.103	1.027	0.110	1.026	0.112	1.025	0.114
$\hat{\pi}_{33}$	-0.500	-0.512	0.048	-0.512	0.049	-0.513	0.052	-0.513	0.053	-0.513	0.052
$\hat{\varphi}_{31}$	-0.500	-0.520	0.149	0.000	0.049	-0.521	0.158	-0.523	0.157	-0.522	0.161
$\hat{\varphi}_{32}$	-1.000	-1.017	0.106	-0.003	0.056	-1.018	0.114	-1.016	0.115	-1.018	0.117
$\hat{\varphi}_{33}$	-0.100	-0.094	0.094	-0.093	0.066	-0.095	0.069	-0.093	0.071	-0.095	0.070
n=300											
		OLS		VECM		FOLSH		FVECH		MLE	
Parameters	True Value	Mean	Std.Dev.	Mean	Std.Dev.	Mean	Std.Dev.	Mean	Std.Dev.	Mean	Std.Dev.
$\hat{\pi}_{11}$	0.000	-0.021	0.042	-0.007	0.039	-0.019	0.043	-0.019	0.043	-0.016	0.037
$\hat{\pi}_{12}$	0.000	-0.006	0.042	-0.007	0.039	-0.005	0.042	-0.005	0.042	-0.004	0.038
$\hat{\pi}_{13}$	0.000	0.004	0.020	0.003	0.020	0.003	0.021	0.003	0.020	0.002	0.019
$\hat{\varphi}_{11}$	-0.300	-0.281	0.073	-0.285	0.074	-0.284	0.074	-0.285	0.073	-0.287	0.073
$\hat{\varphi}_{12}$	0.000	0.004	0.043	0.005	0.078	0.003	0.043	0.003	0.044	0.001	0.041
$\hat{\varphi}_{13}$	0.000	0.000	0.028	-0.502	0.088	0.000	0.028	0.000	0.028	0.001	0.025
$\hat{\pi}_{21}$	0.000	-0.004	0.054	-0.004	0.052	0.000	0.045	-0.001	0.045	-0.001	0.035
$\hat{\pi}_{22}$	0.000	-0.021	0.055	-0.004	0.052	-0.011	0.046	-0.011	0.046	-0.008	0.035
$\hat{\pi}_{23}$	0.000	0.002	0.026	0.002	0.026	0.000	0.022	0.000	0.022	0.000	0.017
$\hat{\varphi}_{21}$	0.000	0.005	0.078	0.004	0.043	0.001	0.070	0.002	0.069	0.002	0.054
$\hat{\varphi}_{22}$	-0.700	-0.683	0.057	-0.690	0.057	-0.688	0.052	-0.689	0.051	-0.692	0.040
$\hat{\varphi}_{23}$	0.000	-0.001	0.033	-1.002	0.059	-0.001	0.028	0.000	0.028	0.000	0.021
$\hat{\pi}_{31}$	1.000	1.003	0.056	1.003	0.056	1.002	0.055	1.002	0.056	1.002	0.057
$\hat{\pi}_{32}$	1.000	1.003	0.056	1.004	0.056	1.003	0.056	1.003	0.056	1.002	0.057
$\hat{\pi}_{33}$	-0.500	-0.502	0.027	-0.502	0.027	-0.502	0.027	-0.502	0.027	-0.501	0.028
$\hat{\varphi}_{31}$	-0.500	-0.502	0.088	0.000	0.028	-0.501	0.089	-0.501	0.089	-0.501	0.088
$\hat{\varphi}_{32}$	-1.000	-1.002	0.059	-0.001	0.032	-1.001	0.060	-1.002	0.059	-1.000	0.060
$\hat{\varphi}_{33}$	-0.100	-0.096	0.037	-0.096	0.037	-0.096	0.038	-0.096	0.038	-0.096	0.038

Table 1 (Continued): Parameter Estimates from Monte Carlo Simulation

n=500											
		OLS		VECM		FOLSH		FVECH		MLE	
Parameters	True Value	Mean	Std.Dev.	Mean	Std.Dev.	Mean	Std.Dev.	Mean	Std.Dev.	Mean	Std.Dev.
$\hat{\pi}_{11}$	0.000	-0.010	0.033	-0.002	0.032	-0.010	0.033	-0.009	0.033	-0.007	0.029
$\hat{\pi}_{12}$	0.000	-0.001	0.033	-0.002	0.031	-0.001	0.034	-0.001	0.034	0.000	0.029
$\hat{\pi}_{13}$	0.000	0.001	0.016	0.001	0.016	0.001	0.016	0.000	0.016	0.000	0.014
$\hat{\varphi}_{11}$	-0.300	-0.297	0.056	-0.299	0.056	-0.296	0.059	-0.297	0.057	-0.297	0.050
$\hat{\varphi}_{12}$	0.000	0.000	0.032	0.003	0.060	0.000	0.032	0.000	0.032	0.000	0.029
$\hat{\varphi}_{13}$	0.000	-0.001	0.022	-0.506	0.066	-0.001	0.022	-0.001	0.022	0.000	0.019
$\hat{\pi}_{21}$	0.000	-0.004	0.040	-0.004	0.040	-0.002	0.035	-0.002	0.035	-0.001	0.026
$\hat{\pi}_{22}$	0.000	-0.015	0.041	-0.004	0.040	-0.009	0.036	-0.009	0.036	-0.005	0.027
$\hat{\pi}_{23}$	0.000	0.002	0.020	0.002	0.020	0.001	0.017	0.001	0.017	0.000	0.013
$\hat{\varphi}_{21}$	0.000	0.003	0.060	0.001	0.031	0.001	0.053	0.001	0.052	0.002	0.043
$\hat{\varphi}_{22}$	-0.700	-0.691	0.044	-0.695	0.044	-0.694	0.038	-0.695	0.037	-0.696	0.029
$\hat{\varphi}_{23}$	0.000	0.001	0.025	-1.002	0.042	0.001	0.022	0.000	0.021	0.000	0.016
$\hat{\pi}_{31}$	1.000	1.004	0.039	1.004	0.039	1.004	0.040	1.003	0.040	1.005	0.044
$\hat{\pi}_{32}$	1.000	1.003	0.040	1.003	0.040	1.003	0.041	1.002	0.041	1.004	0.043
$\hat{\pi}_{33}$	-0.500	-0.502	0.020	-0.502	0.020	-0.502	0.020	-0.501	0.020	-0.502	0.022
$\hat{\varphi}_{31}$	-0.500	-0.506	0.066	-0.001	0.022	-0.504	0.066	-0.504	0.066	-0.506	0.072
$\hat{\varphi}_{32}$	-1.000	-1.002	0.042	0.001	0.025	-1.001	0.043	-1.001	0.043	-1.001	0.045
$\hat{\varphi}_{33}$	-0.100	-0.099	0.028	-0.099	0.028	-0.099	0.028	-0.099	0.028	-0.099	0.031

OLS and VECM under the heteroscedasticity condition still provide us an unbiased estimator, but their standard deviations are larger than the methods that assume MGARCH error structure. Table 2 shows that MLE, FOLSH, and FVECH are more efficient than OLS and VECM. It shows that methods ignoring the MGARCH error structure would result in less efficient estimator. All methods are consistent estimator and the efficiency measured by the Mean Squared Error (MSE) are improving when larger sample size is used.

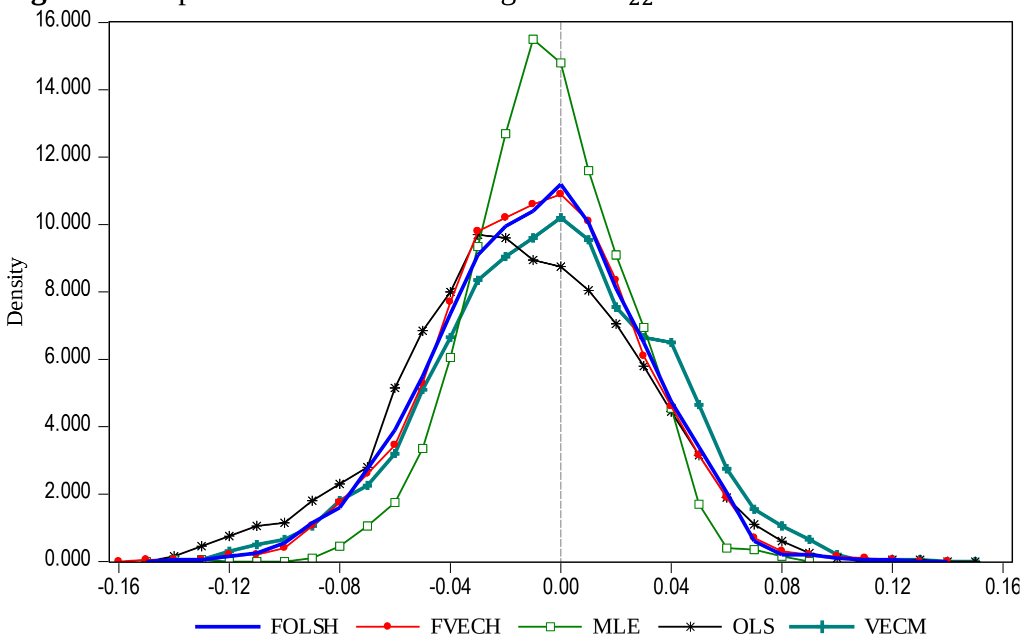
MLE is still the most efficient estimator as shown by the least average MSE in every sample size. However, MLE become computationally demanding when number of parameter is large. Table 2 shows that FGLS-based estimator (FOLSH and FVECH) perform better than OLS and VECM and only slightly inferior to MLE. It suggests that FGLS-based estimator could be useful in overcoming computation burden of the MLE. FGLS-based estimator needs to compute inverse of $\hat{\Omega}$ which is a very large and sparse matrix, but the inversion of that matrix may cause computational problems as mentioned above in Section 4. Such problems can be solved by the suggested method in that section. The algorithm for matrix inversion in most statistical software is still limited only for matrix in small dimension. We already tried to compute $\widehat{\Omega}^{-1}$ using standard command in EViews and MATLAB in our simulation, while *n*<100 FGLS-based estimators perform fairly good that comparable to MLE. However, when *n* becomes larger (i.e. n=300 and n=500), the FGLS-based estimator become poorly inefficient since it produces extreme values for the estimated parameters. All estimated parameters from FGLS-based estimators presented in this paper are based on our matrix inversion procedure. The results based on standard matrix inversion in statistical software are not presented to save space.

Table 2: Average of Mean Squared Error

	OLS	VECM	FOLSH	FVECH	MLE
n=100	0.00970	0.15012	0.00936**	0.00958*	0.00834***
n=300	0.00280	0.14180	0.00255*	0.00253**	0.00218***
n=500	0.00154	0.14104	0.00144*	0.00141**	0.00127***

Note: *** The best estimator, ** 2nd best estimator, * 3rd best estimator

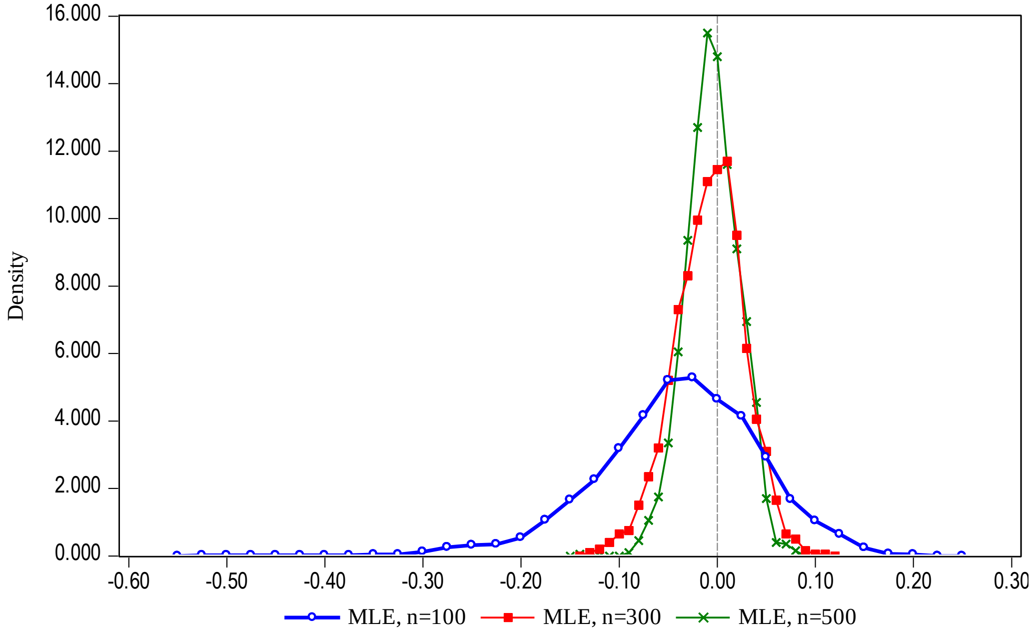
Figure 1: Empirical Distribution Histogram of $\hat{\pi}_{22}$ when n=500



Note: The true value for $\hat{\pi}_{22}$ is 0 as shown by the vertical dashed line

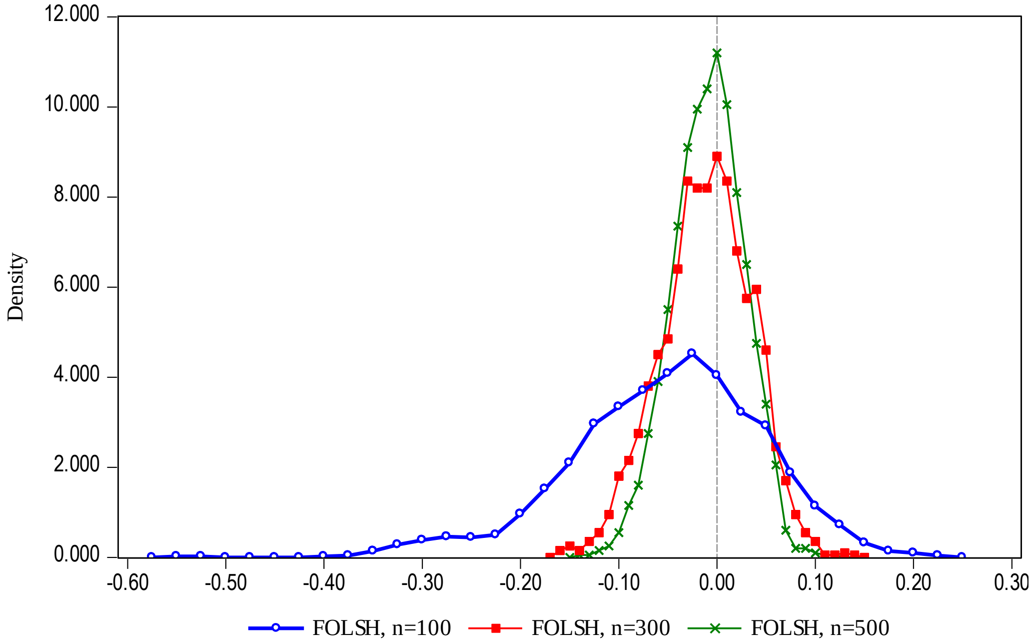
Figures 1 compares the distribution of $\hat{\pi}_{22}$ with *n*=500. The figures show that MLE is the most efficient estimator. FOLSH and FVECH have very similar efficiency as shown by the empirical distribution histogram and relatively are more efficient than OLS and VECM. The figure also shows that OLS estimator is biased to the left although the sample size is large (*n*=500).

Figure 2: Empirical Distribution Histogram of $\hat{\pi}_{22}$ by MLE



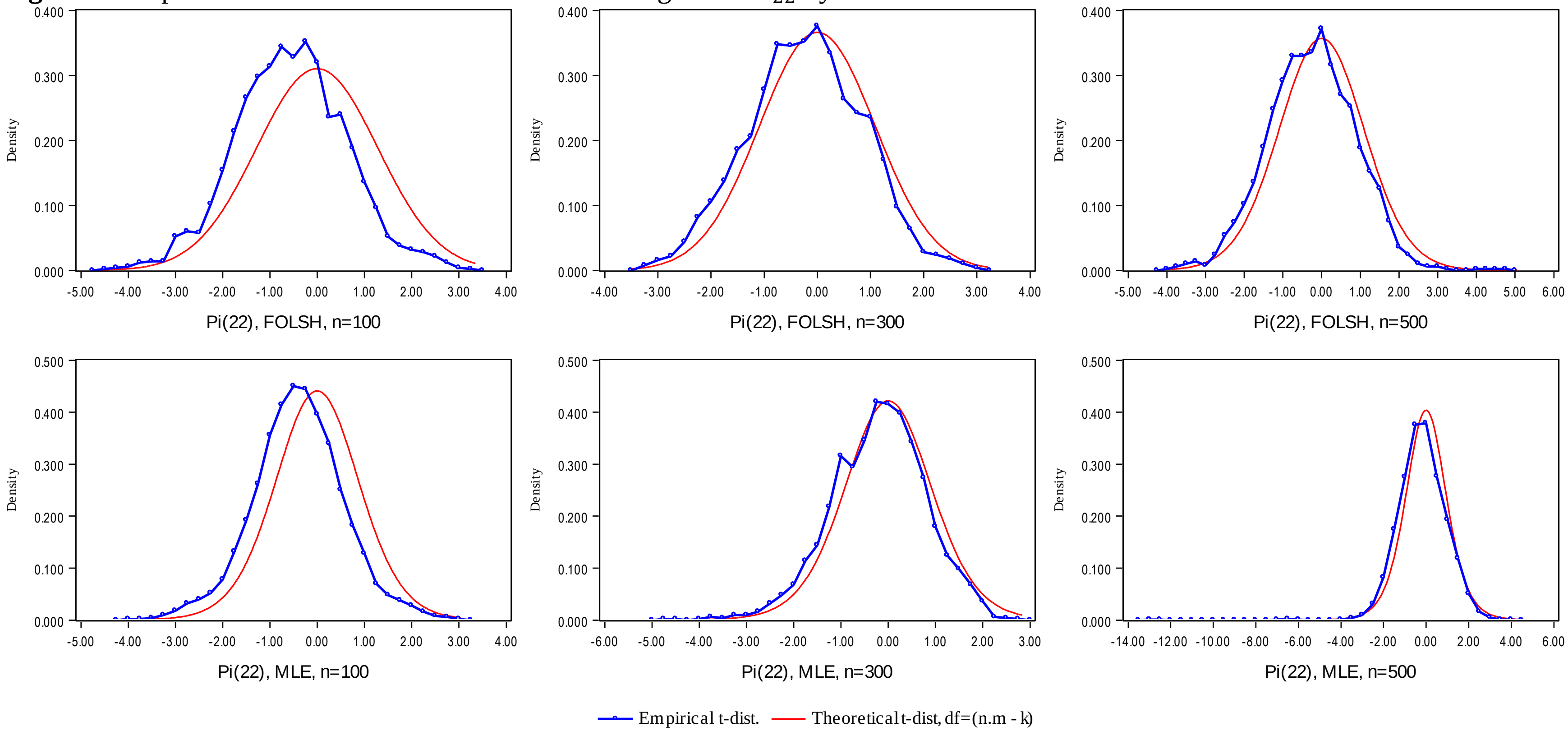
Note: The vertical dashed line indicates the true value of the parameter

Figure 3: Empirical Distribution Histogram of $\hat{\pi}_{22}$ by FOLSH



Note: The vertical dashed line indicates the true value of the parameter.

Figure 4: Empirical and Theoretical *t*-Distribution Histogram of $\hat{\pi}_{22}$ by FOLSH and MLE



Note: degree of freedom = nm-k, where n=number observation, m=number of equation (3), and k=number of parameter (18)

Table 3: Average of Rejection Rate of Null Hypothesis* Test at 5 Percent Significance Level

n=100	$\hat{\pi}_{11}$	$\hat{\pi}_{12}$	$\hat{\pi}_{13}$	$\hat{\varphi}_{11}$	$\hat{\varphi}_{12}$	$\hat{\varphi}_{13}$	$\hat{\pi}_{21}$	$\hat{\pi}_{22}$	$\hat{\pi}_{23}$	$\hat{\varphi}_{21}$	$\hat{\varphi}_{22}$	$\hat{\varphi}_{23}$	$\hat{\pi}_{31}$	$\hat{\pi}_{32}$	$\hat{\pi}_{33}$	$\hat{\varphi}_{31}$	$\hat{\varphi}_{32}$	$\hat{\varphi}_{33}$	Average
OLS	0.151	0.127	0.127	0.142	0.119	0.099	0.135	0.203	0.128	0.118	0.171	0.102	0.107	0.100	0.105	0.095	0.113	0.104	0.125
VECM	0.111	0.107	0.111	0.142	0.110	0.969	0.136	0.131	0.134	0.117	0.155	1.000	0.139	0.086	0.100	1.000	1.000	0.113	0.315
FOLSH	0.024	0.071	0.072	0.097	0.080	0.071	0.061	0.032	0.086	0.088	0.138	0.059	0.080	0.077	0.028	0.035	0.035	0.060	0.066
FVECH	0.021	0.070	0.079	0.099	0.081	0.062	0.052	0.027	0.073	0.092	0.126	0.062	0.085	0.082	0.028	0.040	0.035	0.063	0.065
MLE	0.008	0.031	0.033	0.037	0.028	0.026	0.033	0.022	0.043	0.043	0.070	0.037	0.049	0.043	0.015	0.018	0.012	0.023	0.032
n=300	$\hat{\pi}_{11}$	$\hat{\pi}_{12}$	$\hat{\pi}_{13}$	$\hat{\varphi}_{11}$	$\hat{\varphi}_{12}$	$\hat{\varphi}_{13}$	$\hat{\pi}_{21}$	$\hat{\pi}_{22}$	$\hat{\pi}_{23}$	$\hat{\varphi}_{21}$	$\hat{\varphi}_{22}$	$\hat{\varphi}_{23}$	$\hat{\pi}_{31}$	$\hat{\pi}_{32}$	$\hat{\pi}_{33}$	$\hat{\varphi}_{31}$	$\hat{\varphi}_{32}$	$\hat{\varphi}_{33}$	Average
OLS	0.143	0.111	0.097	0.154	0.093	0.098	0.146	0.165	0.140	0.110	0.151	0.100	0.103	0.113	0.111	0.122	0.112	0.108	0.121
VECM	0.099	0.099	0.099	0.157	0.111	1.000	0.138	0.137	0.138	0.090	0.147	1.000	0.119	0.105	0.104	1.000	1.000	0.115	0.314
FOLSH	0.030	0.054	0.072	0.105	0.068	0.055	0.061	0.030	0.064	0.069	0.085	0.054	0.049	0.056	0.053	0.056	0.057	0.060	0.060
FVECH	0.026	0.048	0.075	0.102	0.070	0.062	0.063	0.031	0.062	0.075	0.087	0.053	0.049	0.055	0.051	0.057	0.054	0.062	0.060
MLE	0.015	0.030	0.041	0.075	0.043	0.038	0.045	0.026	0.053	0.071	0.077	0.042	0.038	0.042	0.048	0.049	0.053	0.046	0.046
n=500	$\hat{\pi}_{11}$	$\hat{\pi}_{12}$	$\hat{\pi}_{13}$	$\hat{\varphi}_{11}$	$\hat{\varphi}_{12}$	$\hat{\varphi}_{13}$	$\hat{\pi}_{21}$	$\hat{\pi}_{22}$	$\hat{\pi}_{23}$	$\hat{\varphi}_{21}$	$\hat{\varphi}_{22}$	$\hat{\varphi}_{23}$	$\hat{\pi}_{31}$	$\hat{\pi}_{32}$	$\hat{\pi}_{33}$	$\hat{\varphi}_{31}$	$\hat{\varphi}_{32}$	$\hat{\varphi}_{33}$	Average
OLS	0.122	0.122	0.110	0.142	0.083	0.101	0.135	0.153	0.134	0.111	0.148	0.109	0.096	0.095	0.094	0.108	0.096	0.104	0.115
VECM	0.110	0.110	0.110	0.147	0.114	1.000	0.137	0.137	0.137	0.089	0.156	1.000	0.106	0.087	0.091	1.000	1.000	0.106	0.313
FOLSH	0.031	0.073	0.072	0.074	0.045	0.055	0.061	0.037	0.076	0.075	0.072	0.069	0.052	0.050	0.036	0.044	0.041	0.040	0.056
FVECH	0.038	0.077	0.067	0.073	0.042	0.053	0.065	0.040	0.070	0.071	0.065	0.063	0.052	0.049	0.036	0.045	0.044	0.041	0.055
MLE	0.023	0.047	0.051	0.062	0.046	0.048	0.061	0.040	0.059	0.086	0.054	0.055	0.047	0.044	0.038	0.039	0.039	0.043	0.049

*The null hypothesis: the estimated parameter = its true value, the alternative hypothesis: the estimated parameter $\neq$ its true value

Figure 2 and 3 show example of empirical distribution of the estimated parameter $\hat{\pi}_{22}$ by MLE and FOLSH respectively, for n=100, 300, and 500. Those figures suggest that both ML estimator and FOLSH are consistent estimators as the estimated parameter more converge to the true value when the sample size is larger. Both MLE and FOLSH tend to be unbiased when sample size is large.

Figure 4 shows example of empirical *t*-statistic distribution for $\hat{\pi}_{22}$. From the figures, both FOLSH and MLE tend to conform to student-*t* distribution when larger sample size is used. The empirical distribution for $\hat{\pi}_{22}$ estimated by FVECH is very similar to that by FOLSH. Table 3 shows that rejection rate of null hypothesis that each parameter is equal to its true value is also close to the significance level (0.05) for parameter estimated by FOLSH, FVECH, and MLE. From the table it is also apparent that estimators that do not consider multivariate GARCH error structure (OLS and VECM) has higher rejection rate compares to those of estimators that consider the error structure (FOLSH, FVECH, and MLE). These findings show us that neglecting the presence of multivariate GARCH error structure will increase the rejection rate or the type I error.

5. EMPIRICAL APPLICATION

Weekly data from July 1997 until July 2011 of US S&P500, Japan Nikkei225 and Malaysia KLSE composite index are collected as a dataset for our model (n=732). The indexes are stated in logarithmic and are measured in US Dollar. Since they are in log index, their first order differences can be regarded as stock market return of the respective markets.

Unit root test indicates that the three time series are non-stationary at level, but they are stationary at their first difference. The Augmented Dickey Fuller (ADF) statistic (τ-*stat*.) for data in level indicates the null hypothesis that the series has unit root cannot be rejected at 10 percent significance level or less. Meanwhile, the τ-stat. for the respective series in the first order difference significantly rejects the null hypothesis of unit root at one percent significance level.

Table 4: Unit Root Test

	Level		1st Differences	
Unit Root Test	ADF τ-*stat.*	P-Value	ADF τ-*stat.*	P-Value
S&P500	-2.4618	0.1254	-29.7881	0.0000
Nikkei225	-2.4258	0.1349	-27.8684	0.0000
KLSE	-0.8080	0.8158	-28.1092	0.0000

Null Hypothesis: Series has unit root

Johansen's cointegration test was performed for the dataset, the results, as presented in Table 5, show that one cointegrating equation is found from tests based on both Trace and Maximum Eigenvalue method.

Estimation of VECM with one cointegrating equation is shown in Table 6, where Y_1, Y_2, and Y_3 correspond to log of S&P500, Nikkei225, and KLSE index respectively. From the table, it shows that coefficients of error correction for cointegrating equation are all significant to show that the stock markets have long run price relationship. In the VAR part, lagged S&P500 return has significant effect to itself and to both Japanese and Malaysian stock market returns. The results indicate that US stock market is still a very dominant market that shares its greater influence to other markets.

In addition, the significant VECM coefficients also indicate that past information (lagged variables of both price and return) can explain the present stock market returns. It implies that the stock markets are neither informationally efficient nor perfectly integrated. The importance of past information may be used for setting arbitrage strategies in the markets to exploit the market inefficiency.

Table 5: Johansen Cointegration Test

Unrestricted Cointegration Rank Test (Trace)				
Hypothesized		Trace	0.05	
No. of CE(s)	Eigenvalue	Statistic	Critical Value	Prob.**
None *	0.0436	38.0006	29.7971	0.0046
At most 1	0.0058	5.4141	15.4947	0.7634
At most 2	0.0016	1.1569	3.8415	0.2821
Unrestricted Cointegration Rank Test (Maximum Eigenvalue)				
Hypothesized		Max-Eigen	0.05	
No. of CE(s)	Eigenvalue	Statistic	Critical Value	Prob.**
None *	0.0436	32.5865	21.1316	0.0008
At most 1	0.0058	4.2572	14.2646	0.8313
At most 2	0.0016	1.1569	3.8415	0.2821

Trace and Max-eigenvalue test indicates 1 cointegrating eqn(s) at the 0.05 level
* denotes rejection of the hypothesis at the 0.05 level
**MacKinnon-Haug-Michelis (1999) p-values

Table 6: Vector Error Correction Model (VECM)

Coint.Eq.	Coef.		
Y_{1t-1}	1.000		
Y_{2t-1}	-0.682		
	(0.090)		
Y_{3t-1}	-0.024		
	(0.054)		
C	-3.700		
E.C. Eq.	ΔY_{1t}	ΔY_{2t}	ΔY_{3t}
Coint.Eq.	-0.024	0.027	0.040
	(0.009)	(0.012)	(0.015)
ΔY_{1t-1}	-0.095	0.214	0.174
	(0.041)	(0.053)	(0.069)
ΔY_{2t-1}	-0.007	-0.076	0.032
	(0.032)	(0.042)	(0.055)
ΔY_{3t-1}	0.012	-0.047	-0.076
	(0.024)	(0.031)	(0.040)
C	0.000	-0.001	0.000
	(0.001)	(0.001)	(0.002)

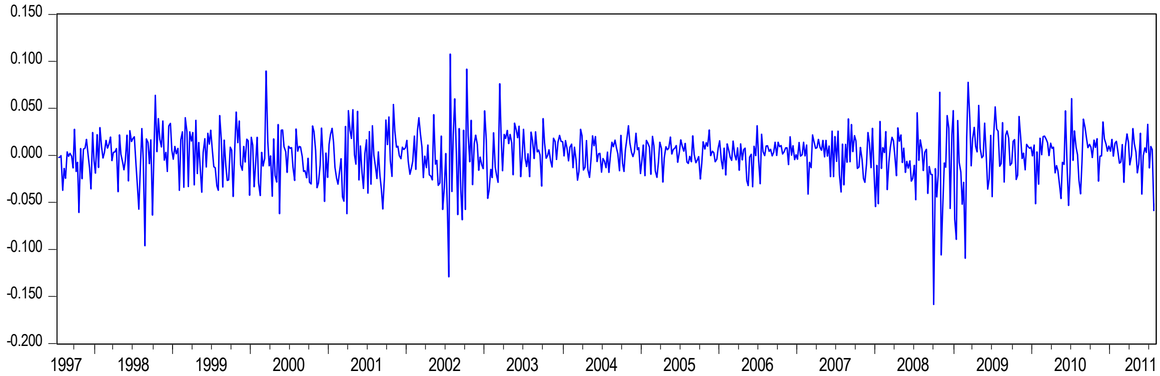
Standard Error in Parenthesis

The residuals of estimated VECM show a non-homoscedastic structure as it is shown in Figure 5. The residual of VECM can be regarded as a market shock or the unexpected return, and from the figure we can observed that during period of 1999-2002 and 2008-2009 the volatility of the US residuals were higher compared to that in the other periods. The two sub-periods are known as the burst of dot-com bubble and the collapse of financial institutions in the US market. The pattern of the Japan residuals is less clear to be connected with some events; however, it is clear that the residuals are also not homoscedastic. Meanwhile, the residuals plot of Malaysian stock market returns show that higher volatility is detected during the Asian financial crisis in 1997-1998 and also during the US financial turmoil in late 2008 until 2009.

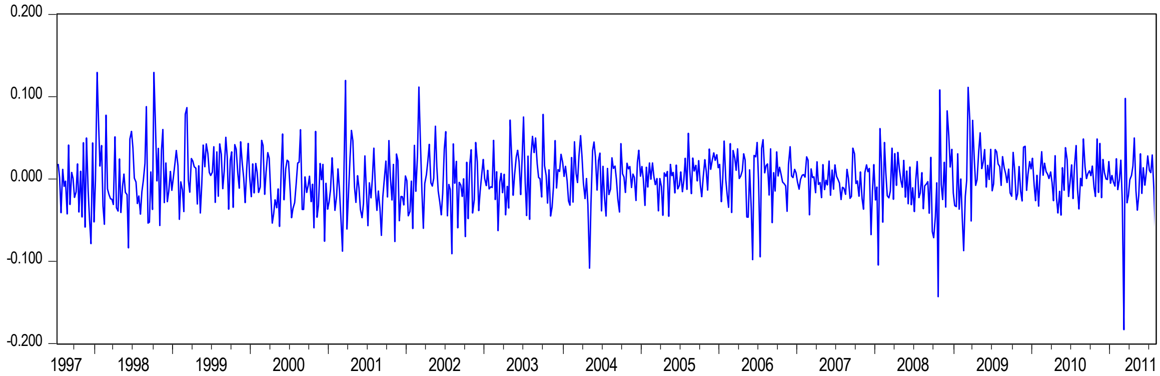
The similar pattern of residuals during a crisis period, i.e. during the collapse of Lehman Brothers in US, indicates the presence of volatility spillover from US to other markets, and thus it become evidence of the correlated structure of the residuals. This phenomenon is often seen in financial market. The latter property of the residuals becomes a motivation to apply SUR type model.

Residuals from each single OLS model are also computed, the results are similar to those of VECM's residuals that they indicate that the residuals are heteroscedastic. The residuals are then used in estimating $\widehat{H}_t$ by Diagonal BEKK. Having the variance-covariance series, we proceed to the next step for constructing matrix $\widehat{\Omega}$ and used it to obtain FGLS estimators.

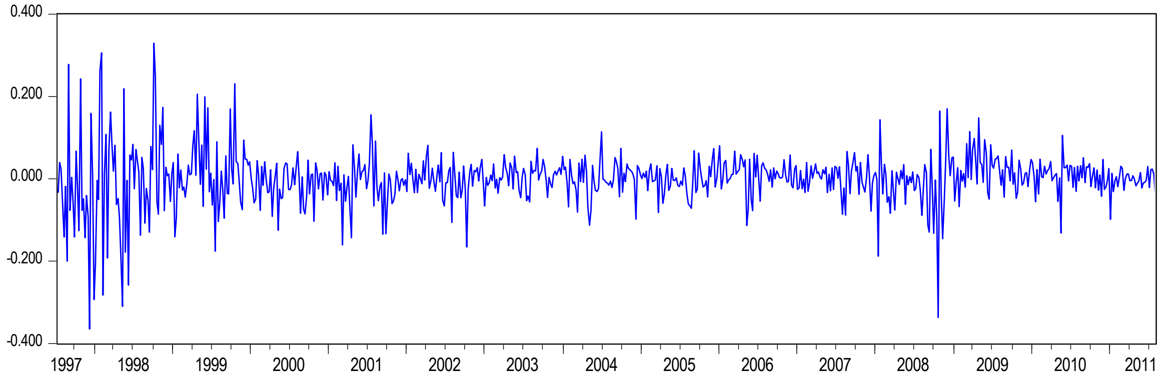
Figure 5: Residuals of VECM



Residuals of D(Ln(S&P500))

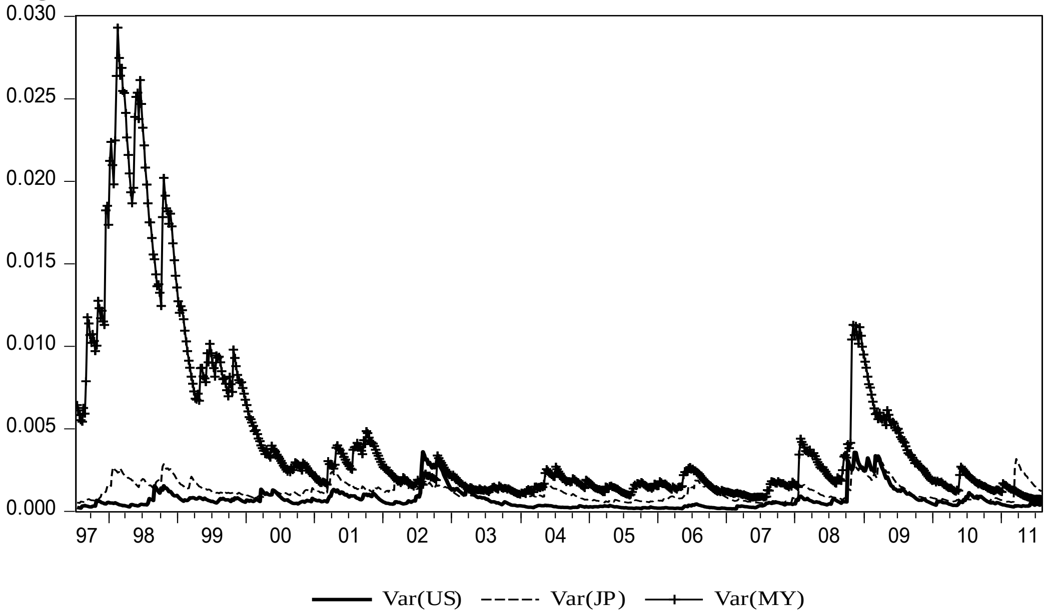


Residuals of D(Ln(Nikkei225))

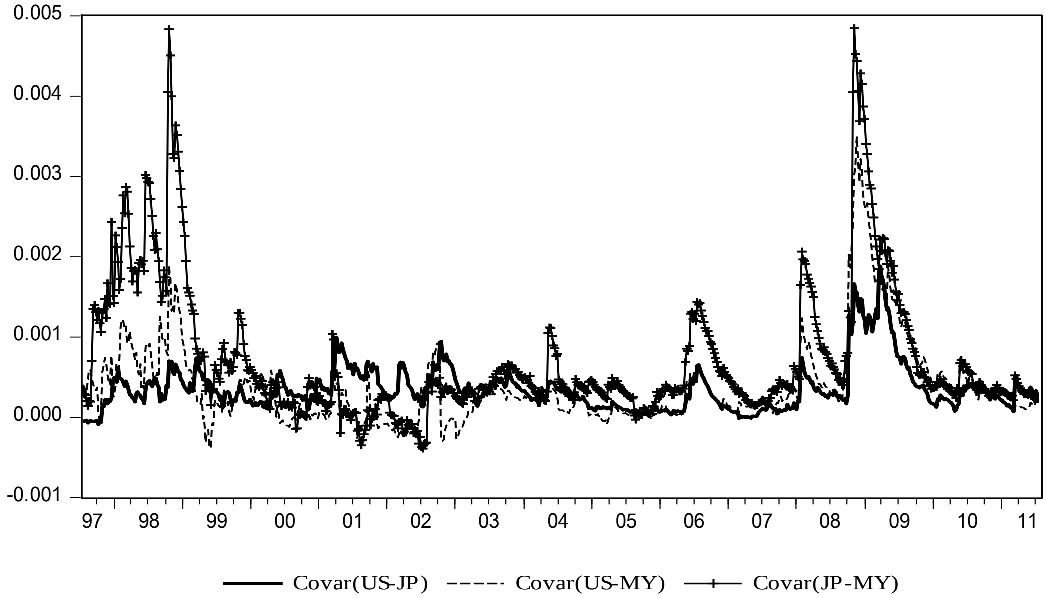


Residuals of D(Ln(KLSE))

Figure 6: Estimated Conditional Variance-Covariance



(a) Conditional Variance



(b) Conditional Covariance

The FGLS estimators, the restated VECM (without GARCH), OLS, and MLE estimated parameters are shown in Table 7. As shown in the table, although the sign and value of the estimated parameters are very similar among the various estimation methods, but the probability of significance are sometime different. Based on the data properties shown in Figure 5 and 6, the GARCH error structure does exists. And based on the simulation results, estimation methods that take into account the GARCH structure are more efficient than those that ignore the structure. Therefore, in the empirical example, the use of such methods (OLS and VECM) might produce wrong conclusion regarding the significance of the estimated parameters. For example, $\hat{\pi}_{32}$ estimated by OLS (and VECM) is significantly different from zero, but it is not significant when it is estimated by FOLSH, FVECH, and MLE. It means that when we estimate the parameter using method that neglecting the MGARCH error structure we would conclude that lagged of Nikkei225 Index (Japanese stock prices) affects Malaysia KLSE returns (Malaysian stock returns), while we should not.

Table 6: Estimated Parameters of OLS, VECM, FOLSH, FVECH, and MLE

Estimated Parameter	OLS		VECM		FOLSH		FVECH		MLE	
	Coef.	S.E.	Coef.	S.E.	Coef.	S.E.	Coef.	S.E.	Coef.	S.E.
$\hat{c}_1^0$	0.139	0.047**	0.089	#	0.138	0.045**	0.195	0.045**	0.139	0.038**
$\hat{\pi}_{11}$	-0.028	0.009**	-0.024	#	-0.029	0.010**	-0.040	0.010**	-0.024	0.007**
$\hat{\pi}_{12}$	0.014	0.007*	0.016	#	0.014	0.004**	0.019	0.004**	0.008	0.004
$\hat{\pi}_{13}$	-0.001	0.003	0.001	#	0.000	0.000**	0.000	0.000**	-0.001	0.002
$\hat{\varphi}_{11}$	-0.093	0.041*	-0.095	0.041*	-0.091	0.159	-0.090	0.061	-0.106	0.040**
$\hat{\varphi}_{12}$	-0.006	0.032	-0.007	0.032	0.007	0.010	0.008	0.005	-0.004	0.024
$\hat{\varphi}_{13}$	0.013	0.024	0.012	0.024	0.038	0.028	0.038	0.017*	0.035	0.015*
$\hat{c}_2^0$	-0.023	0.061	-0.102	#	-0.017	0.025	0.028	0.016*	-0.006	0.048
$\hat{\pi}_{21}$	0.020	0.012	0.027	#	0.015	0.016	0.005	0.003*	0.019	0.009*
$\hat{\pi}_{22}$	-0.024	0.009**	-0.019	#	-0.020	0.031	-0.015	0.009*	-0.023	0.006**
$\hat{\pi}_{23}$	-0.001	0.003	-0.001	#	0.001	0.002	0.001	0.000*	-0.003	0.003
$\hat{\varphi}_{21}$	0.218	0.053**	0.214	0.053**	0.259	0.086**	0.261	0.066**	0.200	0.042**
$\hat{\varphi}_{22}$	-0.075	0.042	-0.076	0.042	-0.094	0.045*	-0.095	0.031**	-0.050	0.038
$\hat{\varphi}_{23}$	-0.046	0.031	-0.047	0.031	-0.056	0.037	-0.057	0.023**	-0.026	0.022
$\hat{c}_3^0$	-0.129	0.079	-0.147	#	-0.109	0.272	-0.052	0.035	-0.017	0.050
$\hat{\pi}_{31}$	0.040	0.016*	0.040	#	0.026	0.041	0.011	0.007*	0.007	0.011
$\hat{\pi}_{32}$	-0.026	0.011*	-0.027	#	-0.014	0.029	-0.004	0.003	-0.003	0.008
$\hat{\pi}_{33}$	-0.005	0.004	-0.001	#	-0.001	0.001	-0.001	0.001	-0.003	0.004
$\hat{\varphi}_{31}$	0.173	0.069*	0.174	0.069*	0.246	0.119*	0.251	0.082**	0.223	0.036**
$\hat{\varphi}_{32}$	0.031	0.055	0.032	0.055	0.006	0.003*	0.002	0.001**	0.011	0.030
$\hat{\varphi}_{33}$	-0.073	0.040	-0.076	0.040	-0.067	0.041	-0.061	0.023**	-0.026	0.037

** significant at 0.01
* significant at 0.05
The Standard error marked by # indicates that the coefficient is computed from loading vector and adjustment vector in the error correction equations, the respective standard error for these parameters are shown in Table 5.

6. CONCLUDING REMARKS

The standard Vector Error correction model (VECM), which is based on normality assumption of error term, is often applied to analyze the real financial time series. However, as shown in the section 5 it is often seen that residuals of this model seem to follow GARCH errors process. From this experience we extend the standard VECM to include GARCH error process. We call such model as VEC-GARCH model. Although the maximum likelihood (ML) estimator is known as the most efficient estimator under the normality assumption, ML estimation is computationally demanding when a model to be estimated is not small. To overcome these disadvantages and to reduce computational burden of ML estimator we consider the generalized least square estimator (GLS) instead of ML estimator. GLS is relatively free from the distributional assumptions.

In this paper we mainly concerns with the GLS representation, the algorithm of it, and the properties of it, we have examined the performance of GLS and MLE in VEC-GARCH model by Monte Carlo simulation and the applicability of it by real data analysis of the financial time series. The Monte Carlo simulation naturally has shown that MLE is still better than the FGLS. However FGLS-based estimators that also consider GARCH error structure are also more efficient than estimators that neglect the error structure. The performance of MLE and FGLS-based estimator in our simulation are only slightly different, yet both are better estimators compare to the OLS and VECM. Thus, the suggested FGLS-based estimator may overcome the disadvantages of MLE, especially in reducing the computational burden.

Our suggested method for the large matrix inversion successfully overcomes the computational problem such as memory size, computer time, and innacurate numerical results. The estimated parameters from the FGLS-based estimator performed in the simulation is as good as the MLE.

There, however, remain several problems in estimating VECM with GARCH errors for the future research as follows: (1) to use realized volatility (RV) instead of multivariate GARCH model, (2) to compare the GLS and MLE under non-normality by Monte Carlo simulation, (3) to carry out theoretical comparisons of asymptotic properties of the GLS and MLE, under normality and non-normality, (4) to examine the performance of VEC model with GARCH errors when it is applied to empirical analysis of financial time series. We have a plan to attack these problems in future.

REFERENCES

Bauwens, L., S. Laurent and J. V. K. Rombouts (2006). Multivaritae GARCH Models: a survey, Journal of applied Econometrics, 21: 79-109.

Bollerslev, T., R.F.Engle and J.M.Wooldridge (1988). A Capital asset pricing model with time varying covariances, *Journal of Political Economy* 96, 116-131.

Engle, R. F. and K. F. Kroner (1995). Multivariate Simultaneous Generalized ARCH. *Econometric theory*, 11, 122-150.

Johansen, S. (1995). *Likelihood-based Inference in Cointegrated Vector autoregressive Models*, Oxford: Oxford University Press.

Johnston, J. and J. DiNardo (2007). *Econometric Methods*, McGraw-Hill Co, Inc.

Silvennoinenn, A. and T. Teräsvirta (2009). Multivariate GARCH models, Handbook of Financial Time Series, ed. by T. G. Andersen, R. A. Davis, J. P. Kreiss and T. Mikosch, New Yor: Springer.

Zellner, A. (1962). An Efficient Method of Estimating Seemingly Unrelated Regressions and Test of Aggregation Bias, *Journal of the American Statistical Association*, 57, 500-509.

PART II

APPLICATION OF SUR-GARCH METHOD IN INTERNATIONAL CAPM TEST

1. MODELS

The proposed test model is aimed to examine the relationship between expected returns of national stock market indexes and the world market portfolio returns. The national stock market indexes are weighted average of the constituent stocks prices based on either market capitalization (e.g. S&P500 Index) or liquidity (e.g. Nikkei 225). The riskless asset is proxied by government securities; 3-month T-Bill.

In previous international CAPM literatures, MSCI world index or other Exchange Traded Fund (ETF) that consists of national market indexes were used as proxy of the world market portfolio. It should be noted that such index weighs the composing asset based on market capitalization or liquidity where the weight is always nonnegative. It means that the world market portfolio consists of assets in long position. Meanwhile CAPM assumes that unrestricted short selling of those assets is allowed. One may argue that we can short sell the index instead of short selling its composing assets. However, the strategy of short selling the world market index does not ensure us that the portfolio is efficient and at tangent of capital market line. To overcome these problems, world market portfolio is constructed following Merton (1972) procedure and Tobin's separation theory (Tobin, 1958) to guarantee that the portfolio is not only mean-variance efficient, but also located at a point which is at tangent of the capital market line.

1.1. Expected return and Conditional Variance-Covariance Matrix of Each Asset

Considering that the stock markets has long-run equilibrium with the other markets and disturbance errors of the estimation model are correlated and heteroscedastic, vector error correction model with GARCH (VEC-GARCH Model) is applied to estimate the expected returns of each national market index and their conditional variance-covariance matrix. The VEC-GARCH model consists of mean equations and variance equations as follows.

The mean equations (the unrestricted VECM) is

$$\boldsymbol{R}_t^d = \widehat{\boldsymbol{C}} + (\widehat{\boldsymbol{\Pi}})\boldsymbol{M}_{t-1}^d + \widehat{\boldsymbol{\Phi}}\boldsymbol{R}_{t-1}^d + \hat{\boldsymbol{\varepsilon}}_t \tag{1}$$

where,

$\boldsymbol{R}_t^d$ = $[\Delta m_{1,t}\ \Delta m_{2,t}\ ...\ \Delta m_{i,t}\ ...\ \Delta m_{N,t}]'$ is vector of first order difference of log national market indexes at time t, where $\Delta m_{i,t} = r_{i,t} = log\left(\frac{m_{i,t}}{m_{i,t-1}}\right)$ is also national market return at time t.

$\boldsymbol{M}_{t-1}^d$ = $[m_{1,t-1}\ ...\ m_{i,t-1}\ ...\ m_{N,t}]'$ is vector of first order lagged of log national market indexes

$\widehat{\boldsymbol{C}}$ = $[\hat{c}_1\ \hat{c}_2\ ...\ \hat{c}_i\ ...\ \hat{c}_N]'$ is vector of constant terms

$\widehat{\boldsymbol{\Pi}}$ = *NxN* matrix of error correction coefficients. When *rank(*$\widehat{\boldsymbol{\Pi}}$*)<N*, $\widehat{\boldsymbol{\Pi}}$ can be decomposed into $\boldsymbol{AB}$ by Granger representation theorem , where $\boldsymbol{A}$ is vector of coefficient of cointegrating equation (adjustment parameters) and, $\boldsymbol{B}$ is vector of cointegrating coefficient.

$\boldsymbol{\Phi}$ = $\begin{bmatrix}\varphi_{11} & \cdots & \varphi_{1N} \\ \vdots & \ddots & \vdots \\ \varphi_{P1} & \cdots & \varphi_{NN}\end{bmatrix}$ is a *NxN* matrix of *VAR* parameters

$\boldsymbol{\varepsilon}_t$ = $[\varepsilon_1\ \varepsilon_2\ ...\ \varepsilon_i\ ...\ \varepsilon_N]$ is the vector of disturbance errors, where $\boldsymbol{\varepsilon}_t \sim (0, \widehat{\boldsymbol{H}}_t)$

and the variance equations (Diagonal BEKK Model, Engle and Kroner (1995)) is

$$\widehat{\boldsymbol{H}}_\mathrm{t} = \widehat{\boldsymbol{\Psi}}\widehat{\boldsymbol{\Psi}}' + [\widehat{\boldsymbol{A}}_1\widehat{\boldsymbol{A}}_1']\odot[\hat{\boldsymbol{\varepsilon}}_{\mathrm{t}-1}\hat{\boldsymbol{\varepsilon}}_{\mathrm{t}-1}'] + [\widehat{\mathbf{A}}_2\widehat{\mathbf{A}}_2']\odot\widehat{\boldsymbol{H}}_{\mathrm{t}-1}, \tag{2}$$

where, $\widehat{\boldsymbol{H}}_t$ is *N*x*N* conditional variance-covariance matrix (its diagonal elements are conditional variances, $[\hat{\sigma}_t^2(r_i)]_{ii}$, and the off-diagonal elements are conditional covariances, $[\hat{\sigma}_t(r_i r_j)]_{ij}$, where $i \neq j$, for i and j=[1 N]), $\widehat{\boldsymbol{\Psi}}\widehat{\boldsymbol{\Psi}}'$, $\widehat{\boldsymbol{A}}_1\widehat{\boldsymbol{A}}_1'$, and $\widehat{\boldsymbol{A}}_2\widehat{\boldsymbol{A}}_2'$ are diagonal matrix of constants, coefficients of ARCH terms, and coefficients of GARCH terms respectively, and $\odot$ is element by element (Hadamard) product operator.

The parameters in the mean equations and the variance equations theoretically can be estimated by maximum likelihood estimator (MLE). However, when the system is large as in our case, MLE often produces inaccurate results because too many parameters need to be estimated such that the optimization of the log likelihood function failed. To overcome this problem, the mean equation (VECM) parameters were estimated as those in Seemingly Unrelated Regression (SUR) system using modified feasible generalized least square (mFGLS) estimator that taking into account the GARCH error structure. This estimation strategy was also used in testing the CAPM and shall be explained later.

For estimating conditional variance of realized return, the mean equation in equation (1) was replaced by $\widetilde{\boldsymbol{R}}_t = \widehat{\boldsymbol{C}} + \hat{\boldsymbol{\varepsilon}}_t$ and the conditional variance-covariance matrix was estimated by Diagonal BEKK. Henceforth, accent " $\tilde{}$ " and " $\breve{}$ " are used for indicating variable based on the realized return and the estimated expected return respectively.

1.2. World Market Portfolio Formation

World market portfolio was constructed by assuming that unrestricted short selling and borrowing at riskless rate in domestic or national market are allowed. The assumptions were made to follow the underlying assumptions in CAPM.

The proportion of each asset in an efficient portfolio was obtained by minimizing objective function of portfolio variance with respect to following constraints: [1] a set of target portfolio expected return and, [2] the sum of proportion of each asset (including riskless asset) is equal to one. When short selling is prohibited, constraint [2] is modified by adding restriction on proportion of each risky asset to vary between 0 to 1, yet in this paper the proportion is unrestricted to indicate that the short selling can be done without any restriction.

Suppose that country i is our focus of analysis and call it home country. Portfolio P consists of riskless asset available at domestic market i and N international risky portfolios $(m_1^d, \dots, m_i^d, \dots m_N^d)$. The rate of return of P is the weighted average of rate of return of its composing assets. Our objective is to construct world market portfolio denoted by *M* that consists of risky portfolios only (proxied by market indexes). Let us define $r_{f,t}$, $\boldsymbol{\omega}_{\mathrm{t}} = (\omega_{1,\mathrm{t}} \dots \omega_{\mathrm{i,t}} \dots \omega_{\mathrm{N,t}})'$, and $\boldsymbol{e}$ as riskless rate of return, vector of proportion of risky assets in portfolio *M* and vector of ones respectively. Constraint [2] implies that $(1 - \boldsymbol{\omega}'\boldsymbol{e})$ is the proportion of riskless asset in portfolio P. Applying constraint [2] to the expected return of risk-free asset and risky assets definition, the expected return of *P* may be stated as:

$$\check{r}_{\mathrm{t}}^{\mathrm{P}} = [r_{\mathrm{f,t}} + \boldsymbol{\omega}_{\mathrm{t}}'(\breve{\boldsymbol{R}}_{\mathrm{t}}^{\mathrm{d}} - r_{\mathrm{f,t}}\boldsymbol{e})] \tag{3}$$

Having conditional variance-covariance matrix $\widehat{\boldsymbol{H}}_t$ from (2), variance of portfolio *P* at time *t* is computed by,

$$\widehat{\sigma}_{\mathrm{t}}^{2}(\check{r}^{\mathrm{P}}) = \boldsymbol{\omega}_{\mathrm{t}}'\widehat{\boldsymbol{H}}_{\mathrm{t}}\boldsymbol{\omega}_{\mathrm{t}}. \tag{4}$$

The optimal weight of the *N* risky assets and risk-free asset was obtained by solving following optimization problem:

$$\min_{\omega_{\mathrm{t}}} \frac{1}{2}\boldsymbol{\omega}_{\mathrm{t}}'\widehat{\boldsymbol{H}}_{\mathrm{t}}\boldsymbol{\omega}_{\mathrm{t}} + \lambda\{\check{r}_{\mathrm{t}}^{\mathrm{P}} - [r_{\mathrm{f,t}} + \boldsymbol{\omega}_{\mathrm{t}}'(\breve{\boldsymbol{R}}_{\mathrm{t}}^{\mathrm{d}} - r_{\mathrm{f,t}}\boldsymbol{e})]\}. \tag{5}$$

The first-order condition of (5) leads to following solution:

$$\boldsymbol{\omega}_{\mathrm{t}}^{*} = \lambda\widehat{\boldsymbol{H}}_{\mathrm{t}}^{-1}(\breve{\boldsymbol{R}}_{\mathrm{t}}^{\mathrm{d}} - r_{\mathrm{f,t}}\boldsymbol{e}). \tag{6}$$

Taking $\boldsymbol{\omega}_t^{*}$ from (6) and apply the $\boldsymbol{e}'\boldsymbol{\omega}_t^{*} = 1$ restriction, we may obtain λ:

$$\begin{gathered}\boldsymbol{e}'\boldsymbol{\omega}_{\mathrm{t}}^{*} = \mathbf{e}'[\lambda\widehat{\boldsymbol{H}}_{\mathrm{t}}^{-1}(\breve{\boldsymbol{R}}_{\mathrm{t}}^{\mathrm{d}} - r_{\mathrm{f,t}}\boldsymbol{e})] = 1 \\ \lambda = m = [\alpha - \delta r_{\mathrm{f,t}}]^{-1}\end{gathered} \tag{7}$$

where $\alpha = \breve{\boldsymbol{R}}_t^{d\prime}\widehat{\boldsymbol{H}}_t^{-1}\boldsymbol{e}$ and $\delta = \boldsymbol{e}'\widehat{\boldsymbol{H}}_t^{-1}\boldsymbol{e}$.

From (6), the expected return of risky portfolio *M* is $\check{r}_t^{M} = \boldsymbol{\omega}_t^{*\prime}\breve{\boldsymbol{R}}_t^{d}$ and the variance of portfolio *P* will be equal to the variance of portfolio *M* defined as $\widehat{\sigma}_t^2(\check{r}^P) = \widehat{\sigma}_t^2(\check{r}^M) = \boldsymbol{\omega}_t^{*\prime}\widehat{\boldsymbol{H}}_t\boldsymbol{\omega}_t^{*}$. Define $\widehat{\boldsymbol{\sigma}}_t(\check{r}^M)$ as *nx1* vector of covariance of the tangency portfolio *M* with each of the risky asset. Then using (6) and (7), we have

$$\widehat{\boldsymbol{\sigma}}_{\mathrm{t}}(\check{r}^{\mathrm{M}}) = \widehat{\boldsymbol{H}}_{\mathrm{t}}\boldsymbol{\omega}_{\mathrm{t}}^{*} = m(\breve{\boldsymbol{R}}_{\mathrm{t}}^{\mathrm{d}} - r_{\mathrm{f,t}}\boldsymbol{e}) \tag{8}$$

Pre-multiply (8) by $\boldsymbol{\omega}_t^{*\prime}$ we have $\widehat{\sigma}_t^2(\check{r}^M)$ restated as

$$\widehat{\sigma}_t^2(\check{r}^M) = \boldsymbol{\omega}_t^{*\prime}\widehat{\boldsymbol{\sigma}}_t(\check{r}^M) = m\boldsymbol{\omega}_t^{*\prime}\left(\breve{\boldsymbol{R}}_t^d - r_{f,t}\boldsymbol{e}\right) = m(\check{r}_t^M - r_{f,t}) \quad (9)$$

Rearranging (8) and substituting in for *m* from (9) we have the CAPM:

$$\left(\breve{\boldsymbol{R}}_t^d - r_{f,t}\boldsymbol{e}\right) = \frac{1}{m}\widehat{\boldsymbol{\sigma}}_t(\check{r}^M) = \frac{\widehat{\boldsymbol{\sigma}}_t(\check{r}^M)}{\widehat{\sigma}_t^2(\check{r}^M)}(\check{r}_t^M - r_{f,t}). \quad (10)$$

The LHS of (10) is the expected excess return from each asset, while on the RHS, $\frac{\widehat{\boldsymbol{\sigma}}_t(\check{r}^M)}{\widehat{\sigma}_t^2(\check{r}^M)} = \widehat{\boldsymbol{\beta}}_t$ is vector of time varying betas of each risky asset, and $(\check{r}_t^M - r_{f,t})$ is the expected market risk premium that prevails for all risky assets. Note that because we are assuming that short selling is unrestricted, $\check{r}_t^M$ is always nonnegative, and the portfolio *M* is always in the efficient frontier of portfolio *P* (the risk-free and risky assets portfolio). However, elements of $\breve{\boldsymbol{R}}_t^d$, the estimated expected return of each asset could be positive or negative. When the expected return of an asset is negative, it will be more likely to be short sold. Thus, $\widehat{\boldsymbol{\sigma}}_t(\check{r}^M)$ is not always positive. As a result we may find that an asset's beta and the beta risk premium is negative.[1]

1.3. Testing Conditional CAPM

The capital asset pricing model in equation (10) will serve as our test model. In addition, because we consider international assets, we must put additional risk factor other than the world market risk (represented by the betas) that indicates the required adjustment for the excess return. In this paper we include exchange rate returns in the model. We can consider the international CAPM being tested in this paper is involving Exchange Traded Funds (ETFs) that track directly the respective stock market indexes. Therefore, like the CAPM test for assets traded in one market, we can ignore the transaction cost of acquiring the cross-border assets. The test model is defined in a system equation as follows:

$$\boldsymbol{ER}_t = \widehat{\widehat{\alpha}} + \widehat{\boldsymbol{\beta}}_t'\widehat{\widehat{\boldsymbol{\theta}}} + \boldsymbol{\Xi}_t'\widehat{\widehat{\boldsymbol{\xi}}} + \boldsymbol{\eta}_t \quad (11)$$

[1] See Pennacchi (2008) pp. 37-60 for more detailed derivation of the market portfolio.

where $\boldsymbol{ER}_t$ and $\widehat{\boldsymbol{\beta}}_t$ are vector of excess returns and market betas as defined in (10), $\boldsymbol{\Xi}_t$ is vector of exchange rate returns for the respective markets. The vectors of estimated coefficients are $\widehat{\widehat{\boldsymbol{\alpha}}}$, $\widehat{\widehat{\boldsymbol{\theta}}}$, and $\widehat{\widehat{\boldsymbol{\xi}}}$.

CAPM is said works well when all elements in $\widehat{\widehat{\boldsymbol{\alpha}}}$ are statistically not different from zero (the test does not reject $H_0^1\colon \widehat{\widehat{\alpha}}_{\mathrm{i}} = 0, \forall i$). However, evaluating $\widehat{\widehat{\alpha}}_{\mathrm{i}}$ individually shall show the applicability of CAPM for pricing that asset. In addition, because of the fully integrated market assumption, we expect that the (beta) market risk premium for every markets are homogenous. However, since short selling is allowed, the negative betas and risk premiums are possible. Thus, the homogeneity test was carried out by taking the absolute values of the premium (the test does not reject $H_0^2\colon |\widehat{\widehat{\theta}}_1| = |\widehat{\widehat{\theta}}_2| = \cdots = |\widehat{\widehat{\theta}}_{\mathrm{N}}|$). Rejection of the null hypothesis indicates that markets are not fully integrated, in other words, the risk is priced differently for different assets; a violation of the law of one price. The elements in $\widehat{\widehat{\boldsymbol{\xi}}}$ show additional risk price required with respect to the exchange rate changes. As exchange rate policies are different across the countries, we expect that the estimated coefficient in $\widehat{\widehat{\boldsymbol{\xi}}}$ will be higher for countries that adopt free float regime than those that adopt fixed exchange rate or dollar pegged regime. Moreover, exchange rates against US Dollar in emerging markets are also tend to be more volatile than those in developed countries, thus it is also expected that the estimated coefficient is significantly different from zero for countries with non-fixed exchange rate regime.

Under fully integrated market assumption the unexpected returns or shocks in one stock market may affect or spill over to the others. Moreover, we also found common cyclicality of business cycles in the stock markets. Therefore, we are assuming that the error terms $\boldsymbol{\eta}_t$ has multivariate GARCH error structure. In order to estimate the parameters, we apply SUR with GARCH (SUR-GARCH) estimation. Estimation from the standard SUR was also presented to see the effect of ignoring the GARCH error structure.

2. ESTIMATION STRATEGY

Equation (1) and (11) can be restated as SUR model. For simplicity, we will use system equation (11) as a sample to explain the estimation strategy.

Let us define $\boldsymbol{ER}_i$ as *T*-vector of excess return of asset-*i*, matrix $\boldsymbol{X}_i = [\boldsymbol{e}, \widehat{\boldsymbol{\beta}}_{\mathrm{i}}, \Xi_{\mathrm{i}}]$ is vector of independent variables, where its respective elements are *T*-vector of ones, *T*-vector of time varying beta for asset-*i*, and *T*-vector of exchange rate changes for market-*i*, and $\boldsymbol{\Gamma}_{\mathrm{i}} = [\boldsymbol{\alpha}_{\mathbf{i}}, \boldsymbol{\theta}_{\mathbf{i}}, \boldsymbol{\xi}_{\mathrm{i}}]$ is vector of coefficients for equation-*i*. Then, equation-*i* in the system equation (11) can be restated as follows:

$$\boldsymbol{ER}_{\mathrm{i}} = \boldsymbol{X}_{\mathrm{i}}\boldsymbol{\Gamma}_{\mathrm{i}} + \boldsymbol{\eta}_{\mathrm{i}}, \qquad \mathrm{i} = 1, \dots, \mathrm{N} \tag{12}$$

where $\boldsymbol{\eta}_i$ is *T*-vector of the disturbance errors for the equation. In stacked model, the system equation (11) can be restated as follows:

$$\begin{bmatrix} \boldsymbol{ER}_1 \\ \boldsymbol{ER}_2 \\ \vdots \\ \boldsymbol{ER}_N \end{bmatrix} = \begin{bmatrix} \boldsymbol{X}_1 & 0 & \dots & 0 \\ 0 & \boldsymbol{X}_2 & \dots & 0 \\ \vdots & \vdots & & \vdots \\ 0 & 0 & \dots & \boldsymbol{X}_N \end{bmatrix} \begin{bmatrix} \boldsymbol{\Gamma}_1 \\ \boldsymbol{\Gamma}_2 \\ \vdots \\ \boldsymbol{\Gamma}_{\mathrm{N}} \end{bmatrix} + \begin{bmatrix} \boldsymbol{\eta}_1 \\ \boldsymbol{\eta}_2 \\ \vdots \\ \boldsymbol{\eta}_{\mathrm{P}} \end{bmatrix},$$

In general, the corresponding matrices define the following system,

$$\boldsymbol{Y} = \boldsymbol{X\Gamma} + \boldsymbol{\eta}. \tag{13}$$

VECM in system equation (1) also can be stated similar to the above system equation by redefining the $\boldsymbol{X}$ and $\boldsymbol{\Gamma}$ accordingly. To reduce the number of parameters needs to be estimated in the VEC-GARCH, we first estimate the VECM (without GARCH), create series of cointegrating equation (we assume that there is only one cointegrating equation), and use it as new variable in a VAR system. Thus, the $\boldsymbol{X}_i$ for system equation (1) defined as $\boldsymbol{X}_i = [\boldsymbol{e}, \boldsymbol{CE}, \boldsymbol{R}^d_{i,t-1}]$, where $\boldsymbol{CE}$ is *T*-vector of cointegrating series that applied for every *i*.

2.1. Feasible Generalized Least Square (FGLS) SUR Estimation

FGLS or also known as Zellner's estimator (Zelner, 1962), assumes that $E[\boldsymbol{\eta}_i | \boldsymbol{X}_1, \boldsymbol{X}_2, \dots, \boldsymbol{X}_N] = 0$ (strict exogeneity of $\boldsymbol{X}_i$), and $E[\boldsymbol{\eta}_{\mathrm{i}}\boldsymbol{\eta}'_{\mathrm{i}} | \boldsymbol{X}_1, \boldsymbol{X}_2, \dots, \boldsymbol{X}_N] = \sigma_{ii}\boldsymbol{I}_T$ (homoscedasticity). As stock markets are assumed to be fully integrated, the disturbances might be correlated across equations. Therefore, $E\left[\eta_{it}\eta'_{js}\middle|\boldsymbol{X}_1,\boldsymbol{X}_2,\ldots,\boldsymbol{X}_N\right]=\sigma_{ij}$ for $t=s$ and 0 for $t\neq s$. The σ_{ij} is covariance between disturbances *i* and *j*; it is *ij*th element of variance-covariance matrix $\boldsymbol{\Sigma}$. Let us also define $\boldsymbol{\Omega}=\boldsymbol{\Sigma}\otimes\mathbf{I}$. The generalized least square estimator under the covariance structures assumption is

$$\hat{\boldsymbol{\Gamma}}=[\boldsymbol{X}'(\hat{\boldsymbol{\Sigma}}\otimes\boldsymbol{I})^{-1}\boldsymbol{X}]^{-1}[\boldsymbol{X}'(\hat{\boldsymbol{\Sigma}}\otimes\boldsymbol{I})^{-1}\boldsymbol{Y}]. \tag{14}$$

Because σ_{ij} is generally unknown, it is estimated by $\hat{\sigma}_{ij}=s_{ij}=\frac{\hat{\boldsymbol{\eta}}'_i\hat{\boldsymbol{\eta}}_j}{T}$ where $\hat{\boldsymbol{\eta}}_i$ is vector of residuals in equation *i*. By doing so, the estimated variance-covariance matrix $\hat{\boldsymbol{\Sigma}}$ can be computed. The FGLS estimator requires inversion of matrix $\hat{\boldsymbol{\Sigma}}$, so that the matrix must have a non-zero discriminant.

The standard errors of the parameters were estimated by taking the square root of elements in sampling variances:

$$\mathrm{Var}\left[\hat{\boldsymbol{\Gamma}}\middle|\boldsymbol{X}\right]=\hat{\sigma}^2(\boldsymbol{X}'\hat{\boldsymbol{\Omega}}^{-1}\boldsymbol{X})^{-1} \tag{15}$$

where, $\hat{\sigma}^2=\frac{\hat{\boldsymbol{\eta}}'_i\hat{\boldsymbol{\eta}}_j}{T-N}=\frac{(\boldsymbol{Y}-\boldsymbol{X}\hat{\boldsymbol{\Gamma}})'\hat{\boldsymbol{\Omega}}^{-1}(\boldsymbol{Y}-\boldsymbol{X}\hat{\boldsymbol{\Gamma}})}{T-N}$.

The join hypotheses of H_0^1 and H_0^2, were tested by Wald coefficient test with *J* degree of freedom, where *J* is *N* and *N-1* respectively. The restriction is defined by $\boldsymbol{R}\hat{\boldsymbol{\Gamma}}=\boldsymbol{q}$, where $\boldsymbol{R}$ is (*JxK*) matrix of restriction with *K* is number of the parameters in $\hat{\boldsymbol{\Gamma}}$, and $\boldsymbol{q}$ is *J*-vector of the true values. The Wald statistic is $\chi^2[J]$ distributed and computed by

$$\mathrm{W[J]}=(\boldsymbol{R}\hat{\boldsymbol{\Gamma}}-\boldsymbol{q})'[\boldsymbol{R}\hat{\sigma}^2(\boldsymbol{X}'\hat{\boldsymbol{\Omega}}^{-1}\mathbf{X})^{-1}\boldsymbol{R}'](\boldsymbol{R}\hat{\boldsymbol{\Gamma}}-\mathbf{q}). \tag{16}$$

2.2. Modified Feasible Generalized Least Square (mFGLS) SUR-GARCH Estimation

Considering that system equation (1) and (11) are estimated in fully integrated markets and there were shocks and crises during the observation periods that spilled over among the samples, the multivariate GARCH error structure should be considered. To do so, following is steps to include the error structure for estimating the parameters in the models:

[1] Estimate the mean equations by first ignoring the GARCH error structure and obtain the residuals.

[2] Use the residuals to estimates conditional variance-covariance matrix by using Diagonal BEKK model. At every observation t, we have $\widehat{\boldsymbol{H}}_t$ with elements of $\hat{h}_{ij,t}$.

[3] Use the variance-covariance matrix from step 2 to construct $\widehat{\boldsymbol{\Omega}}$. Note that $\widehat{\boldsymbol{\Omega}}$ in (14) is defined as $\widehat{\boldsymbol{\Sigma}}\otimes\boldsymbol{I}$ where the diagonal elements are the vector of variances of each equation (which is a constant variance) and the off-diagonal elements are all zeros (there is no covariance across the equations). The modified $\widehat{\boldsymbol{\Omega}}$ at this step is considering the heteroscedasticity and covariance across the equations. To illustrate it in a simple example, for N= 3, $\widehat{\boldsymbol{\Omega}}$ is:

$$\widehat{\boldsymbol{\Omega}} = \begin{bmatrix} \widehat{\boldsymbol{\Omega}}_{11} & \widehat{\boldsymbol{\Omega}}_{12} & \widehat{\boldsymbol{\Omega}}_{13} \\ \widehat{\boldsymbol{\Omega}}_{21} & \widehat{\boldsymbol{\Omega}}_{22} & \widehat{\boldsymbol{\Omega}}_{23} \\ \widehat{\boldsymbol{\Omega}}_{31} & \widehat{\boldsymbol{\Omega}}_{32} & \widehat{\boldsymbol{\Omega}}_{33} \end{bmatrix}$$

where, $\widehat{\boldsymbol{\Omega}}_{\mathrm{ij}}$ is a NxN diagonal matrix where its main diagonal elements are elements of N-vector of $h_{ij,t}$ and zeros on the off-diagonal elements, and $\widehat{\boldsymbol{\Omega}}_{\mathrm{ij}} = \widehat{\boldsymbol{\Omega}}_{\mathrm{ji}}$, i.e.,

$$\widehat{\boldsymbol{\Omega}}_{ij} = \begin{bmatrix} \hat{h}_{ij,1} & & & & \\ & \ddots & & 0 & \\ & & \hat{h}_{ij,t} & & \\ & 0 & & \ddots & \\ & & & & \hat{h}_{ij,n} \end{bmatrix}$$

Thus we have,

$$\widehat{\boldsymbol{\Omega}} = \left[\begin{array}{ccc|ccc|ccc}
\begin{matrix}\hat{h}_{11,1} & & & \\ & \ddots & 0 & \\ & & \hat{h}_{11,t} & \\ & 0 & \ddots & \\ & & & \hat{h}_{11,n}\end{matrix} & & & \begin{matrix}\hat{h}_{12,1} & & & \\ & \ddots & 0 & \\ & & \hat{h}_{12,t} & \\ & 0 & \ddots & \\ & & & \hat{h}_{12,n}\end{matrix} & & & \begin{matrix}\hat{h}_{13,1} & & & \\ & \ddots & 0 & \\ & & \hat{h}_{13,t} & \\ & 0 & \ddots & \\ & & & \hat{h}_{13,n}\end{matrix} & & \\ \hline
\begin{matrix}\hat{h}_{21,1} & & & \\ & \ddots & 0 & \\ & & \hat{h}_{21,t} & \\ & 0 & \ddots & \\ & & & \hat{h}_{21,n}\end{matrix} & & & \begin{matrix}\hat{h}_{22,1} & & & \\ & \ddots & 0 & \\ & & \hat{h}_{22,t} & \\ & 0 & \ddots & \\ & & & \hat{h}_{22,n}\end{matrix} & & & \begin{matrix}\hat{h}_{23,1} & & & \\ & \ddots & 0 & \\ & & \hat{h}_{23,t} & \\ & 0 & \ddots & \\ & & & \hat{h}_{23,n}\end{matrix} & & \\ \hline
\begin{matrix}\hat{h}_{31,1} & & & \\ & \ddots & 0 & \\ & & \hat{h}_{31,t} & \\ & 0 & \ddots & \\ & & & \hat{h}_{31,n}\end{matrix} & & & \begin{matrix}\hat{h}_{32,1} & & & \\ & \ddots & 0 & \\ & & \hat{h}_{32,t} & \\ & 0 & \ddots & \\ & & & \hat{h}_{32,n}\end{matrix} & & & \begin{matrix}\hat{h}_{33,1} & & & \\ & \ddots & 0 & \\ & & \hat{h}_{33,t} & \\ & 0 & \ddots & \\ & & & \hat{h}_{33,n}\end{matrix} & &
\end{array}\right]$$

k k

where $\hat{h}_{ij,t}$ follow multivariate MGARCH(1,1) process.

After obtaining an estimate $\widehat{\boldsymbol{\Omega}}$, we have the modified FGLS,

$$\widehat{\boldsymbol{\Gamma}} = [\boldsymbol{X}'\widehat{\boldsymbol{\Omega}}^{-1}\boldsymbol{X}]^{-1}\boldsymbol{X}'\widehat{\boldsymbol{\Omega}}^{-1}\boldsymbol{Y} \qquad (17)$$

Note that inverting a large and sparse matrix $\widehat{\boldsymbol{\Omega}}$ often causes computational problems such as memory size, computer time, and inaccurate numerical results. To avoid those problems we propose the following algorithm: After estimating MGARCH process we construct a relatively small matrix $\widehat{\boldsymbol{H}}_{\mathrm{t}}$ and its inverse $\widehat{\boldsymbol{H}}_{\mathrm{t}}^{-1}$ at each time *t* such that,

$$\widehat{\boldsymbol{H}}_{\mathrm{t}} = \begin{bmatrix} \hat{h}_{11,t} & \hat{h}_{12,t} & \hat{h}_{13,t} \\ \hat{h}_{21,t} & \hat{h}_{22,t} & \hat{h}_{23,t} \\ \hat{h}_{31,t} & \hat{h}_{32,t} & \hat{h}_{33,t} \end{bmatrix}, \text{and } \widehat{\boldsymbol{H}}_{\mathrm{t}}^{-1} = \begin{bmatrix} \hat{h}_t^{11} & \hat{h}_t^{12} & \hat{h}_t^{13} \\ \hat{h}_t^{21} & \hat{h}_t^{22} & \hat{h}_t^{23} \\ \hat{h}_t^{31} & \hat{h}_t^{32} & \hat{h}_t^{33} \end{bmatrix}$$

where $\widehat{\boldsymbol{H}}_{\mathrm{t}}$ and $\hat{h}_{ij,t}$ are estimated variance covariance of MGARCH.

Replacing $\hat{h}_{ij,t}$ with $\hat{h}_t^{ij}$ in $\widehat{\boldsymbol{\Omega}}$ we have easily obtain $\widehat{\boldsymbol{\Omega}}^{-1}$ without inverting a large matrix $\widehat{\boldsymbol{\Omega}}$.

Having the modified $\widehat{\boldsymbol{\Omega}}$ and $\widehat{\boldsymbol{\Gamma}}$ the hypotheses can be tested using the Wald Test as described previously. The performance of the modified FGLS estimator has been examined by carrying out Monte Carlo simulation. The modified FGLS estimator is still an unbiased estimator and it is more efficient and consistent than the standard FGLS estimator when multivariate GARCH error structure does exist (Maekawa and Setiawan, 2012).

3. DATA

Stock market index from 12 economies were collected with its respective currency. The indexes represent 6 developed stock markets: United States S&P500 (US), Germany DAX (GE), Hong Kong Hang Seng (HK), Japan Nikkei225 (JP), Singapore Strait Times (SI) and FTSE100 (UK), and 6 emerging markets: Argentina MerVal (AR), Brazil BOVESPA (BR), China SSEC (CH), Indonesia IDX composite (ID), Malaysia KLSE composite (MA), and Mexico IPC (ME). The market indexes are exchange rate adjusted, with US Dollar as the home currency.

The dataset starts from July 1997 to July 2012 and in weekly basis for avoiding non-synchronous trading time effect. Data were collected from Yahoo Finance service through its website. Because weekly data is used, and it is assumed that portfolios rebalancing are done weekly, the returns are not including dividends. Most of the stock market indexes are value-weighted indexes and the remaining are equally weighted index and top performers' index. However, the indexes used in this paper are assumed sufficient in representing the market portfolio in the respective markets because the indexes used to be regarded as the market references. As the US is regarded as home country, US 3-month T-Bills is used as the risk-free rate.

4. FINDINGS

4.1. Data Properties

Based on Augmented Dickey-Fuller (ADF) Tests and Common Unit Root Tests performed for data in level (log market index) and its first difference (return), the results indicate that all series in level are non-stationary (except for Indonesia and Malaysia when intercept and trend are included), but all series in its first difference are stationary.

Granger causality test for returns of the US Dollar adjusted market indexes were performed, and the results are shown in Table 1. It indicates that US stock market Granger-caused the other markets (except for China, Malaysia and Mexico). The result implies that US stock market is still very dominant and has greater influence to other markets in the world.

Table 1: Granger Causality Test for Stock Markets Returns

	US	GE	HK	JP	SI	UK	AR	BR	CH	ID	MA	ME
US		**0.031**	**0.000**	**0.014**	**0.001**	**0.006**	**0.078**	**0.000**	0.072	**0.000**	0.091	0.139
GE	0.579		**0.010**	0.696	**0.024**	0.942	0.107	**0.008**	**0.007**	**0.001**	**0.014**	**0.744**
HK	0.293	0.638		0.432	**0.042**	0.338	0.067	**0.006**	0.091	**0.000**	**0.027**	0.355
JP	0.243	0.500	0.157		**0.049**	0.522	0.509	0.277	0.120	**0.004**	**0.016**	0.583
SI	0.548	0.892	0.753	0.807		0.501	0.110	0.055	0.082	**0.000**	0.092	0.133
UK	0.253	0.784	0.052	0.411	**0.018**		0.156	**0.003**	**0.017**	**0.012**	**0.039**	0.894
AR	0.279	0.178	0.088	0.462	0.071	0.644		0.194	**0.017**	0.076	0.349	0.614
BR	0.976	0.107	0.180	0.368	0.201	0.229	0.743		0.083	**0.001**	0.423	0.413
CH	0.126	0.495	0.085	0.703	0.286	0.246	0.663	0.589		0.389	0.954	0.923
ID	0.717	0.423	0.196	0.687	0.115	0.913	0.421	0.474	0.853		0.615	0.819
MA	0.925	0.995	0.879	0.897	**0.006**	0.339	0.561	0.098	**0.044**	0.106		0.672
ME	0.928	0.275	0.112	0.454	0.081	0.159	0.184	0.318	0.438	**0.000**	0.262	

The numbers represent *p*-value on Granger Causality *F*-Statistic with lag-1 of the stock markets returns. The table is read 'row' Granger-cause 'column'.

Table 2: Johansen's Cointegration Test

Trace Test				
Hypothesized No. of CE(s)	Eigenvalue	Trace Statistic	0.05 Critical Value	Prob.**
None *	0.110	464.063	374.908	0.000
At most 1 *	0.103	372.930	322.069	0.000
At most 2 *	0.087	288.022	273.189	0.010
At most 3	0.060	216.213	228.298	0.157
Maximum Eigenvalue Test				
Hypothesized No. of CE(s)	Eigenvalue	Max-Eigen Statistic	0.05 Critical Value	Prob.**
None *	0.1096	91.1335	80.8703	0.0048
At most 1 *	0.1025	84.9072	74.8375	0.0047
At most 2 *	0.0874	71.8092	68.8121	0.0256
At most 3	0.0596	48.2450	62.7521	0.5701

Trace test and Max-eigenvalue test indicates 3 cointegrating equations at the 0.05 level.
* denotes rejection of the hypothesis at the 0.05 level, **MacKinnon-Haug-Michelis (1999) p-values

The cointegration test in Table 2 shows that there are three cointegrating equations. It shows that there is long-term equilibrium relationship among the market indices. However, for simplicity and reducing computational burden in VECM estimation, only one cointegrating equation was applied. The cointegrating equation in the VECM is shown in Table 3. Using the cointegration equation, VECM and VEC-GARCH Model parameters are shown in Table 4 and 5.

Table 3: Cointegrating Equation in Vector Error Correction Model (VECM)

	log(US)	log(GE)	log(HK)	log(JP)	log(SI)	log(UK)	log(AR)	log(BR)	log(CH)	log(ID)	log(MA)	log(ME)	Const.
Coeff.	1.000	0.094	0.526	-0.371	0.021	-0.528	-0.138	-0.008	-0.022	0.364	-0.996	-0.023	2.027
S.E.		-0.170	-0.182	-0.127	-0.234	-0.167	-0.061	-0.091	-0.055	-0.071	-0.140	-0.082	-1.002
t-stat.		0.554	**2.884**	**-2.916**	0.090	**-3.153**	**-2.282**	-0.083	-0.397	**5.117**	**-7.131**	-0.282	**2.024**

Number in bold face indicates the coefficient is significant at 5% level.

Table 4: Vector Error Correction Model (VECM)

	$\breve{R}_{US,t}$	$\breve{R}_{GE,t}$	$\breve{R}_{HK,t}$	$\breve{R}_{JP,t}$	$\breve{R}_{SI,t}$	$\breve{R}_{UK,t}$	$\breve{R}_{AR,t}$	$\breve{R}_{BR,t}$	$\breve{R}_{CH,t}$	$\breve{R}_{ID,t}$	$\breve{R}_{MA,t}$	$\breve{R}_{ME,t}$
Coint.Eq.	**-0.027**	-0.022	0.011	**0.027**	0.021	-0.017	-0.002	-0.019	**-0.025**	**0.043**	**0.095**	-0.011
S.E.	**-0.009**	-0.014	-0.013	**-0.012**	-0.013	-0.010	-0.019	-0.021	**-0.012**	**-0.022**	**-0.014**	-0.015
$\breve{R}_{US,t-1}$	-0.034	0.159	**0.218**	**0.203**	**0.185**	**0.171**	0.071	**0.289**	-0.023	0.107	-0.049	0.182
S.E.	-0.062	-0.092	**-0.088**	**-0.078**	**-0.088**	**-0.069**	-0.128	**-0.138**	-0.083	-0.148	-0.093	-0.103
$\breve{R}_{GE,t-1}$	0.006	**-0.178**	0.043	-0.064	-0.004	-0.066	0.046	0.020	0.080	0.090	0.091	-0.002
S.E.	-0.045	**-0.067**	-0.065	-0.057	-0.064	-0.050	-0.094	-0.101	-0.060	-0.108	-0.068	-0.076
$\breve{R}_{HK,t-1}$	-0.004	0.021	-0.050	0.012	0.041	0.028	0.088	0.137	0.018	0.080	0.017	0.036
S.E.	-0.042	-0.063	-0.060	-0.053	-0.060	-0.047	-0.087	-0.094	-0.056	-0.101	-0.063	-0.070
$\breve{R}_{JP,t-1}$	-0.033	0.018	0.021	-0.064	0.029	0.002	-0.037	-0.052	0.005	0.022	0.064	-0.072
S.E.	-0.036	-0.053	-0.051	-0.045	-0.051	-0.040	-0.074	-0.080	-0.048	-0.086	-0.054	-0.060
$\breve{R}_{SI,t-1}$	0.007	-0.005	-0.002	-0.015	**-0.128**	-0.004	0.066	-0.040	-0.007	**0.199**	**0.012**	0.105
S.E.	-0.042	-0.062	-0.060	-0.053	**-0.059**	-0.046	-0.086	-0.093	-0.056	**-0.100**	**-0.063**	-0.070
$\breve{R}_{UK,t-1}$	-0.049	-0.062	-0.057	-0.007	0.025	**-0.177**	-0.039	0.038	0.021	-0.193	0.034	-0.110
S.E.	-0.062	-0.092	-0.088	-0.078	-0.088	**-0.069**	-0.128	-0.138	-0.082	-0.147	-0.093	-0.103
$\breve{R}_{AR,t-1}$	-0.027	0.024	0.033	0.011	0.028	-0.009	-0.001	0.027	0.051	-0.027	0.017	0.022
S.E.	-0.023	-0.034	-0.032	-0.029	-0.032	-0.025	-0.047	-0.050	-0.030	-0.054	-0.034	-0.038
$\breve{R}_{BR,t-1}$	0.012	0.034	-0.004	0.009	-0.017	0.012	-0.037	**-0.130**	0.022	0.042	-0.020	-0.050
S.E.	-0.024	-0.036	-0.034	-0.030	-0.034	-0.027	-0.050	**-0.054**	-0.032	-0.057	-0.036	-0.040
$\breve{R}_{CH,t-1}$	-0.044	-0.035	-0.059	-0.009	-0.050	-0.043	-0.045	-0.006	0.002	-0.105	0.006	-0.012
S.E.	-0.028	-0.042	-0.040	-0.035	-0.040	-0.031	-0.058	-0.062	-0.037	-0.067	-0.042	-0.046
$\breve{R}_{ID,t-1}$	0.009	-0.020	-0.033	-0.017	0.019	0.001	-0.052	0.007	-0.020	-0.047	**-0.060**	-0.013
S.E.	-0.018	-0.027	-0.026	-0.023	-0.026	-0.020	-0.037	-0.040	-0.024	-0.043	**-0.027**	-0.030
$\breve{R}_{MA,t-1}$	0.003	0.003	0.007	0.012	0.105	0.019	0.005	0.043	0.057	-0.010	0.025	-0.017
S.E.	-0.029	-0.043	-0.041	-0.037	-0.041	-0.032	-0.060	-0.065	-0.039	-0.069	-0.044	-0.048
$\breve{R}_{ME,t-1}$	0.023	-0.019	-0.025	-0.029	-0.021	0.008	0.033	-0.084	-0.085	0.058	-0.016	-0.027
S.E.	-0.036	-0.053	-0.051	-0.045	-0.051	-0.040	-0.074	-0.079	-0.048	-0.085	-0.054	-0.059

Number in bold face indicates the coefficient is significant at 5% level.

Table 5: VEC-GARCH Model By Modified FGLS Estimator

	$\check{R}_{US,t}$	$\check{R}_{GE,t}$	$\check{R}_{HK,t}$	$\check{R}_{JP,t}$	$\check{R}_{SI,t}$	$\check{R}_{UK,t}$	$\check{R}_{AR,t}$	$\check{R}_{BR,t}$	$\check{R}_{CH,t}$	$\check{R}_{ID,t}$	$\check{R}_{MA,t}$	$\check{R}_{ME,t}$
Coint.Eq.	**-0.021**	**-0.025**	0.019	**0.030**	**0.025**	**-0.014**	0.001	**-0.041**	-0.017	**0.035**	**0.095**	-0.011
S.E.	**0.006**	**0.008**	0.010	**0.008**	**0.009**	**0.007**	0.010	**0.016**	0.011	**0.017**	**0.012**	0.011
$\check{R}_{US,t-1}$	**-0.106**	0.044	**0.149**	0.131	**0.165**	0.083	0.005	-0.016	-0.056	**0.217**	-0.012	0.000
S.E.	**0.050**	0.071	**0.066**	0.068	**0.061**	0.053	0.100	0.111	0.078	**0.100**	0.060	0.082
$\check{R}_{GE,t-1}$	0.032	**-0.117**	0.094	-0.043	0.034	-0.059	0.076	0.033	0.080	0.055	**0.094**	0.044
S.E.	0.037	**0.056**	0.048	0.050	0.045	0.040	0.074	0.081	0.057	0.070	**0.044**	0.058
$\check{R}_{HK,t-1}$	0.018	0.076	-0.051	0.034	0.017	**0.102**	0.109	**0.279**	0.018	0.037	0.016	0.059
S.E.	0.031	0.044	0.046	0.043	0.041	**0.033**	0.060	**0.074**	0.051	0.073	0.045	0.053
$\check{R}_{JP,t-1}$	-0.029	0.025	-0.031	-0.060	0.023	0.007	-0.010	-0.078	-0.013	0.018	0.004	-0.089
S.E.	0.028	0.040	0.038	0.042	0.035	0.029	0.058	0.065	0.044	0.060	0.036	0.047
$\check{R}_{SI,t-1}$	-0.004	-0.042	-0.027	-0.009	**-0.107**	-0.049	0.009	-0.085	-0.019	0.064	0.016	0.039
S.E.	0.032	0.046	0.049	0.046	**0.046**	0.034	0.066	0.079	0.051	0.078	0.048	0.057
$\check{R}_{UK,t-1}$	0.028	0.061	-0.018	0.051	-0.038	-0.048	0.017	0.199	0.057	-0.172	-0.054	0.042
S.E.	0.050	0.073	0.064	0.068	0.059	0.055	0.101	0.112	0.078	0.095	0.059	0.080
$\check{R}_{AR,t-1}$	-0.015	**0.021**	0.029	0.016	**0.048**	-0.003	0.035	0.032	0.052	-0.027	0.025	0.048
S.E.	0.018	**0.028**	0.023	0.027	**0.022**	0.019	0.044	0.041	0.028	0.034	0.021	0.029
$\check{R}_{BR,t-1}$	-0.002	0.012	-0.006	0.001	-0.013	-0.011	0.009	-0.096	0.005	**0.065**	0.010	-0.020
S.E.	0.019	0.028	0.025	0.027	0.023	0.020	0.040	0.045	0.030	**0.039**	0.024	0.031
$\check{R}_{CH,t-1}$	**-0.068**	**-0.071**	**-0.074**	**-0.069**	**-0.073**	**-0.083**	**-0.145**	**-0.129**	0.005	**-0.118**	0.002	**-0.097**
S.E.	**0.020**	**0.029**	**0.027**	**0.029**	**0.025**	**0.022**	**0.041**	**0.046**	0.036	**0.043**	0.026	**0.034**
$\check{R}_{ID,t-1}$	0.014	-0.008	-0.043	-0.009	0.027	0.002	0.003	0.043	-0.031	0.030	**-0.063**	0.013
S.E.	0.014	0.019	0.023	0.021	0.021	0.015	0.030	0.037	0.022	0.041	**0.024**	0.026
$\check{R}_{MA,t-1}$	0.028	0.051	**0.085**	0.015	**0.132**	**0.056**	0.044	0.071	0.069	0.069	**0.089**	0.028
S.E.	0.023	0.031	**0.035**	0.030	**0.032**	**0.023**	0.044	0.056	0.036	0.058	**0.040**	0.040
$\check{R}_{ME,t-1}$	0.031	-0.024	-0.002	-0.025	0.014	0.014	-0.035	-0.099	-0.056	-0.002	0.010	-0.048
S.E.	0.028	0.040	0.038	0.040	0.035	0.029	0.058	0.066	0.045	0.061	0.035	0.049

Number in bold face indicates the coefficient is significant at 5% level.

4.2. Expected Return of Risky Asset

The estimated parameters and their standard errors in the VECM and VEC-GARCH model are different. Because MGARCH error structure is assumed, the estimated expected returns are based on the VEC-GARCH model.

The statistics of the estimated expected return and realized return are presented in Table 6. In general, emerging stock markets such as Brazil, China, and Mexico had higher expected return, yet they were also more volatile than those in the developed markets. Several economic crises and recessions took place during the observation period, such that the averages of expected returns in most countries were negative. The long period of recession in Japan was causing both its realized and expected returns are negative. In emerging markets, only stock market in Argentina that consistently has negative realized and expected return.

Table 6: Annualized Weekly Statistics

Realized Return	US	GE	HK	JP	SI	UK	AR	BR	CH	ID	MA	ME
Mean	0.024	0.034	0.015	-0.030	0.033	0.004	-0.029	0.044	0.056	0.020	0.014	0.096
Std.Dev.	0.178	0.265	0.255	0.225	0.255	0.199	0.367	0.397	0.238	0.430	0.275	0.295
Expected Return*	US	GE	HK	JP	SI	UK	AR	BR	CH	ID	MA	ME
Mean	0.008	-0.001	-0.005	-0.014	-0.010	0.002	-0.006	0.000	0.002	-0.005	-0.031	-0.001
Std.Dev.	0.026	0.033	0.048	0.035	0.061	0.031	0.057	0.081	0.037	0.081	0.076	0.039

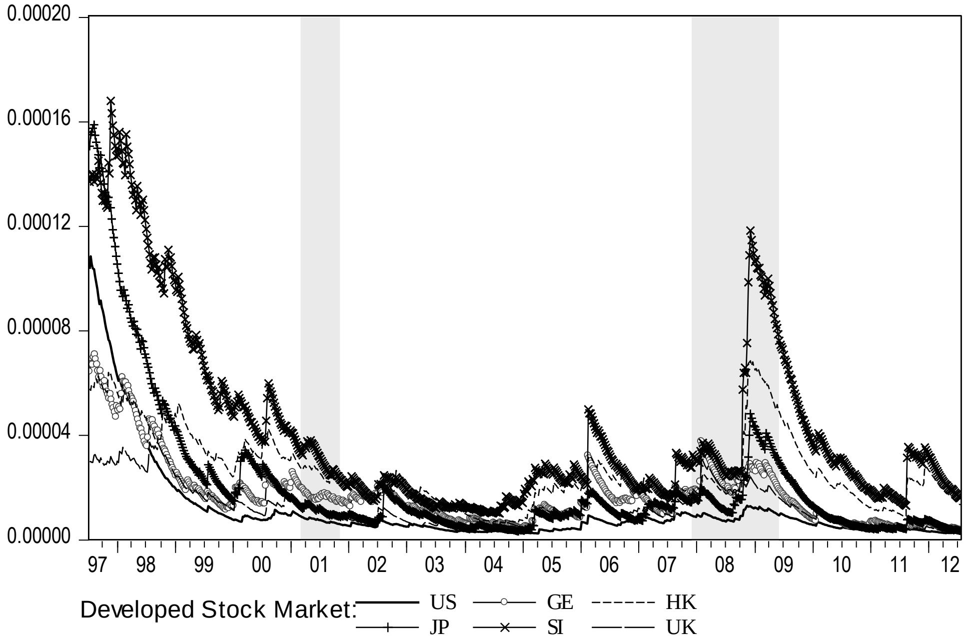
*The estimated expected return was estimated by VEC-GARCH using modified FGLS estimator

4.3. Conditional Variance of Risky Asset

Conditional variances for each market expected return were estimated using Diagonal BEKK model of Engle and Kroner (1995) as specified in equation (2) and the results are shown in Figure 1 and 2 for the developed and emerging markets respectively.

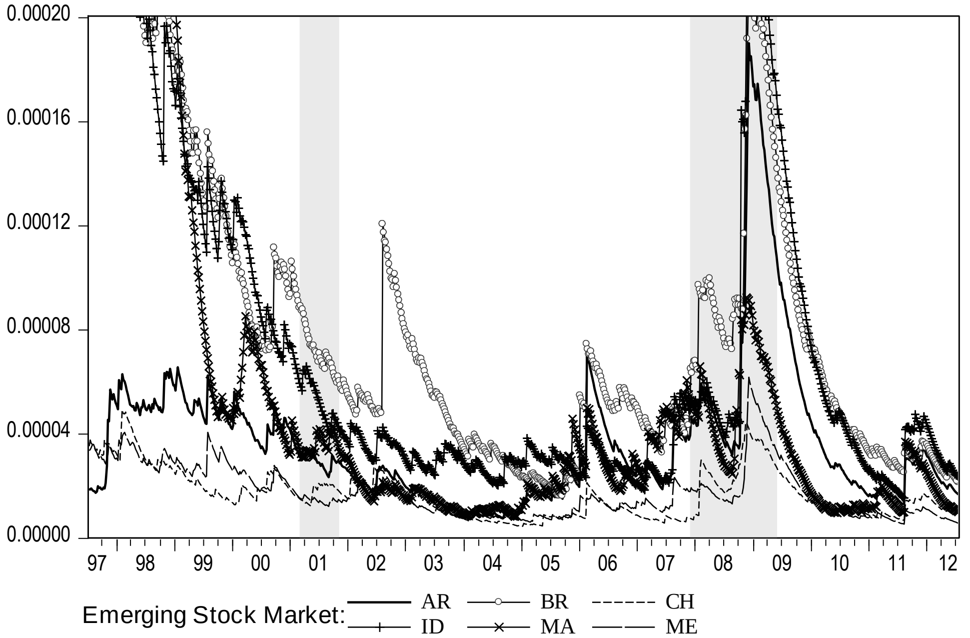
The figures show that the conditional variances were increasing during period of crises, yet the magnitudes varied across the samples. For example, non-Asian developed and emerging stock markets were less affected by the Asian financial crisis in 1997-1998. However, the US financial crisis in 2008-2009 seems spilled over to other markets and the Asian markets were becoming more volatile in that period. Emerging markets apparently show higher volatility than that for the developed markets. The different magnitudes of volatility indicate that there is opportunity to obtain lower diversifiable risk by investing in those markets; this could be the driving factor of stronger price comovements and stock market integration.

Figure 1: Conditional Variance of Estimated Expected Return in Developed Markets



Note: Shaded area is the US recession period (based on NBER Business Cycle Dating Committee report, last update was on September 20, 2010).

Figure 2: Conditional Variance of Estimated Expected Return in Emerging Markets



Note: The scale was trimmed to conform to the Figure 1. Shaded area is the US recession period (based on NBER Business Cycle Dating Committee report, last update was on September 20, 2010).

4.4. Test of International CAPM

The *ex-ante* and *ex-post* test for the CAPM were carried out under both SUR (without GARCH) and SUR-GARCH. The results are shown in Table 7 and 8 respectively. In *ex-ante* test (Table 7), the null hypotheses H_0^1 and H_0^2 are all rejected, they indicate that CAPM does not work well for the international assets and the market risk premiums are heterogeneous across the markets. However, individual test of the hypothesis that $\hat{\alpha}_i = 0$ (*t*-test) show that CAPM can be applied for pricing of all market indexes (except for the Malaysian market), even under the SUR-GARCH test, all alphas are not statistically different from zero. It is susceptible that the rejection of H_0^1 were caused by large differences in the standard errors[2]. This is an indication that the market risk premium adjustments across the markets were so vary during the observation period. The results suggest that removing some markets from the sample might alter the verdict that the CAPM does not fit well for international asset pricing.

The homogeneity test of the market risk premiums are also rejected in both tests. It suggests that the stock markets were not fully integrated yet. Market risk is priced higher in Asian stock markets such as in Singapore, Indonesia, and Malaysia, than that in other markets. Meanwhile, in the US and the Japan, the market risk premium is lower than that in the other markets.

Note that all market risk premiums are nonnegative. This is the expected result. It indicates that the constructed world market portfolio is always in the efficient frontier and is at the tangency of capital market line.

[2] The differences in the standard errors cannot be seen in the tables because the numbers were rounded to only three decimals.

Table 7: *Ex-Ante* International Dynamic Beta CAPM Test

	SUR				SUR-GARCH			
	Coef.	S.E.	t-Stat.	Prob.	Coef.	S.E.	t-Stat.	Prob.
$\hat{\alpha}_{US}$	0.000	0.000	1.885	0.059	0.000	0.000	0.279	0.781
$\hat{\theta}_{US}$	**0.021**	**0.000**	**51.835**	**0.000**	**0.021**	**0.000**	**74.689**	**0.000**
$\hat{\alpha}_{GE}$	0.000	0.000	1.144	0.253	0.000	0.000	-1.152	0.252
$\hat{\theta}_{GE}$	**0.022**	**0.000**	**65.176**	**0.000**	**0.022**	**0.000**	**95.359**	**0.000**
$\hat{\xi}_{GE}$	-0.002	0.003	-0.596	0.552	-0.001	0.002	-0.564	0.574
$\hat{\alpha}_{HK}$	0.000	0.000	0.180	0.857	0.000	0.000	1.747	0.084
$\hat{\theta}_{HK}$	**0.022**	**0.000**	**70.061**	**0.000**	**0.022**	**0.000**	**104.130**	**0.000**
$\hat{\xi}_{HK}$	0.066	0.056	1.175	0.240	**0.063**	**0.031**	**2.032**	**0.045**
$\hat{\alpha}_{JP}$	**0.000**	**0.000**	**-2.233**	**0.026**	0.000	0.000	0.165	0.869
$\hat{\theta}_{JP}$	**0.022**	**0.000**	**71.136**	**0.000**	**0.021**	**0.000**	**105.469**	**0.000**
$\hat{\xi}_{JP}$	0.002	0.003	0.829	0.407	0.002	0.002	0.951	0.344
$\hat{\alpha}_{SI}$	0.000	0.000	-0.349	0.727	0.000	0.000	0.293	0.770
$\hat{\theta}_{SI}$	**0.024**	**0.000**	**67.676**	**0.000**	**0.023**	**0.000**	**101.945**	**0.000**
$\hat{\xi}_{SI}$	0.018	0.010	1.765	0.078	**0.012**	**0.006**	**2.037**	**0.044**
$\hat{\alpha}_{UK}$	0.000	0.000	1.751	0.080	0.000	0.000	0.043	0.966
$\hat{\theta}_{UK}$	**0.022**	**0.000**	**71.038**	**0.000**	**0.022**	**0.000**	**103.266**	**0.000**
$\hat{\xi}_{UK}$	0.001	0.003	0.465	0.642	0.002	0.002	0.822	0.413
$\hat{\alpha}_{AR}$	0.000	0.000	-0.122	0.903	0.000	0.000	0.053	0.958
$\hat{\theta}_{AR}$	**0.023**	**0.000**	**81.410**	**0.000**	**0.022**	**0.000**	**121.855**	**0.000**
$\hat{\xi}_{AR}$	0.000	0.003	-0.098	0.922	0.000	0.002	-0.143	0.886
$\hat{\alpha}_{BR}$	0.000	0.000	0.820	0.412	0.000	0.000	-1.226	0.223
$\hat{\theta}_{BR}$	**0.023**	**0.000**	**62.376**	**0.000**	**0.022**	**0.000**	**92.981**	**0.000**
$\hat{\xi}_{BR}$	0.000	0.005	-0.007	0.994	0.001	0.003	0.253	0.801
$\hat{\alpha}_{CH}$	0.000	0.000	-0.298	0.766	0.000	0.000	-1.898	0.061
$\hat{\theta}_{CH}$	**0.022**	**0.000**	**75.680**	**0.000**	**0.021**	**0.000**	**102.483**	**0.000**
$\hat{\xi}_{CH}$	-0.040	0.023	-1.710	0.087	-0.011	0.015	-0.749	0.455
$\hat{\alpha}_{ID}$	0.000	0.000	-0.721	0.471	0.000	0.000	0.426	0.671
$\hat{\theta}_{ID}$	**0.024**	**0.000**	**67.258**	**0.000**	**0.023**	**0.000**	**103.701**	**0.000**
$\hat{\xi}_{ID}$	0.003	0.002	1.471	0.141	0.003	0.001	1.943	0.055
$\hat{\alpha}_{MA}$	**-0.001**	**0.000**	**-2.937**	**0.003**	0.000	0.000	0.765	0.446
$\hat{\theta}_{MA}$	**0.024**	**0.000**	**53.605**	**0.000**	**0.023**	**0.000**	**91.641**	**0.000**
$\hat{\xi}_{MA}$	0.008	0.008	0.929	0.353	0.004	0.005	0.774	0.441
$\hat{\alpha}_{ME}$	0.000	0.000	-0.348	0.728	0.000	0.000	-1.380	0.171
$\hat{\theta}_{ME}$	**0.022**	**0.000**	**69.672**	**0.000**	**0.022**	**0.000**	**99.530**	**0.000**
$\hat{\xi}_{ME}$	0.000	0.003	-0.126	0.900	0.000	0.002	-0.177	0.860
Coefficients Wald Test								
Null Hypothesis		d.f.	Chi-sq	Prob.		d.f.	Chi-sq	Prob.
$\hat{\alpha}_i = 0, \forall i$		12	**30.843**	**0.002**		12	**26.463**	**0.009**
$\lvert\hat{\gamma}_1\rvert = \lvert\hat{\gamma}_2\rvert = \cdots = \lvert\hat{\gamma}_N\rvert$		11	**69.525**	**0.000**		11	**174.795**	**0.000**

Number in bold face indicates the coefficient is significant at 5% level.

The effect of changes in currency exchange rates seemed to be absorbed by the market risk factor. Under the SUR-GARCH test, the additional required rate of return to compensate the exchange rate risk is only applied for assets from Hong Kong and Singapore stock market. This result indicates that the market risk (beta) is still the only relevant risk factor in the International CAPM (provided that un-integrated stock markets were removed from the sample such that H_0^1 could not be rejected).

Table 8: *Ex-Post* International Dynamic Beta CAPM Test

	SUR				SUR-GARCH			
	Coef.	S.E.	t-Stat.	Prob.	Coef.	S.E.	t-Stat.	Prob.
$\hat{\alpha}_{US}$	0.000	0.001	0.199	0.842	0.000	0.000	0.670	0.504
$\hat{\theta}_{US}$	-0.019	0.003	-6.893	**0.000**	-0.015	0.002	-8.040	**0.000**
$\hat{\alpha}_{GE}$	0.000	0.001	0.219	0.827	0.000	0.001	0.472	0.638
$\hat{\theta}_{GE}$	-0.017	0.003	-6.124	**0.000**	-0.011	0.002	-5.722	**0.000**
$\hat{\xi}_{GE}$	0.644	0.051	12.604	**0.000**	0.652	0.036	18.327	**0.000**
$\hat{\alpha}_{HK}$	0.000	0.001	-0.346	0.729	0.000	0.001	-0.222	0.825
$\hat{\theta}_{HK}$	-0.006	0.003	-1.792	0.073	-0.002	0.002	-0.793	0.430
$\hat{\xi}_{HK}$	1.300	0.830	1.566	0.117	1.782	0.485	3.672	**0.000**
$\hat{\alpha}_{JP}$	-0.001	0.001	-1.316	0.188	-0.002	0.001	-2.260	**0.026**
$\hat{\theta}_{JP}$	-0.011	0.003	-3.260	**0.001**	-0.007	0.002	-3.027	**0.003**
$\hat{\xi}_{JP}$	0.658	0.062	10.635	**0.000**	0.623	0.045	13.819	**0.000**
$\hat{\alpha}_{SI}$	-0.001	0.001	-0.737	0.461	0.000	0.001	-0.007	0.994
$\hat{\theta}_{SI}$	-0.003	0.003	-0.977	0.329	0.000	0.002	0.023	0.981
$\hat{\xi}_{SI}$	1.227	0.102	11.996	**0.000**	1.112	0.061	18.143	**0.000**
$\hat{\alpha}_{UK}$	0.000	0.001	0.236	0.813	0.000	0.000	0.150	0.881
$\hat{\theta}_{UK}$	-0.018	0.002	-7.196	**0.000**	-0.012	0.002	-7.459	**0.000**
$\hat{\xi}_{UK}$	0.653	0.040	16.178	**0.000**	0.661	0.027	24.050	**0.000**
$\hat{\alpha}_{AR}$	-0.002	0.002	-0.928	0.353	-0.002	0.001	-1.898	0.061
$\hat{\theta}_{AR}$	0.008	0.004	2.189	**0.029**	0.008	0.003	3.051	**0.003**
$\hat{\xi}_{AR}$	0.345	0.072	4.759	**0.000**	0.462	0.077	6.027	**0.000**
$\hat{\alpha}_{BR}$	-0.001	0.002	-0.408	0.683	0.001	0.001	0.591	0.556
$\hat{\theta}_{BR}$	-0.005	0.003	-1.717	0.086	-0.005	0.002	-2.877	**0.005**
$\hat{\xi}_{BR}$	1.037	0.054	19.222	**0.000**	1.050	0.035	30.399	**0.000**
$\hat{\alpha}_{CH}$	0.000	0.001	-0.350	0.726	-0.001	0.001	-0.647	0.519
$\hat{\theta}_{CH}$	0.002	0.004	0.568	0.570	0.005	0.003	1.558	0.122
$\hat{\xi}_{CH}$	0.853	0.736	1.159	0.247	1.389	0.524	2.652	**0.009**
$\hat{\alpha}_{ID}$	0.000	0.002	-0.078	0.938	0.003	0.001	2.919	**0.004**
$\hat{\theta}_{ID}$	-0.001	0.003	-0.424	0.671	0.001	0.002	0.476	0.635
$\hat{\xi}_{ID}$	0.978	0.035	28.243	**0.000**	1.064	0.032	33.572	**0.000**
$\hat{\alpha}_{MA}$	0.000	0.001	-0.276	0.783	0.001	0.001	1.404	0.163
$\hat{\theta}_{MA}$	-0.004	0.004	-1.014	0.310	0.002	0.002	0.674	0.502
$\hat{\xi}_{MA}$	1.093	0.081	13.481	**0.000**	1.090	0.061	17.976	**0.000**
$\hat{\alpha}_{ME}$	0.002	0.001	1.391	0.164	0.003	0.001	3.718	**0.000**
$\hat{\theta}_{ME}$	-0.007	0.003	-2.307	**0.021**	-0.007	0.002	-3.845	**0.000**
$\hat{\xi}_{ME}$	1.128	0.071	15.891	**0.000**	1.067	0.042	25.520	**0.000**
Coefficients Wald Test								
Null Hypothesis		d.f.	Chi-sq	Prob.		d.f.	Chi-sq	Prob.
$\hat{\alpha}_i = 0, \forall i$		12	9.442	0.665		12	**45.876**	**0.000**
$\lvert\hat{\gamma}_1\rvert = \lvert\hat{\gamma}_2\rvert = \cdots = \lvert\hat{\gamma}_N\rvert$		11	**36.816**	**0.000**		11	**148.947**	**0.000**

Number in bold face indicates the coefficient is significant at 5% level.

The *ex-post* test presented in Table 8 examined whether CAPM was applied in the market for pricing the international assets. The test was using realized or actual returns. From the table, CAPM fits well in pricing the assets when multivariate GARCH error structure was ignored (the test under SUR). Testing the alphas individually shall support the finding. However, from the data properties of the sample, the multivariate GARCH error structure does exist. Ignoring the error structure proved that the latter conclusion was inaccurate; when GARCH error structure was considered, the CAPM does not fit well! This result is inline with previous findings, for example in Lewellen and Nagel (2006) and Wu and Chiou

(2007). The latter paper used Kalman filter method in testing the CAPM. Thus, this paper shows the applicability of SUR-GARCH model using the modified FGLS estimator that is simpler than other method such as the Kalman filter.

As the previous analysis in the *ex-ante* test, removing some stock market indexes might make CAPM works. Under the SUR-GARCH test, CAPM only failed to work for pricing the Japanese and the Indonesian stock market returns. This research shows that constructing world market portfolio from the twelve indexes leads to inapplicable CAPM. Finding the stock market indexes that make CAPM works is subject to future research.

The realized market risk premiums are also heterogeneous. Some of the market risk premiums are negative. They indicate that those indexes returns have negative covariance with the world market portfolio. This is as a result of allowing the short selling. Moreover, this result also suggests that the stock markets were not fully integrated yet. The price of the market risk for the assets varies. In addition, exchange rate risk is also priced differently for the market indexes. It indicates that beta risk is not the only risk factor considered by investors. Multi factors CAPM may be applied in the markets.

5. CONCLUSIONS

Conditional International CAPM was tested under assumptions of unrestricted short selling and borrowing at riskless rate such that the constructed world market portfolio is a mean-variance efficient portfolio at the tangency of the capital market line. Previous researches that attempt to test the CAPM for international assets were using readily available index that was subject to the critique that the market portfolio was not mean-variance efficient. By constructing the world market portfolio that meets with the assumptions, we could estimates not only the expected return and conditional variance of that portfolio, but also the conditional covariance between the world market portfolio and its composing portfolios (national stock market indices), such that time varying betas for each international asset can be estimated. Thus, all assumptions used in building the CAPM were all met so that the test of the model can be emphasized on the estimated parameters of the model.

The test of the international asset-pricing model was carried out by assuming that the markets were fully integrated so that covariance among the disturbance terms is not zero. Furthermore, because crises, recession, and other kind of market shocks happened during the observation period in 1997:7 until 2012:7, heteroscedasticity of the errors is expected, and the sample data properties said so. To consider the covariance and the heteroscedasticity, the time varying beta CAPM was tested under SUR-GARCH model. In *ex-ante* test CAPM is rejected, but individual analysis of the assets showed that CAPM may be applied for all of the assets. It indicates that the risk premium adjustments were done during the period of analysis, and the levels of adjustments varied across the assets. The market risk is also priced differently for each assets, it indicates that the markets were not fully integrated yet. However, exchange rate risk is not significantly affecting the expected excess return. The latter shows that theoretically CAPM is correct, that the only risk factor worth to consider is the market risk (beta). However, in the real world, the *ex-post* tests show that CAPM had failed in pricing the international assets and it suggests that other risk factors might exist.

The findings are subject to the stock market index selection for constructing the world market portfolio. In addition, CAPM might also work under sub-sample periods, for example under non-recession period. Finding the structural break point under SUR-GARCH also a challenge in the econometric side. These issues should be addressed in the future research.

REFERENCES

Bekaert, G., and Harvey, C. R. (1995). Time-Varying World Market Integration. The Journal of Finance, 50:2, pp. 403–444.

Black, F. (1972). Capital market equilibrium with restricted borrowing, *Journal of Business,* Vol. 45, pp. 444–55.

Bollerslev, T., Engle, R. F. and Wooldridge, J. M. (1988). Capital asset pricing model with time-varying covariances, *Journal of Political Economy*, 96, 116–31.

Breitung, Jörg (2000). The Local Power of Some Unit Root Tests for Panel Data, in B. Baltagi (ed.), *Advances in Econometrics, Vol. 15: Nonstationary Panels, Panel Cointegration, and Dynamic Panels*, Amsterdam: JAI Press, p. 161–178.

Chan, Louis K.C., Yasushi Hamao and Josef Lakonishok. (1991). Fundamentals and Stock Returns in Japan, *Journal of Finance*, 46:5, pp. 1739–789.

Chen, Z., and Knez, P. J. (1995). Measurement of Market Integration and Arbitrage, *The Review of Financial Studies*, 8:2, pp.287–325.

Das, S. and R. Uppal, (2004). Systemic Risk and International Portfolio Choice, *Journal of Finance*, 59:6, pp. 2809-2834.

deBondt, Werner F. M. and Richard H. Thaler, (1987). Further Evidence on Investor Overreaction and Stock Market Seasonality, *Journal of Finance*. 42:3, pp. 557–81.

Engel, C., and Rodrigues, A. P., (1989). Tests of international CAPM with time-varying covariances. *Journal of Applied Econometrics*, Vol. 4, 119–138.

Engle, Robert F. and K. F. Kroner, (1995). Multivariate Simultaneous Generalized ARCH, *Econometric Theory*, 11, pp. 122-150.

Fama, E. F., and French, K. R. (1992). The cross-section of expected stock returns, *Journal of Finance*, pp. 427–465.

Fama, E. F., and French, K. R., (2004). The Capital Asset Pricing Model: Theory and Evidence, *Journal of Economic Perspectives*, 18, pp. 25–46.

Fama, E. F., and MacBeth, J. D., (1973). Risk, return, and equilibrium: Empirical tests, *The Journal of Political Economy*, 81(3), pp. 607–636.

Fama, E.F. and French, K.R., (1998). Value Versus Growth: The International Evidence. *Journal of Finance*, 53:6, pp. 1975–999.

Fernandez, V. P. (2005). The international CAPM and a wavelet-based decomposition of value at risk, *Studies in Nonlinear Dynamics & Econometrics*, 9:4, pp.1–35.

Fischer Black and Robert Litterman, (1992). Global Portfolio Optimization, *Financial Analysts Journal*, 48:5, pp. 28-43

French, K. R., and Poterba, J. M. (1991). Investor Diversification and International Equity Markets, *The American Economic Review*, 81(2), pp. 222–226.

Gibbons, M. R., Ross, S. A., and Shanken, J. (1989). A test of the efficiency of a given portfolio. Econometrica, *Journal of the Econometric Society*, pp. 1121–1152.

Greene, W. H. (2008). *Econometric Analysis*, Prentice Hall, Boston.

Hadri, Kaddour (2000). Testing for Stationarity in Heterogeneous Panel Data, *Econometric Journal*, 3, pp. 148–161.

Haugen, R. A., and Baker, N. L. (1991). The efficient market inefficiency of capitalization-weighted stock portfolios. *The Journal of Portfolio Management*, (Spring), 35–40.

Kothari, S. P., Jay Shanken and Richard G. Sloan. (1995). Another Look at the Cross-Section of Expected Stock Returns, *Journal of Finance*, 50:1, pp. 185–224.

Kumar, P., Sorescu, S. M., Boehme, R. D., and Danielsen, B. R. (2008). Estimation Risk, Information, and the Conditional CAPM: Theory and Evidence, *Review of Financial Studies*, 21:3, pp. 1037–1075.

Levin, A., C. F. Lin, and C. Chu (2002). Unit Root Tests in Panel Data: Asymptotic and Finite-Sample Properties, *Journal of Econometrics,* 108, 1–24.

Levy, H. (2010). The CAPM is Alive and Well: A Review and Synthesis. European Financial Management, 16(1), pp. 43–71.

Levy, M. (2008). CAPM risk-return tests and the length of the sampling period, Working Paper, Hebrew University.

Lewellen, J., and Nagel, S. (2006). The Conditional CAPM does not explain asset-pricing anomalies, *Journal of Financial Economics*, 82, 289-314.

Lintner, J., (1965). The Valuation of Risk Assets and the Selection of Risky Investments m Stock Portfolios and Capital Budgets, *Review of Economics and*

Statistics, pp. 13-37.
MacKinnon, J.G., Haug, A.A., and Michelis, L., (1999), Numerical Distribution Functions of Likelihood Ratio Tests for Cointegration, *Journal of Applied Econometrics*, 14, pp. 563-577.
Maekawa, K., and Setiawan, K., (2012), Estimation of Vector Error Correction Model with GARCH Errors, *Proceeding on SMU-ESSEC Symposium on Empirical Finance & Financial Econometrics, 8-9 June 2012, Singapore.*
Merton, R.C., (1972). An analytic derivation of the efficient portfolio frontier, *Journal of Financial and Quantitative Analysis*, 7, pp. 1851–72.
Pennachi, G. (2008). *Theory of Asset Pricing*, Pearson, Boston.
Sharpe, W., (1964). Capital Asset Prices: A Theory of Market Equilibrium under Conditions of Risk. *Journal of Finance*, pp. 425-442.
Tobin, J. (1958). Liquidity Preference as Behavior Towards Risk, *The Review of Economic Studies*, 67, pp. 65–86.
Tsuji, C. (2009). Can We Resurrect the CAPM in Japan? Evaluating Conditional Asset Pricing Models by Incorporating Time-varying Price of Risk. *Research in Applied Economics*, 1:1.
Wu, P. S., & Chiou, J. S. (2007). Multivariate test of Sharpe-Lintner CAPM with Time-Varying Beta. *Applied Financial Economics Letters*, 3(5), 335–341.
Wu, H. (2008). International Asset Pricing Models: A Forecasting Evaluation, *International Research Journal of Finance and Economics*, 15, pp. 175–184.
Zellner, A. (1962). An efficient method of estimating seemingly unrelated regressions and tests for regression bias, *Journal of the American Statistical Association*, 57, 348–6.