Entrepreneurial skills, entrepreneurial technology and firm growth.*

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Abstract

One recent empirical regularity is that firm-growth is negatively related to firm's age. Besides, employment-age profiles are flatter in less developed economies, but it is also observed flatter employment-age profiles among fast growing economies rather than in slow growing economies. This paper develops an occupational choice life-cycle model based on Guner et al. (2015), where entrepreneurs' skills determine entrepreneurial technology in a similar way to Poschke (2015). We consider that exogenous technological advances imply a higher degree of complexity. Entrepreneurs' skills determine the degree of complexity they can manage. As in Guner et al. (2015), entrepreneurs invest in their skills over their life-cycle. But, unlike Guner et al. (2015), entrepreneurs' skills depreciation depends also on the rhythm at which skills of newborn entrepreneurs are growing. The empirical implication of the stationary equilibrium concerning firm growth is consistent with these observed facts.

Keywords: Total Factor Productivity, entrepreneurs, skills, overlapping generations, depreciation.

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1 Introduction

One recent empirical regularity is that young firms grow more than older firms (see, among others, Barba et al. (2014), Coad et al. (2013), Dunne et al. (1989), Evans (1987a, 1987b), Fariñas and Moreno (2000), Haltiwanger et al. (2012), López-García and Puente (2012)). This empirical regularity is observed among most European countries and the U.S.. That is, firm-growth (measured by the increase in the number of employees) is negatively related to firm's age, once it has been controlled for some other factors. Furthermore, Hsieh and Klenow (2014) find flatter employment-age profiles in less developed economies than in more developed economies. And, Caunedo and Yurdagul (2016) find flatter employment-age profiles in fast growing economies and steeper ones in slow growing economies. At micro level, there is some empirical work relating managers' abilities and firm growth (see, among others, Queiro (2015)). In this paper we focus on the role of entrepreneurs' skills on firm growth, and we show that higher growth rate of technological advances makes entrepreneurials' skills more easily depreciated with age. Consequently, we can explain flatter employment-age profiles not only in more developed economies but also in faster growing economies.

In this paper we consider a model based on Guner et al. (2015) and Poschke (2015). As in Guner et al. (2015), we consider an occupational choice overlapping generations economy in which a single output good is produced by heterogeneous plants or production units. Heterogeneous firms coexist because each firm faces a "span of control" or diminishing returns to scale of the production function á la Lucas (1978). As in Poschke (2015), we consider that there are exogenous technological advances in the economy and that new advances imply a higher degree of complexity. Entrepreneurs' skills determine the degree of complexity they can manage and, hence, the degree of adoption of new technologies. As in Guner et al. (2015), there are overlapping generations of finitely-lived entrepreneurs and they invest in their skills over their life-cycle[1]. However, unlike Guner et al. (2015), entrepreneurs' skills investment will depend on their return on managing a more complex technology and on the depreciation of their relative skill with respect to newborn entrepreneurs' skills. Therefore, faster exogenous growth of technological advances can lead to a higher growth of firms' productiv-

[1]Family succession is not allowed.

ity, but also may imply a higher depreciation of old entrepreneurs abilities. This will affect the age-profile of firms' productivity growth. We show that employment-age profiles can be steeper in more developed economies but also flatter in faster growing economies.

Our paper is related to a vast recent literature regarding plant size distribution characteristics. Firstly, we should mention one of the first papers of this literature, named Lucas (1978). Lucas (1978) builds a quite simple occupational choice economy populated by a constant distribution of one period life-time individuals differing in their ability or talent. Lucas (1978) studies the relationship between the economy's level of per capita wealth and its corresponding average firm size. From a theoretical point of view, one of the main conclusions is that if the elasticity of substitution among input factors is less than one, we should observe a positive relationship between per capita wealth and average firm size. This is so because a higher per capita wealth leads to a higher wage rate and, consequently, a higher opportunity cost of becoming a manager. Therefore, the entrepreneurship rate decreases and the average firm size increases. Lucas (1978) makes a simple estimation using time series of the U.S. data corroborating this empirical prediction.

Recently there are some papers analyzing not only the observed differences in the static plant size distribution characteristics but also differences in firm dynamics (in particular, employment age profile) across countries. From a static point of view, there is some consensus that richer economies have a higher fraction of large establishments, higher employment share in large plants, higher skewness of firm size, lower entrepreneurship rate or higher average firm size [See, among others, García-Santana and Ramos (2014), Poschke (2015)]. More recently, there are several papers analyzing employment and productivity growth over the firms' life cycle. As we have mentioned above, there are some empirical regularities concerning firm growth depending on firm's age. In particular, young firms grow more than older firms (see, among others, Barba et al. (2014), Coad et al. (2013), Dunne et al. (1989), Evans (1987a, 1987b), Fariñas and Moreno (2000), Haltiwanger et al. (2012), López-García and Puente (2012). Regarding the relationship between firm growth and the aggregates of an economy, Hsieh and Klenow (2014) find flatter employment-age and productivity-age profiles in less developed countries (India and México) than in more developed economies (the US). And Caunedo and Yurdagul (2016) find flatter employment-age profiles in fast growing economies and steeper ones in slow growing economies among a broad set of countries.

Concerning the empirical work that focus on the factors determining differences in firm growth, there are some papers claiming that managers' abilities is one significant factor. Barba et al. (2014) analyze firms' life cycle employment growth in France, Italy and Spain. They find that a common characteristic to these three countries is that firms grow at a decreasing rate along their life cycle (young firms grow faster than old firms) and that one of the factors that determines the firms' life cycle employment growth are the age of chief executive officers (CEOs). Queiro (2015) finds, among a sample of Portuguese firms, that one additional "*year of manager education increases firm growth by around 0.3-0.4 percentage points*" [Queiro (2015), p. 2], and that "*moving from the distribution of manager education in Portugal to that of the U.S. would raise aggregate productivity by about 20 percent*" [Queiro (2015), abstract]. He also shows, for a sample of 50 countries, that the use of new technologies is more frequent among firms with more educated managers. Poschke (2015) also claims that technological advances do not affect all firms equally because managers differ in their skills and, consequently, in their relative cost in implementing new technologies. On other line of research, some authors claim that individual plants' productivities can be capturing differences in management practices [See Bloom et al. (2012, 2014), Van Reenen (2015), Caliendo et al. (2015) or Garicano (2015), among others]. Caliendo et al. (2015) and Garicano et al. (2015) claim that managers' skills determine the efficient use of knowledge to make decisions, whenever they are required to solve complicate production problems or reorganize production. Bloom et al. (2012, 2014) and Van Reenen (2015) claim that management practices are more relevant than differences in technological innovation. They remark that "*managerial talent will show up as TFP (if properly measured because two firms with the same inputs will produce more output with the better manager*" [See Bloom et al. (2012), p. 9].

In this paper we consider a model based on Guner et al. (2015) and Poschke (2015). Both papers develop an occupational choice version of Lucas (1978) span-of-control model. In a similar way to Poschke (2015), we consider that managers' skills determine entrepreneurial technology. As in Atkeson and Kehoe (2015), we consider that the economy's technology frontier level is growing at an exogenous technological change but, as in Poschke

(2015), we assume that only the more qualified entrepreneurs will be able to produce at the technology frontier level. Poschke (2015) points out that there is a quite vast literature focusing on the relevance of workers' skills in adopting new technologies. However, entrepreneur's skill can be crucial in adopting new technological advances, '*employees need to apply a given technology, while entrepreneurs need to choose and coordinate the technologies used in a firm's production process.... If entrepreneurs want to benefit from the new possibilities put on the menu by technological advances, they need to keep up with technological developments.'* [Poschke (2015), p. 5]. Under Poschke (2015), and as in Guner et al. (2015), entrepreneurs can increase their skills by investing in their skill accumulation. Guner et al. (2015) show that earnings of managers, relative to non-managers, grow with age among most high-income countries. Consequently, Guner et al. (2015) consider entrepreneurs' investment to mimic the higher growth of managers' earnings relative to non-managers. But, unlike Guner et al. (2015), technological progress is characterized by an increase in the degree of complexity in producing the final output, and only more talented entrepreneurs will be able to produce with a higher degree of specialization, similar to Poschke (2015). Poschke (2015) considers that the degree of complexity is determined by the degree of employees' specialization. A higher variety of workers' skills imply a higher degree of complexity to coordinate them in the production process. Unlike Poschke (2015), and similar to Comin and Hobijn (2010), we consider that the degree of complexity is determined by the amount of differentiated types of capital goods. Unlike Caunedo and Yurdagul (2016), we can explain flatter employment-age profiles in fast growing economies under the absence of any shock affecting the size of the technology improvement[2]. We can explain flatter employment-age profiles in fast growing economies because new entrepreneurs are born with a higher knowledge of new technologies, old entrepreneurs' skills may depreciate more rapidly with respect to the new managers' skills in faster growing economies.

[2]Caunedo and Yurdagul (2016) consider a model with innovative and non-innovative firms, whose relative proportion is endogenously determined. Only succesful innovative firms improve their technology. The main factor driving their results is the cross-country difference in their probability of productivity improvement. In our model, we consider that technology improvements grow at an exogenous rate, but these advances do not affect all firms. At the firm level, the adoption of new technologies will depend on managers' skills. Entrepreneurs can increase their skills along their active life-cycle, but their relative productivity with new born generations' skills may decrease.

The paper is organized as follows. In the next section, Section 2 describes the basic model and characterize the balanced growth path solution. Section 3 presents some numerical results and section 4 concludes.

2 The benchmark model.

Our model is based on Guner et al. (2015) and Poschke (2015) and describes a life-cycle occupational choice economy à la Lucas (1978) with no uncertainty. The economy is populated by overlapping generations households that live a finite horizon lifetime. There is no population growth, and without loss of generality, we normalize the size of population at each period of time to 1. At each period of time, a new generation of households is born with an initial endowment of entrepreneurial skill. The initial endowment of entrepreneurial skill has two components: *i)* an exogenous component determined by a distribution, which is constant through time, and *ii)* an endogenous component determined by the growth rate of the economy's technology level.

From the production side, a single final output good is produced by heterogeneous producers that have access to a diminishing returns to scale technology and differ in their managerial skill. Similar to Poschke, the firm's level of technology will depend on the entrepreneur's skill. Individuals' managerial skill will determine its entrepreneurial technology in a similar spirit to Poschke (2015). In particular, entrepreneurial skill will determine the degree of specialization in the production process, that is, *the complexity of the firm's production process* [Poschke (2015), p. 21]. Poschke (2015) considers that technological advances imply *that individual firms have to cope with increasing complexity of technology* [Poschke (2015), p. 4]. And, even though all firms access to a more productive technology, the implementation of a higher degree of complexity will have a higher cost for some firms than for others. Poschke (2015) assumes that *keeping up with advancing technology is costly for entrepreneurs* [Poschke (2015), p. 5]. Consequently, *while advances in the technological frontier give all firms access to a more productive technology, they do not affect all firms equally. Some firms can implement new technologies at lower cost, and therefore take more advantage of them. As a result, some firms remain close to the frontier and use a production process involving many, highly specialized inputs, while others fall behind the* *frontier, use a simpler production process, and fall behind in terms of relative productivity* [Poschke (2015), p.4]. As in Poschke (2015), we will assume that the exogenous technology frontier level, A_t, determines the number of differentiated types of capital that can be used in the production process. As in Guner et al. (2015) we will consider that the individuals' wage rate is the same for all workers, and that do not depend on their skill. Therefore, unlike Poschke (2015), we will not consider that it is required different type of workers' skill for each type of differentiated type of capital.

Firstly, we solve the static entrepreneurs' decision on output. *Secondly*, we solve the household problem for entrepreneurs and for workers. *Thirdly*, we define the balanced growth path for the economy. *Finally,* as in Atkeson and Kehoe (2005), we show that higher productivity implies higher firm size, taking into account that in our economy there is not any type of distortion.

2.1 Entrepreneurs' profit maximization (Static)

All households are born with an initial endowment of entrepreneurial skill, $z_{1,t} = \xi_t z_1$, where z_1 is drawn from an exogenous stationary distribution. The component ξ_t captures the type of managerial skill that can grow without bound because can be inherited across generations, and it is common to all households of the same generation. As we will see later, the component ξ_t has to be growing because we are assuming that the exogenous component of technology frontier level is growing and, in order to a balanced growth path exist, ξ_t must also grow.

Households are born with no bequests and decide, at the beginning of their life, whether to work during their active lifetime period or become an entrepreneur. As in Guner et al. (2015), once they make their decision, they cannot change it at any other time. In equilibrium all workers receive the same wage rate per unit of time. All households make their consumption-saving decisions and they do not value leisure. Furthermore, all entrepreneurs, during their active lifetime period, make their investment decisions on skill accumulation.

Entrepreneurs: As in Lucas (1978), we assume that all entrepreneurs' technology exhibit a diminishing returns to scale with respect to private input factors ('span-of-control' technology). Similar to Poschke (2015), entrepreneurs differ in the complexity of the firm's production process they can carry out. Each entrepreneur produces the same single final consumption good y using the following technology:

$$y_{z,t} = \left\{ n_{z,t}^{(1-\alpha)} \left[\left(\int_0^{B_{z,t}} k_{i,z,t}^{(\sigma-1)/\sigma} di \right)^{\sigma/(\sigma-1)} \right]^{\alpha} \right\}^{\gamma}, \tag{1}$$

where, $\gamma < 1$ denotes the span-of-control, $B_{z,t}$ measures the degree of complexity of the technology (a higher number of types of capital), which depends on the technology frontier level, A_t, and on the entrepreneur's managerial skill, z, $k_{i,z,t}$ denotes the quantity of type of capital i is demanded by entrepreneur z at time t and $n_{z,t}$ is the quantity of labor demanded by entrepreneur z at time t. As in Poschke (2015), we assume that for each entrepreneur the cost of using a production process increases with the degree of complexity[3]. Consequently, the optimal choice of types of capital, $B_{z,t}$, will be an increasing function of entrepreneur's skill. The elasticity of substitution among the different types of capital is given by $\sigma > 1$[4].

Rewriting the production function

$$y_{z,t} = \left[n_{z,t}^{(1-\alpha)} M_{zt}^{\alpha} \right]^{\gamma}, \tag{2}$$

where

$$M_{z,t} = \left[\int_0^{B_{z,t}} k_{i,z,t}^{(\sigma-1)/\sigma} di \right]^{\sigma/(\sigma-1)},$$

we can solve the maximization of profits of entrepreneur z in two stages. Firstly, for a given value of $M_{z,t}$, the firm chooses the combination of $k_{i,z,t}$ that minimize the cost of obtaining the level $M_{z,t}$. Secondly, the firm chooses the optimal combination of labor, $n_{z,t}$, and $M_{z,t}$ that maximizes profits.

From the first step, we get that:

$$k_{i,z,t} = \left[\frac{r_t + \delta}{\lambda} \right]^{-\sigma} M_{z,t}, \tag{3}$$

where λ is the marginal cost of one more unit of $M_{z,t}$, assuming that all different types of capital are hired at the same interest rate $(r_t+\delta)$. Therefore,

[3] As in Poschke (2015) or Akcigit et al. (2014), we assume that entrepreneur's time endowment is equal to 1, and that the cost of managing production processes with a higher degree of complexity (a higher number of types of capital) is inversely related to their skills. Because revenues are increasing in $B_{z,t}$, entrepreneurs will choose to maximize the value of $B_{z,t}$ given their skills and time endowment.

[4] And as in Poschke (2015), we assume that the different inputs are gross substitutes. Poschke (2014, p. 21) claims that because the degree of specialization differs across firms, it is reasonable to assume that differentiated inputs are gross substitutes ($\sigma > 1$)).

we are also assuming that the technology is the same for all types of capital, and that it is a linear technology that produces one unit of each type of capital from one unit of forgone output. Consequently, all types of capital are rented at the competitive rate of $(r_t + \delta)$ using a within-period capital rental or "credit" contract.

Substituting $k_{i,z,t}$ in $M_{z,t}$, we have that,

$$\lambda = B_{z,t}^{-\frac{1}{\sigma-1}}(r_t + \delta). \tag{4}$$

As in Poschke (2015), the marginal cost of $M_{z,t}$ decreases with the firm's degree of complexity, $B_{z.t}$.

Secondly, we have that entrepreneurs maximize profits:

$$\pi_{z,t} = \max_{\{M_{z,t}, n_{z,t}\}} \left[\left(n_{z,t}^{(1-\alpha)} M_{z,t}^{\alpha} \right)^{\gamma} - (r_t + \delta) B_{z,t}^{-\frac{1}{\sigma-1}} M_{z,t} - w_t n_{z,t} \right],$$

where we have taken into account that, at each period t, all workers receive the same wage rate (w_t) and that the constant marginal cost of $M_{z,t}$ is given by $B_{z,t}^{-\frac{1}{\sigma-1}}(r_t + \delta)$.

The first order conditions are given by the following expressions:

$$n_{z,t} = (1-\alpha)\gamma \frac{y_{z,t}}{w_t}, \tag{5}$$

and

$$M_{z,t} = \alpha\gamma \frac{y_{z,t}}{(r_t + \delta)} B_{z,t}^{\frac{1}{\sigma-1}}. \tag{6}$$

Therefore, the optimal demand for $M_{z,t}$ and $n_{z,t}$ in terms of factors' prices, technology level (degree of complexity) and parameters can be obtained by solving the previous first order conditions (5) and (6), and taking into account the production function (2):

$$M_{z,t} = k_y B_{z,t}^{\frac{[1-\gamma(1-\alpha)]}{(1-\gamma)(\sigma-1)}} \left(\frac{1}{r_t + \delta} \right)^{(1-\gamma(1-\alpha))/(1-\gamma)} \left(\frac{1}{w_t} \right)^{\gamma(1-\alpha)/(1-\gamma)}, \tag{7}$$

$$n_{z,t} = n_y B_{z,t}^{\frac{\alpha\gamma}{(1-\gamma)(\sigma-1)}} \left(\frac{1}{r_t + \delta} \right)^{\alpha\gamma/(1-\gamma)} \left(\frac{1}{w_t} \right)^{(1-\alpha\gamma)/(1-\gamma)}, \tag{8}$$

and

$$y_{z,t} = y_y \left[B_{z,t}^{\frac{\alpha}{(1-\gamma)(\sigma-1)}} \left(\frac{1}{r_t+\delta}\right)^{\frac{\alpha}{(1-\gamma)}} \left(\frac{1}{w_t}\right)^{\frac{(1-\alpha)}{(1-\gamma)}} \right]^{\gamma}, \tag{9}$$

where the parameters k_y, n_y and y_y are defined as follows:

$$k_y \equiv \gamma^{(1/(1-\gamma))}\alpha^{(1-\gamma(1-\alpha))/(1-\gamma)}(1-\alpha)^{\gamma(1-\alpha)/(1-\gamma)},$$

$$n_y \equiv \gamma^{(1/(1-\gamma))}\alpha^{\alpha\gamma/(1-\gamma)}(1-\alpha)^{(1-\alpha\gamma)/(1-\gamma)},$$

and

$$y_y \equiv \left[\gamma^{\frac{1}{(1-\gamma)}}\alpha^{\frac{\alpha}{(1-\gamma)}}(1-\alpha)^{\frac{(1-\alpha)}{(1-\gamma)}}\right]^{\gamma}.$$

The wage rate, w_t, and the interest rate, $r_t + \delta$, are the same across firms, and from equations (3) and (4), we have that the amount of each type of capital demanded by each firm is given by the following expression:

$$k_{i,z,t} = B_{z,t}^{\frac{\sigma}{1-\sigma}} M_{z,t}.$$

Therefore, the total amount of capital used by entrepreneur z is given by $K_{z,t} = B_{z,t}k_{i,z,t} = B_{z,t}B_{z,t}^{\frac{\sigma}{1-\sigma}}M_{z,t} = B_{z,t}^{\frac{1}{1-\sigma}}M_{z,t}$ and the production function can be expressed as follows:

$$y_{z,t} = \left[n_{z,t}^{(1-\alpha)}M_{zt}^{\alpha}\right]^{\gamma} = B_{z,t}^{\frac{\alpha\gamma}{\sigma-1}}\left[n_{z,t}^{(1-\alpha)}K_{zt}^{\alpha}\right]^{\gamma}$$

Finally, the maximum profits can be written as follows:

$$\pi_{z,t} = B_{z,t}^{\frac{\alpha\gamma}{(1-\gamma)(\sigma-1)}}\Pi(r_t, w_t) = (1-\gamma)y_{z,t}, \tag{10}$$

where $\Pi = (1-\gamma)\left[\gamma^{\frac{1}{(1-\gamma)}}\alpha^{\frac{\alpha}{(1-\gamma)}}(1-\alpha)^{\frac{(1-\alpha)}{(1-\gamma)}}\left(\frac{1}{r_t+\delta}\right)^{\frac{\alpha}{(1-\gamma)}}\left(\frac{1}{w_t}\right)^{\frac{(1-\alpha)}{(1-\gamma)}}\right]^{\gamma}$.

As in Poschke (2015), a high elasticity of substitution among the differentiated inputs, σ, lowers profits from using more differentiated types of capital.

It can be shown that the ratio of capital per worker across firms is given by the following relationship:

$$\frac{K_{z,t}}{n_{z,t}} = \frac{B_{z,t}k_{i,z,t}}{n_{z,t}} = \frac{B_{z,t}^{\frac{1}{1-\sigma}}M_{z,t}}{n_{z,t}},$$

where $K_{z,t}$ denotes the total amount of capital used by entrepreneur z, $B_{z,t}$ denotes the number of types of capital used by entrepreneur z, and $k_{i,z,t}$ is the amount of each type of capital used by firm z.

Taking into account (5), (6) and (3), we have that:

$$\frac{K_{z,t}}{n_{z,t}} = \frac{B_{z,t}^{\frac{1}{1-\sigma}} \alpha\gamma \frac{y_{z,t}}{(r_t+\delta)} B_{z,t}^{\frac{1}{\sigma-1}}}{(1-\alpha)\gamma \frac{y_{z,t}}{w_t}} = \frac{\alpha w_t}{(1-\alpha)(r_t+\delta)} \tag{11}$$

that is, capital per worker must be the same across firms. As expected, since there are no distortions in the economy, capital per worker does not depend on firm's productivity.

Also, taking into account (5) and (11), we have that:

$$\frac{K_{z,t}}{y_{z,t}} = \frac{\frac{K_{z,t}}{n_{z,t}}}{\frac{y_{z,t}}{n_{z,t}}} = \frac{\frac{\alpha w_t}{(1-\alpha)(r_t+\delta)}}{\frac{w_t}{(1-\alpha)\gamma}} = \frac{\alpha\gamma}{r_t+\delta}$$

that is, we also have that the ratio of capital over output across firms does not depend on firm's productivity.

Finally, taking into account individual optimal $K_{z,t}$, $n_{z,t}$ and $y_{z,t}$, individual $TFP_{z,t}$ is given by

$$TFP_{z,t} = \frac{y_{z,t}}{K_{z,t}^{\alpha\gamma} n_{z,t}^{(1-\alpha)\gamma}} = B_{z,t}^{\frac{\alpha\gamma}{(\sigma-1)}}$$

therefore, individual $TFP_{z,t}$ is directly proportional to $B_{z,t}$[5].

Regarding the degree of complexity, $B_{z,t}$, we assume that it depends positively on both, the managerial skill level, z, and technology frontier level, A_t, as follows:

$$B_{z,t} = \left(\frac{z_{j,t}}{\overline{z}_t}\right)^{\varphi} A_t$$

where $\overline{z}_t$ is the maximum contemporaneous level of entrepreneurs' ability and is proportional to ξ_t. As we mentioned earlier, the entrepreneur technology depends on its relative ability with respect to the most able entrepreneur. The elasticity of the degree of complexity with respect to the entrepreneurs'

[5] In this paper, firm's TFP is determined by the amount of differentiated types of capital goods adopted. This is the *variety effect* mentioned in Comin and Hobijn (2010), that is the increase in productivity that arises by the range of types of capital goods used in the production process.

ability is determined by the parameter φ. The value of φ will affect to the return of investment in skills[6].

Consequently, individual $TFP_{z,t}$ depends on relative entrepreneurial skill and on the technology frontier level, as follows:

$$TFP_{z,t} = \left(\frac{z_{j,t}}{\overline{z}_t}\right)^{\frac{\varphi\alpha\gamma}{(\sigma-1)}} A_t^{\frac{\alpha\gamma}{(\sigma-1)}} \tag{12}$$

where A_t denotes the technology frontier level at time t.

2.2 Households' problem

At each period t a large number of finitely-lived households of measure one are born. There is no population growth. Households live for J periods, start their retirement period at the age of J_R and do not value leisure. Households' income depends on profits (for entrepreneurs) or on wage income (for workers) up to period $J_R - 1$. Entrepreneurs do not invest in their skills the last period before becoming retired (that is, in period $J_R - 1$). During their retirement period, households live only on their savings. All households can save in risk-free assets, s, whose rate of return is r. Households' indirect utility function depends on their initial endowment in managerial skill (z_1) and on the technology frontier level at the period they were born, that is on t. Since there are not bequests, households' initial wealth is assumed to be zero. Also, we assume that households cannot die with debt and, given that the utility increases in consumption, households will consume all their wealth in their last period of life.

Consequently, entrepreneurs' budget constraints are given by

$$c_{j,t+j-1} + x_{j,t+j-1} + s_{j+1,t+j} = \pi_{z,t+j-1} + (1+r_{t+j-1})s_{j,t+j-1}, \qquad j \in [1, J_R - 2], \tag{13}$$

$$c_{J_R-1,t+J_R-2} + s_{J_R,t+J_R-1} = \pi_{z,t+J_R-1} + (1+r_{t+J_R-1})s_{J_R-1,t+J_R-2}, \tag{14}$$

[6] Poschke (2015) assumes that the elasticity of the entrepreneurial technology with respect to the level of economy-wide technology is increasing in managers' ability. Consequently, more qualified entrepreneurs benefit more from the economy-wide technology level than low qualified entrepreneurs. This is what Poschke (2015) names as "skill-biased change in entrepreneurial technology". In our case, the elasticity of the entrepreneurial technology with respect to the level of economy-wide technology is 1. Therefore, we don't have this mechanism.

$$c_{j,t+j-1}+s_{j+1,t+j}=(1+r_{t+j-1})s_{j,t+j-1}, \qquad j\in[J_R,J], \tag{15}$$

and the law of motion for their skills by

$$z_{j+1,t+j}=(1-\delta_z)z_{j,t+j-1}+h(z_{j,t+j-1},x_{j,t+j-1}), \qquad j\in[1,J_R-2]. \tag{16}$$

The budget constraints depend on whether entrepreneurs are active or retired. If they are active ((13) and (14)), their income comes from the profits they earn ($\pi_{z,t+j-1}$) and the return of their accumulated savings. They consume ($c_{j,t+j-1}$), accumulate savings ($s_{j+1,t+j}$) and invest in their skills ($x_{j,t+j-1}$) if they are not going to retire the next period (13). The law of motion for the entrepreneurs' skills (16), as in Guner et al. (2015), depends on the current skills and on the amount of income invested. Entrepreneurs' skills, in absolute terms, may depreciate over time if not investment is done to keep their knowledge constant. If they are retired, their only source of income comes from their savings (15).

Then, assuming that utility function at each period is given by

$$u(c_{j,t+j-1})=\ln(c_{j,t+j-1}),$$

the problem of an entrepreneur born at time t, can be written as follows:

$$\begin{aligned} V^e_{1,t}\left(A_t,z_{1,t}\right) &= \max\sum_{j=1}^{J}\beta^{j-1}\ln(c_{j,t+j-1}) \\ & \quad s.t.\ \text{Equations (13), (14), (15), (16) and} \\ s_{1,t} &= s_{J+1,t+J}=0 \end{aligned}$$

The first order conditions are given by

$$c_{j+1,t+j} = \beta(1+r_{t+j})c_{j,t+j-1} \quad j\in[1,J-1] \tag{17}$$

$$1+r_{t+j} = \pi_{z_{j+1},t+j}h_{x_j,t+j-1}+\frac{\left(1+h_{z_{j+1},t+j}\right)h_{x_j,t+j-1}}{h_{x_{j+1},t+j}} \qquad j\in[1,J_R-3] \tag{18}$$

$$1+r_{t+J_R-1} = \pi_{z_{J_R-1},t+J_R-2}h_{x_{J_R-2},t+J_R-2} \tag{19}$$

and the corresponding budget constraints plus the law of motion for their skills, where, as in Guner et al. (2015), $\pi_{z_{j+1},t+j}=\frac{\partial\pi(z_{j+1,t+j},r_{t+j},w_{t+j})}{\partial z_{j+1,t+j}}$,

$$h(z_{j,t+j-1},x_{j,t+j-1})=(1-\delta_\theta)^j z^{\theta_1}_{j,t+j-1}x^{\theta_2}_{j,t+j-1} \qquad \text{with } 0<\theta_1,\theta_2<1,$$

$\frac{\partial h(z_{j,t},x_{j,t+j-1})}{\partial x_{j,t+j-1}} = h_{x_{j,t+j-1}} = (1-\delta_\theta)^j \theta_2 z_{j,t+j-1}^{\theta_1} x_{j,t+j-1}^{\theta_2-1}$,

$\frac{\partial h(z_{j+1,t+j},x_{j+1,t+j})}{\partial z_{j+1,t+j}} = h_{z_{j+1,t+j}} = (1-\delta_\theta)^{j+1} \theta_1 z_{j+1,t+j}^{\theta_1-1} x_{j+1,t+j}^{\theta_2}$ and δ_θ denotes that the learning ability decreases with age.

The above conditions are standard: (17) are the Euler equations, (18) and (19) are the non-arbitrage conditions between saving in the risk-free asset or investing in skill accumulation. We can see that in the non-arbitrage condition between saving in the risk-free asset or investing in skill accumulation, the benefits of investing in managerial skill have two components: *(i)* the increase in next period's profit because of higher managerial skill, and *(ii)* because we are assuming that there is not full depreciation of entrepreneurial skill, entrepreneurs will enjoy of a higher entrepreneurial skill from next period. This second component does not arise in the entrepreneur's last period of investment in skill accumulation. Nevertheless, as shown in Cai (2011), if $h(z_{j,t+j-1}, x_{j,t+j-1})$ is linear in $z_{j,t+j-1}$ (that is, if $\theta_1 = 1$), investment in skill accumulation does not depend on initial entrepreneurial skill. And, as mentioned by Guner et al. (2015), taking into account that the marginal benefits of investing in skill accumulation are decreasing in $x_{j,t+j-1}$, and the marginal cost of saving in the risk-free asset is constant, there always be an interior solution.

Summarizing, for every cohort t there are $2(J+ J_R-2)$ unknowns ($J+ J_R-2$ control variables $\left[\{c_{j,t+j-1}\}_{j=1}^{J}, \{x_{j,t+j-1}\}_{j=1}^{J_R-2}\right]$ and $J+ J_R-2$ state variables $\left[\{s_{j+1,t+j}\}_{j=1}^{J}, \{z_{j+1,t+j}\}_{j=1}^{J_R-2}\right]$) to be solved with $2(J+ J_R-2)$ equations: $J-1$ Euler equations, J_R-2 non-arbitrage conditions between investing in skill accumulation or saving in the risk-free asset, J_R-2 laws of motion for the managerial skill, J budget constraints, and $s_{J+1,t+J}=0$.

Given that the only source of ex-ante heterogeneity among households born at the same period t is the entrepreneurial skill level they are born with, all their optimal decisions depend ultimately on their respective initial entrepreneurial skill level (that is, the whole life-cycle profile for $\{c_{j,t+j-1} = c_j(z_{1,t+j-1})\}_{j=1}^{J}$, $\{s_{j+1,t+j} = s_{j+1}(z_{1,t})\}_{j=1}^{J}$ and $\{x_{j,t+j+1} = x_j(z_{1,t})\}_{j=1}^{J_R-2}$, depend only on their ex-ante source of heterogeneity, $z_{1,t}$. For a given initial level of entrepreneurial skill $z_{1,t}$, the optimal decisions of households of the same cohort are identical, since there is no other source of heterogeneity).

Workers: All workers will receive the same equilibrium wage rate at every period, independently on their initial skill's endowment. The problem faced by each worker born at time t can be written as follows:

$$\begin{aligned} V_t^w &= \max \sum_{j=1}^{J} \beta^{j-1} \ln(c_{j,t+j-1}) \\ s.t. \quad c_{j,t+j-1} + s_{j+1,t+j} &= w_{t+j-1} + (1 + r_{t+j-1}) s_{j,t+j-1}, \qquad j \in [1, J_R - 1] \\ c_{j,t+j-1} + s_{j+1,t+j} &= (1 + r_{t+j-1}) s_{j,t+j-1}, \qquad j \in [J_R, J] \\ s_{1,t} &= s_{J+1,t+J} = 0 \end{aligned}$$

where the budget constraints depend on whether workers are active or retired, but this time workers do not invest in their own skills.

The first order conditions can be written as follows:

$$c_{j+1,t+j} = \beta(1 + r_{t+j}) c_{j,t+j-1}, \qquad j \in [1, J-1]$$

plus the corresponding budget constraints.

2.3 Growth rates along the Balanced Growth Path

Along the balanced growth path (BGP), all variables in per capita terms grow at a constant growth rate except the fraction of entrepreneurs (or workers), the rate of return of capital and the labor force, which do not grow. In particular, output, labor income, profits, consumption, savings, aggregate capital and investment in skills grow at the same constant growth rate.

From each firm's optimal output level (9) we have that:

$$(1 + g) = (1 + g_y) = (1 + g_w)^{-\frac{(1-\alpha)\gamma}{1-\gamma}} (1 + g_B)^{\frac{\alpha\gamma}{(\sigma-1)(1-\gamma)}} \tag{20}$$

and from the first order condition that determines the demand for labor (8), we can see that:

$$(1 + g_w) = (1 + g_y) \tag{21}$$

which implies that:

$$1 + g = (1 + g_B)^{\frac{\alpha\gamma}{(\sigma-1)(1-\alpha\gamma)}} \tag{22}$$

From the first order condition that determines the demand for each type of capital (7), we can obtain that:

$$(1+g_{k_i}) = (1+g_B)^{\frac{[1-\sigma(1-\alpha\gamma)]}{(\sigma-1)(1-\alpha\gamma)}} \tag{23}$$

And, taking into account (22), we have that:

$$(1+g_{k_i}) = (1+g)^{\frac{[1-\sigma(1-\alpha\gamma)]}{\alpha\gamma}}$$

And the growth rate of the aggregate stock of capital in the economy is given by:

$$1+g_K = (1+g_B)(1+g)^{\frac{[1-\sigma(1-\alpha\gamma)]}{\alpha\gamma}} = (1+g_B)^{\frac{\alpha\gamma}{(\sigma-1)(1-\alpha\gamma)}} = 1+g$$

Profits must grow at the same rate as labor income. From equation (10), we have that:

$$(1+g_\pi) = (1+g_B)^{\frac{\alpha\gamma}{(\sigma-1)(1-\gamma)}}(1+g_w)^{-\frac{\gamma(1-\alpha)}{(1-\gamma)}}$$

$$(1+g_\pi) = (1+g_B)^{\frac{\alpha\gamma}{(\sigma-1)(1-\alpha\gamma)}} = 1+g$$

From the households' budget constraints, we must have that age-profile consumption, savings and managerial skill investment must grow at the same constant growth rate. In particular:

$$\begin{aligned}(1+g_x) &= (1+g_\pi) = (1+g_w) && (24)\\ &= (1+g) = (1+g_B)^{\frac{\alpha\gamma}{(\sigma-1)(1-\alpha\gamma)}} && (25)\end{aligned}$$

Where the degree of complexity depends on the technology level and on the managerial skill:

$$B_{z,t} = \left(\frac{z_{j,t}}{\overline{z}_t}\right)^{\varphi} A_t$$

where $\overline{z}_t$ is proportional to ξ_t, and $A_t = \overline{A}(1+g_A)^t$. Consequently,

$$(1+g_B) = (1+g_A)$$

And individual TFP, $TFP_{z,t}$, grows at the rate:

$$g_{TFP} = (1+g)^{(1-\alpha\gamma)} - 1 = (1+g_A)^{\frac{\alpha\gamma}{(\sigma-1)}} - 1. \tag{26}$$

Finally, as in Guner et al. (2015), along the balanced growth path there must be a relationship between $1+g_z$ and $1+g_A$. The relationship can be obtained either by the law of motion of the entrepreneurial's skill (16) or by the non-arbitrage condition of entrepreneurs' optimal managerial skill investment, (18) or (19). In particular, from condition (19) we can obtain the following relationship:

$$(1+g_z) = (1+g_\xi) = (1+g_A)^{\frac{\alpha\gamma}{(\sigma-1)(1-\gamma)(1-\theta_1)}} (1+g)^{\frac{(\theta_2-1)(1-\gamma)-\gamma(1-\alpha)}{(1-\gamma)(1-\theta_1)}} \tag{27}$$

Substituting this expression (27) in the economy growth rate (24), we obtain that

$$(1+g_\xi) = (1+g_A)^{\frac{\theta_2\alpha\gamma}{(1-\theta_1)(\sigma-1)(1-\alpha\gamma)}} = (1+g)^{\frac{\theta_2}{(1-\theta_1)}} = (1+g_z) \tag{28}$$

That is, new generations are born with higher skills depreciating the old generations relative skill.

In order to be able to solve the stationary version of the model, we have to normalize all those variables whose constant growth rate along the BGP is positive. Consequently, output, wage income, profits, consumption, savings for a given age, aggregate capital, aggregate consumption and managerial skill investment are normalized by $(1+g)^t$. Each type of physical capital will be normalized by $(1+g)^{\frac{[1-\sigma(1-\alpha\gamma)]}{\alpha\gamma}t}$, the degree of complexity will be normalized by $(1+g)^{\frac{(\sigma-1)(1-\alpha\gamma)}{\alpha\gamma}t}$. And the fraction of workers or the labor force are not normalized because they are constant along the BGP:

The normalized variables will be denoted by the "$\sim$" symbol. Along the balanced growth path, all these normalized variables remain constant. As in Guner et al. (2015), each new cohort of individuals are born with a common component $\xi(t)$ that grows over time at the rate g_z, and a random component z_1 whose distribution, cdf $F(z)$ and density $f(z)$ on $[0, z^{\max}]$, is constant. Therefore, the normalized component is simply z_1 for each individual. And the normalized component for every entrepreneur's skill is given by $z_j(z_1)$.

Therefore, rewriting the first order conditions in terms of the normalized variables, we have a stationary system. The threshold value for z_1 that determines the fraction of workers, among households at the age-1, can be obtained such that: $\widetilde{V}^w = \widetilde{V}_1^e(z_1^*)$, where $\widetilde{V}_1^e(z_1^*)$ is an increasing function of z_1, therefore the threshold value for z_1^* will be uniquely determined and will be constant for any period t.

2.4 Stationary Equilibrium along the BGP

We assume an exogenous, and constant, distribution of initial endowment of the random component of managerial skills z_1. In particular, as in Guner et al. (2015), we assume that z_1 follows a log-normal distribution on $[0, z^{\max}]$.

For a given distribution of initial endowment of managerial skills, z_1, and no population growth, we can obtain the aggregates of labor, capital, investments in skills and final consumption good.

In equilibrium, normalized prices $(r, \widetilde{w})$ will be determined such that all markets clear. In particular, in the labor market we have that:

$$N = \sum_{j=1}^{J_R-1} \mu_j \left[\int_{z_1^*}^{z^{\max}} n(z_j(z_1), r, \widetilde{w}) f(z_1) dz_1 \right] = F(z_1^*) \sum_{j=1}^{J_R-1} \mu_j$$

where N is the per capita endogenous stationary equilibrium level of workers, μ_j denotes the mass of the age-j individuals, $z_j(z_1)$ denotes the normalized age-j managerial skill, which depends on the initial managerial skill, z_1, $f(z_1)$ denotes de density function for z_1, $F(z_1)$ the accumulative distribution function for initial managerial skill z_1 and $n(z_j(z_1), r, \widetilde{w})$ denotes the amount of labor hired by each entrepreneur taking into account his/her managerial skill level. Consequently, the left hand side is the aggregate labor demand and the right hand side is the aggregate labor supply.

The market for capital clears when the supply of capital per capita equals the demand of capital per capita,

$$\begin{aligned} \widetilde{K} &= \sum_{j=1}^{J_R-1} \mu_j \int_{z_1^*}^{z^{\max}} \widetilde{B}\left[z_j(z_1)\right] \widetilde{k}\left[z_j(z_1), r, \widetilde{w}\right] f(z_1) dz_1 \\ &= \sum_{j=1}^{J_R-1} \mu_j \int_{z_1^*}^{z^{\max}} \widetilde{K}\left[z_j(z_1), r, \widetilde{w}\right] f(z_1) dz_1 \\ &= \sum_{j=1}^{J-1} \mu_j \left[\int_{z_1^*}^{z^{\max}} \widetilde{s}_j\left[z_j(z_1)\right] f(z_1) dz_1 \right] + F(z_1^*) \sum_{j=1}^{J-1} \mu_j \widetilde{s}_j \end{aligned}$$

By the Walras' Law, the market for the output clears.

The equilibrium for the output good can be written as follows:

$$\begin{aligned} \widetilde{Y} &= \widetilde{C} + \widetilde{I}, \\ \widetilde{Y} &= \sum_{j=1}^{J_R-1} \mu_j \int_{z_1^*}^{z^{\max}} \widetilde{y}(z_j(z_1), r, \widetilde{w}) f(z_1) dz_1 = \sum_{j=1}^{J_R-1} \mu_j \widetilde{y}_j(r, \widetilde{w}), \end{aligned}$$

$$
\begin{aligned}
\widetilde{C} &= F(z_1^*)\sum_{j=1}^{J}\mu_j\widetilde{c}_j + \sum_{j=1}^{J}\mu_j\left[\int_{z_1^*}^{z^{\max}}\widetilde{c}_j(z_j(z_1))f(z_1)dz_1\right], \\
\widetilde{I} &= (\delta+g)\widetilde{K}+\widetilde{X},
\end{aligned}
$$

$$
\widetilde{X} = \sum_{j=1}^{J_R-2}\mu_j\left[\int_{z_1^*}^{z^{\max}}\widetilde{x}_1(z_j(z_1))f(z_1)dz_1\right].
$$

2.4.1 Relationship between plant size and entrepreneurial skill along the BGP

As in Atkeson and Kehoe (2005), we link algebraically entrepreneurials' skills to the size of entrepreneurs' plant as measured by its number of employees. Taking into account that labor and capital are freely mobile across plants, we express the allocation of capital and labor across plants in terms of the aggregate capital and aggregate labor, respectively.

Firstly, considering the equilibrium allocation of labor across plants:

$$
\begin{aligned}
N &= \sum_{j=1}^{J_R-1}\mu_j\left[\int_{z_1^*}^{z^{\max}} n(z_j(z_1),r,\widetilde{w})f(z_1)dz_1\right] \\
&= \sum_{j=1}^{J_R-1}\mu_j n_j
\end{aligned}
$$

where n_j is the per capita amount of workers hired by age-j entrepreneurs,

$$
n_j = \left[\int_{z_1^*}^{z^{\max}} n(z_j(z_1),r,\widetilde{w})f(z_1)dz_1\right],
$$

we have that

$$
N = n_y\left(\frac{1}{r+\delta}\right)^{\frac{\alpha\gamma}{(1-\gamma)}}\left(\frac{1}{\widetilde{w}}\right)^{\frac{(1-\alpha\gamma)}{(1-\gamma)}}\widetilde{A}^{\frac{\alpha\gamma}{(\sigma-1)(1-\gamma)}}\sum_{j=1}^{J_R-1}\mu_j\left[\int_{z_1^*}^{z^{\max}}\widetilde{Z}_{z_j(z_1)}^{\frac{\varphi\alpha\gamma}{(\sigma-1)(1-\gamma)}}f(z_1)dz_1\right]
$$

$$
N = n_y\left(\frac{1}{r+\delta}\right)^{\frac{\alpha\gamma}{(1-\gamma)}}\left(\frac{1}{\widetilde{w}}\right)^{\frac{(1-\alpha\gamma)}{(1-\gamma)}}\widetilde{A}^{\frac{\alpha\gamma}{(\sigma-1)(1-\gamma)}}\widetilde{Z}
$$

where N is the stationary equilibrium proportion of workers among the whole population, and $\widetilde{Z}$, similar to Guner et al. (2015), is the *aggregate* *normalized entrepreneurial quality*:

$$\begin{aligned}\widetilde{Z} &= \sum_{j=1}^{J_R-1} \mu_j \left[\int_{z_1^*}^{z^{\max}} \left(\frac{z_j(z_1)}{\overline{z}(1+g_z)^{j-1}} \right)^{\frac{\varphi\alpha\gamma}{(\sigma-1)(1-\gamma)}} f(z_1) dz_1 \right] \\ &= \sum_{j=1}^{J_R-1} \mu_j \int_{z_1^*}^{z^{\max}} \widehat{Z}_{z_j(z_1)}^{\frac{\varphi\alpha\gamma}{(\sigma-1)(1-\gamma)}} f(z_1) dz_1 = \sum_{j=1}^{J_R-1} \mu_j \widetilde{Z}_j \end{aligned} \tag{29}$$

where $\widetilde{Z}_{z_j(z_1)} = \frac{z_j(z_1)}{\overline{z}(1+g_z)^{j-1}}$ denotes the *individual normalized entrepreneurial quality,* which depends on age-j entrepreneur's skills and on how much age-j entrepreneur's skill depreciates due to new born generations' skills, and $\widetilde{Z}_j$ is defined as follows

$$\widetilde{Z}_j \equiv \int_{z_1^*}^{z^{\max}} \widetilde{Z}_{z_j(z_1)}^{\frac{\varphi\alpha\gamma}{(\sigma-1)(1-\gamma)}} f(z_1) dz_1$$

Analogously, from the equilibrium allocation of aggregate capital across plants, we have that:

$$\widetilde{K} = \sum_{j=1}^{J_R-1} \mu_j \int_{z_1^*}^{z^{\max}} \widetilde{K}\left[z_j(z_1), r, \widetilde{w}\right] f(z_1) dz_1$$

$$\widetilde{K} = k_y \left(\frac{1}{r+\delta} \right)^{\frac{(1-\gamma(1-\alpha))}{(1-\gamma)}} \left(\frac{1}{\widetilde{w}} \right)^{\frac{\gamma(1-\alpha)}{(1-\gamma)}} \widetilde{A}^{\frac{\alpha\gamma}{(\sigma-1)(1-\gamma)}} \widetilde{Z}$$

Finally, as in Atkeson and Kehoe (2005), we can show that the size of an entrepreneur's plant as measured by its number of employees is positively related to its entrepreneurial skill relative to contemporaneous entrepreneurs' skills:

$$\frac{n(z_j(z_1), r, \widetilde{w})}{N} = \frac{\left(\frac{z_j(z_1)}{\overline{z}(1+g_z)^{j-1}} \right)^{\frac{\varphi\alpha\gamma}{(\sigma-1)(1-\gamma)}}}{\widetilde{Z}} = \frac{\widetilde{Z}_{z_j(z_1)}^{\frac{\varphi\alpha\gamma}{(\sigma-1)(1-\gamma)}}}{\widetilde{Z}}, \tag{30}$$

Consequently, the fraction of employment of an entrepreneur's plant, relative to aggregate employment, will be positively related to its *individual normalized entrepreneurial quality relative to aggregate normalized entrepreneurial quality.*

Concerning aggregate capital per capita, it can be shown that

$$\widetilde{K}(z_j(z_1), r, \widetilde{w}) = \frac{n(z_j(z_1), r, \widetilde{w})}{N} \widetilde{K}, \tag{31}$$

regarding aggregate output per capita,

$$\widetilde{y}(z_j(z_1), r, \widetilde{w}) = \frac{n(z_j(z_1), r, \widetilde{w})}{N} \widetilde{Y} \tag{32}$$

and, finally, concerning entrepreneur's profits

$$\widetilde{\pi}(z_j(z_1), r, \widetilde{w}) = (1-\gamma)\widetilde{y}(z_j(z_1), r, \widetilde{w}) = (1-\gamma)\frac{n(z_j(z_1), r, \widetilde{w})}{N}\widetilde{Y} \tag{33}$$

where $\widetilde{K}$ and $\widetilde{Y}$ denote the stationary aggregate capital per capita and the stationary aggregate output per capita, respectively, and $\widetilde{K}(z_j(z_1), r, \widetilde{w})$, $\widetilde{y}(z_j(z_1), r, \widetilde{w})$ and $\widetilde{\pi}(z_j(z_1), r, \widetilde{w})$ denote the stationary entrepreneurs' capital, the stationary entrepreneurs' output, and the stationary entrepreneurs' profits, respectively. Therefore, capital per worker must be equal across plants (and the same to aggregate capital-output ratio, as expected because there are no distortions in this economy), but higher entrepreneurial skill will imply a higher entrepreneurial size (higher proportion of employment and higher proportion of capital). And, similar to Atkeson and Kehoe (2005), entrepreneurs' profits is positively related to entrepreneurs' skills (and hence, to their entrepreneurial technology).

We are able to obtain the above expressions (30), (31) and (32) because, we can check that the entrepreneurs' production function can be written as follows:

$$\begin{aligned}\widetilde{y}(z_j(z_1), r, \widetilde{w}) &= \widetilde{B}\left[z_j(z_1)\right]^{\frac{\alpha\gamma}{\sigma-1}}\left[n(z_j(z_1), r, \widetilde{w})^{(1-\alpha)}\widetilde{K}(z_j(z_1), r, \widetilde{w})^{\alpha}\right]^{\gamma} \qquad (34)\\ &= \widetilde{B}\left[z_j(z_1)\right]^{\frac{\alpha\gamma}{\sigma-1}} F\left[n(z_j(z_1), r, \widetilde{w}), \widetilde{K}(z_j(z_1), r, \widetilde{w})\right]^{\gamma}\end{aligned}$$

and as in Atkeson and Kehoe (2005), $F[.,.]$ exhibits constant returns to scale with respect to all capital and labor employed by each entrepreneur.

2.4.2 Output and age-output profile along the BGP

From the above expressions (30), (31) and (32), we have that employment, capital and output for each age-j entrepreneur can be expressed as

$$\begin{aligned}n_j &= \int_{z_1^*}^{z^{\max}} n(z_j(z_1), r, \widetilde{w}) f(z_1) dz_1\\ &= \frac{N}{\widetilde{Z}}\int_{z_1^*}^{z^{\max}} \widetilde{Z}_{z_j(z_1)}^{\frac{\varphi\alpha\gamma}{(\sigma-1)(1-\gamma)}} f(z_1) dz_1 = \frac{N}{\widetilde{Z}}\widetilde{Z}_j, \qquad (35)\end{aligned}$$

$$\begin{aligned}\widetilde{K}_j &= \int_{z_1^*}^{z^{\max}} \widetilde{K}\left[z_j(z_1), r, \widetilde{w}\right] f(z_1) dz_1\\ &= \int_{z_1^*}^{z^{\max}} \frac{n(z_j(z_1), r, \widetilde{w})}{N}\widetilde{K} f(z_1) dz_1 = \left(\frac{n_j}{N}\right)\widetilde{K},\end{aligned}$$

and

$$
\begin{aligned}
\widetilde{y}_j &= \int_{z_1^*}^{z^{\max}} \widetilde{y}\left[z_j(z_1), r, \widetilde{w}\right] f(z_1) dz_1 \\
&= \int_{z_1^*}^{z^{\max}} \frac{n(z_j(z_1), r, \widetilde{w})}{N} \widetilde{Y} f(z_1) dz_1 = \left(\frac{n_j}{N}\right) \widetilde{Y},
\end{aligned}
$$

where

$$
\widetilde{Z}_j = \int_{z_1^*}^{z^{\max}} \widetilde{Z}_{z_j(z_1)}^{\frac{\varphi\alpha\gamma}{(\sigma-1)(1-\gamma)}} f(z_1) dz_1,
$$

and taking into account that

$$
\widetilde{Z}_{z_j(z_1)} = \frac{z_j(z_1)}{\overline{z}(1+g_z)^{j-1}},
$$

which depends on how much age-j entrepreneur's skill depreciates due to new born generations' skills.

Also, output at every age-j is given by

$$
\widetilde{y}_j(r, \widetilde{w}) = \int_{z_1^*}^{z^{\max}} \widetilde{y}(z_j(z_1), r, \widetilde{w}) f(z_1) dz_1
$$

where, each entrepreneur output can be written as

$$
\begin{aligned}
\widetilde{y}(z_j(z_1), r, \widetilde{w}) &= \widetilde{B}_{z_j(z_1)}^{\frac{\alpha\gamma}{\sigma-1}} \left[n_{z_j(z_1)}^{(1-\alpha)} \widetilde{K}_{z_j(z_1)}^{\alpha}\right]^{\gamma} \\
&= \widetilde{A}^{\frac{\alpha\gamma}{(\sigma-1)}} \widetilde{Z}_{z_j(z_1)}^{\frac{\varphi\alpha\gamma}{(\sigma-1)}} \left[n_{z_j(z_1)}^{(1-\alpha)} \widetilde{K}_{z_j(z_1)}^{\alpha}\right]^{\gamma} \qquad (36)
\end{aligned}
$$

Consequently, we can also express the level of output at age-j as follows. Taking into account (36) and (31), we have that:

$$
\widetilde{y}_j(r, \widetilde{w}) = \int_{z_1^*}^{z^{\max}} \widetilde{y}(z_j(z_1), r, \widetilde{w}) f(z_1) dz_1
$$

$$
= \widetilde{A}^{\frac{\alpha\gamma}{(\sigma-1)}} \left(\frac{\widetilde{K}}{N}\right)^{\alpha\gamma} \int_{z_1^*}^{z^{\max}} \widetilde{Z}_{z_j(z_1)}^{\frac{\varphi\alpha\gamma}{(\sigma-1)}} n_{z_j(z_1)}^{\gamma} f(z_1) dz_1
$$

And, from (30),

$$
\widetilde{A}^{\frac{\alpha\gamma}{(\sigma-1)}} \left(\frac{\widetilde{K}}{N}\right)^{\alpha\gamma} \int_{z_1^*}^{z^{\max}} \widetilde{Z}_{z_j(z_1)}^{\frac{\varphi\alpha\gamma}{(\sigma-1)}} n_{z_j(z_1)}^{\gamma} f(z_1) dz_1
$$

$$
= \widetilde{A}^{\frac{\alpha\gamma}{(\sigma-1)}} \left(\frac{\widetilde{K}}{N}\right)^{\alpha\gamma} \int_{z_1^*}^{z^{\max}} \widetilde{Z}_{z_j(z_1)}^{\frac{\varphi\alpha\gamma}{(\sigma-1)}} \left[\frac{\widetilde{Z}_{z_j(z_1)}^{\frac{\varphi\alpha\gamma}{(\sigma-1)(1-\gamma)}}}{\widetilde{Z}} N\right]^{\gamma} f(z_1) dz_1
$$

$$= \widetilde{A}^{\frac{\alpha\gamma}{(\sigma-1)}} \widetilde{Z}_j^{(1-\gamma)} \left[\widetilde{K}_j^{\alpha} n_j^{(1-\alpha)}\right]^{\gamma} \tag{37}$$

And per capita aggregate output is given by

$$\begin{aligned} \widetilde{Y} &= \sum_{j=1}^{J_R-1} \mu_j \widetilde{y}_j(r, \widetilde{w}) = \sum_{j=1}^{J_R-1} \mu_j \widetilde{A}^{\frac{\alpha\gamma}{(\sigma-1)}} \widetilde{Z}_j^{(1-\gamma)} \left[\widetilde{K}_j^{\alpha} n_j^{(1-\alpha)}\right]^{\gamma} \\ &= \widetilde{A}^{\frac{\alpha\gamma}{(\sigma-1)}} \sum_{j=1}^{J_R-1} \mu_j \widetilde{Z}_j^{(1-\gamma)} \left[n_j \left(\frac{\widetilde{K}}{N}\right)^{\alpha}\right]^{\gamma} \\ &= \widetilde{A}^{\frac{\alpha\gamma}{(\sigma-1)}} \widetilde{Z}^{(1-\gamma)} \left[\widetilde{K}^{\alpha} N^{(1-\alpha)}\right]^{\gamma}. \end{aligned} \tag{38}$$

2.4.3 TFP at the firm level, at age level and on aggregate

From (34), or from (36), we can see that each entrepreneur's normalized TFP is given by

$$\begin{aligned} \widetilde{TFP}_z = \frac{\widetilde{y}(z_j(z_1), r, \widetilde{w})}{\widetilde{K}(z_j(z_1), r, \widetilde{w})^{\alpha\gamma} n(z_j(z_1), r, \widetilde{w})^{(1-\alpha)\gamma}} &= \widetilde{B}\left[z_j(z_1)\right]^{\frac{\alpha\gamma}{\sigma-1}} \\ &= \widetilde{A}^{\frac{\alpha\gamma}{(\sigma-1)}} \widetilde{Z}_{z_j(z_1)}^{\frac{\varphi\alpha\gamma}{(\sigma-1)}} \end{aligned}$$

Also, we can compute normalized TFP by entrepreneurs' age-j taking into account (37)

$$\widetilde{TFP}_j = \frac{\widetilde{y}_j}{\widetilde{K}_j^{\alpha\gamma} n_j^{(1-\alpha)\gamma}} = \widetilde{A}^{\frac{\alpha\gamma}{(\sigma-1)}} \widetilde{Z}_j^{(1-\gamma)} \tag{39}$$

And aggregate normalized TFP, $\widetilde{TFP}$, taking into account (38):

$$\widetilde{TFP} = \frac{\widetilde{Y}}{\widetilde{K}^{\alpha\gamma} N^{(1-\alpha)\gamma}} = \widetilde{A}^{\frac{\alpha\gamma}{(\sigma-1)}} \widetilde{Z}^{(1-\gamma)} \tag{40}$$

which depends on both normalized technology frontier level, $\widetilde{A}$, and on the aggregate *normalized entrepreneurial quality level, $\widetilde{Z}$.* Consequently, aggregate normalized TFP, $\widetilde{TFP}$, is partly endogenously determined by entrepreneurial quality.

In order to analyze how $\widetilde{TFP}_j$ affects the aggregate $\widetilde{TFP}$, we express the aggregate normalized TFP as follows, taking into account (35), (39) and (40),

$$\widetilde{TFP} = \sum_{j=1}^{J_R-1} \mu_j \left(\frac{n_j}{N}\right)^{\gamma} \widetilde{TFP}_j \tag{41}$$

Therefore, under no distortions in the economy, the more efficient $\widetilde{TFP}_j$ has a higher weight in the aggregate $\widetilde{TFP}$. And, the aggregate normalized TFP, $\widetilde{TFP}$, depends on technology frontier level and upon for how much old entrepreneurs' skills depreciate with respect to new entrepreneurs' skills.

Summarizing, in Table I we have firm's life-cycle plant size and firm's life-cycle normalized TFP. Furthermore, from equation (33) we can show that $\frac{\widetilde{\pi}_{j+1}}{\widetilde{\pi}_j} = \frac{n_{j+1}}{n_j}$. Consequently, the shape of the law of motion for the skill accumulation plays an important role on firm growth. Furthermore, taking into account that newborn agents' skills grow at the rate g_z, and that by equation (28), $(1+g_z) = (1+g)^{\frac{\theta_2}{(1-\theta_1)}}$, θ_1 and θ_2 also play a role in the depreciation of old entrepreneurs' skills. In particular, if $\theta_1+\theta_2 > 1$, newborn agents' skills grow at a higher rate than the growth rate of the economy, g. Hence, $\theta_2 > 0$, not only implies that relative managers' earnings, with respect to non-managers' earnings, increase with income and that richer economies will exhibit steeper managers' earnings, as mentioned in Guner et al. (2015), but also that it is more likely to have flatter employment-age and TFP-age profiles in faster growing economies.

TABLE I: FIRMS DYNAMICS OVER THEIR LIFE-CYCLE.

	Expression
$\frac{n_{j+1}}{n_j} =$	$\frac{\widetilde{Z}_{j+1}}{\widetilde{Z}_j} = \frac{\left[\int_{z_1^*}^{z^{\max}} \left(\frac{z_{j+1}(z_1)}{\overline{z}(1+g_z)^j}\right)^{\frac{\varphi\alpha\gamma}{(\sigma-1)(1-\gamma)}} f(z_1)dz_1\right]}{\left[\int_{z_1^*}^{z^{\max}} \left(\frac{z_j(z_1)}{\overline{z}(1+g_z)^{j-1}}\right)^{\frac{\varphi\alpha\gamma}{(\sigma-1)(1-\gamma)}} f(z_1)dz_1\right]}$
$\frac{\widetilde{TFP}_{j+1}}{\widetilde{TFP}_j} =$	$\left(\frac{\widetilde{Z}_{j+1}}{\widetilde{Z}_j}\right)^{(1-\gamma)} = \left[\frac{\int_{z_1^*}^{z^{\max}} \left(\frac{z_{j+1}(z_1)}{\overline{z}(1+g_z)^j}\right)^{\frac{\varphi\alpha\gamma}{(\sigma-1)(1-\gamma)}} f(z_1)dz_1}{\int_{z_1^*}^{z^{\max}} \left(\frac{z_j(z_1)}{\overline{z}(1+g_z)^{j-1}}\right)^{\frac{\varphi\alpha\gamma}{(\sigma-1)(1-\gamma)}} f(z_1)dz_1}\right]^{(1-\gamma)}$

3 Some numerical results

In this section we make some numerical exercises to analyze the implications of the model in terms of the observed facts. Concerning firm size distribution and managers' age-earnings profiles, we consider the US characteristics provided by Guner et al. (2015).

3.1 Parameters

One period corresponds to 10 years. Households start at the age of 20. Households live as workers or entrepreneurs during the first four periods, and as retirees during their last two periods (from 60 years old to 80 years old).

We choose some parameters to mimic some characteristics of the firm size distribution in the US economy, and some others are taken from the literature. Guner et al. (2015) use the 2004 U.S. Economic Census to collect information about the U.S. plant size and employment distribution. They find that the average plant size is approximately 17.86, the fraction of plants that employ less than 10 employees is about 72.5%, the fraction of plants that employ more than 100 employees is 2.7%. Concerning the distribution of plants' employment share, they find that the employment share of the total employment working in small plants (with less than 10 employees) is 15%. As in Guner et al. (2015) or Poschke (2015), we consider that managers' skills follow a lognormal distribution, where μ_z denotes the mean of $log(z_1)$ and σ_z is the standard deviation of $log(z_1)$. And we assume the same parameter values as in Guner et al. (2015). The share of capital in the economy is determined by the firms' returns to scale (γ) and the importance of capital (α). The importance of capital (α) is chosen to generate a capital income of one third, and the value of γ is close to the findings of previous papers for the U.S. economy (Guner et al. (2015), $\gamma = 0.77$, Buera et al. (2011), $\gamma = 0.77$, Atkeson and Kehoe (2007), Caunedo and Yurdagul (2016), Midrigan and Xu (2013), $\gamma = 0.85$, Cagetti and De Nardi (2006), $\gamma = 0.88$). Concerning the skill accumulation technology, θ_1, θ_2 and δ_θ are calibrated. We choose the values for θ_1, θ_2, σ and $\widetilde{A}$ to replicate the main characteristics of the US plant size distribution. Finally, we assume the same capital share in output and depreciation rate of capital as in Guner et al. (2015). And taking into account that the reported investment to output ratio for the period 1960-2000 in Guner et al. (2015) is 0.178, the capital output ratio is 2.656. The subjective discount factor is consistent with an annual interest rate of 5%.

Table II: Parameter Values

Name of the parameter	Parameter	Value (annualized)
Exogenous parameters		
Subjective discount factor	β	0.946
Mean log-managerial ability	μ_z	0
Dispersion in log-managerial ability	σ_z	2.65
Span of control	γ	0.8
Importance of capital	α	$0.3256/\gamma$
Capital depreciation rate	δ	0.067
Skill depreciation rate	δ_z	0
Calibrated parameters		
Skill accumulation technology	θ_1	0.96
Skill accumulation technology	θ_2	0.5
Skill depreciation rate	δ_θ	0.07
Elasticity of substitution types capital	σ	2.5
Normalized level of technology frontier	$\widetilde{A}$	600
Exogenous technological progress	g_A	0.0635

Table III: Observed Data and Model results

Statistics	U.S. Data	Model
GDP per capita growth rate	0.02	0.02
Capital-output ratio	2.656	2.65
Fraction of small plants (<10 workers)	0.725	0.689
Fraction of medium plants (>20 & <50)	0.093	0.091
Employment share of medium plants (>20 & <50)	0.167	0.165
Mean firm size	17.22	17.9

3.2 Some Numerical Experiments

We analyze the effect of changes in some exogenous parameters on the average and employment age-profile [7].

We start by showing that the empirical implications of considering economies that differ in their normalized level of the technology frontier level (normalized by the same constant growth rate equal to 2%) are consistent with the data.

[7] The assumption of a Cobb-Douglas function with respect to $n_{z,t}$ and M_{zt} ensures that the firm size distribution (in terms of number of employees) will be constant along a balanced growth path.

In the following Figure 1 we show that the employment-age profile in highly developed rich economies (with higher level of normalized technology frontier $\widetilde{A}$) is steeper than less developed poor economies (with lower $\widetilde{A}$).

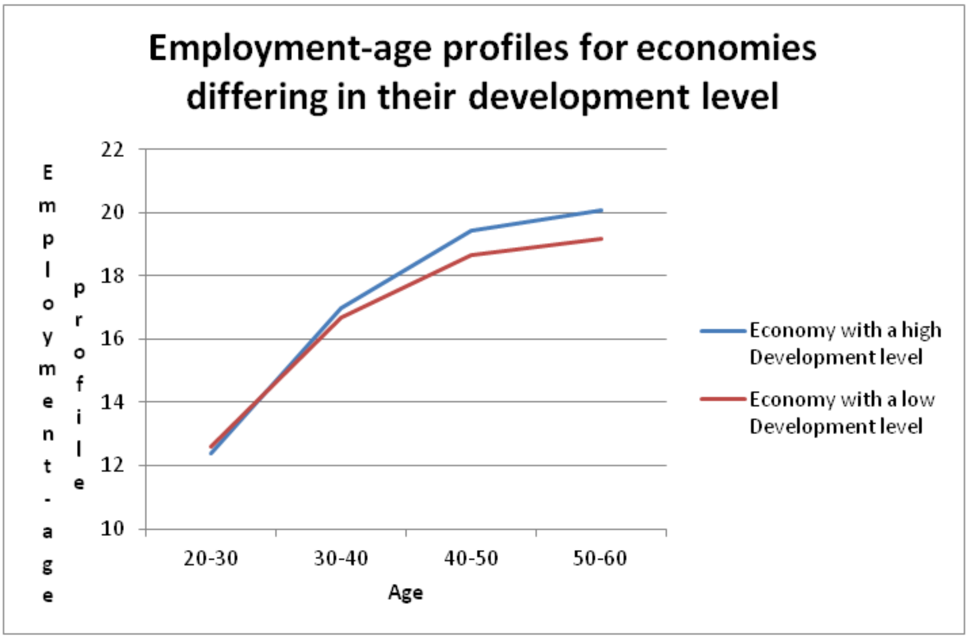

Figure 1.- Economies differing in their technology frontier level.

In Table IV, we also can see that our model is consistent with the observed facts that in less developed economies, firms start with a lower size, the average firm size is smaller, the entrepreneurship rate is higher and the skewness of firm size is also lower than in more developed economies.

TABLE IV: ECONOMIES DIFFERING IN THEIR DEVELOPMENT LEVEL.

	Benchmark ($\widetilde{A} = 600$)	Lower development ($\widetilde{A} = 350$)
Skewness of firm size	4.4908	4.3661
Firm size age-1	12.3976	12.588
Firm size (age-2/age-1)	1.370	1.325
Firm size (age-3/age-2)	1.566	1.484
Firm size (age-4/age-3)	1.620	1.524
Average firm size	17.219	16.782
Entrepreneurship rate	5.49%	5.62%

Concerning TFP-age profile, as expected is lower in poor economies than in richer economies, and also flatter.

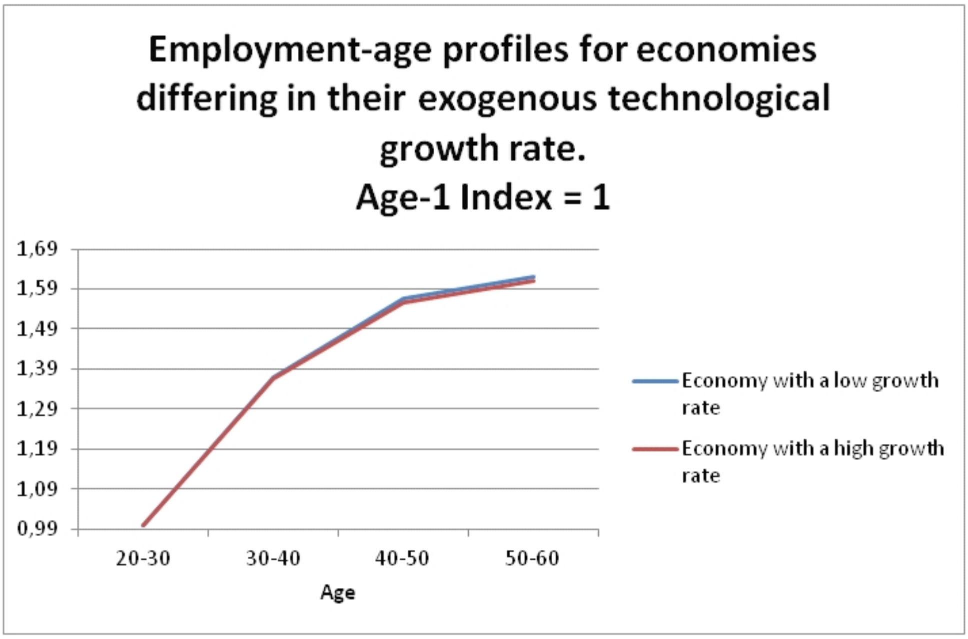

Figure 1: Figure 2.- Economies differing in their exogenous growth rate.

If we consider a higher exogenous technological process, managers' skills are more easily depreciated by skills of new generations. We find that employment-age profile is flatter, for a given level of development ($\widetilde{A}$). We show the results in Table V and in Figure 2. As we can see in Table V, with a higher exogenous technological change (higher exogenous growth of the technology frontier level), newborn firms (entrants) have a higher average firm size, but the employment-age profile is slightly flatter.

TABLE V: ECONOMIES DIFFERING IN THEIR EXOGENOUS GROWTH RATE.

	Benchmark ($g_A = 6.35\%$)	Higher growth ($g_A = 12.96\%$)
Skewness of firm size	4.4906	4.4715
Firm size age-1	12.397	12.463
Firm size (age-2/age-1)	1.370	1.364
Firm size (age-3/age-2)	1.566	1.555
Firm size (age-4/age-3)	1.620	1.607

4 Conclusions

One recent empirical regularity is that young firms grow more than older firms or that firm-growth (measured by the increase in the number of employees) is negatively related to firm's age, once it has been controlled for some other factors (see, among others, Barba et al. (2014), Coad et al. (2013), Dunne et al. (1989), Evans (1987a, 1987b), Fariñas and Moreno (200), Haltiwanger et al. (2012), López-García and Puente (2012)). Furthermore, Hsieh and Klenow (2014) find flatter employment-age profiles in less developed economies than in more developed economies. And, Caunedo and Yurdagul (2016) find flatter employment-age profiles in fast growing economies and steeper ones in slow growing economies. Consequently, it seems that, ceteris paribus, young firms grow much more than old firms in less developed countries, but also in countries with higher aggregate productivity growth rate. At micro level, there is some empirical work relating the age of chief executive officers (CEOs) to firms' life cycle growth (Barba et al. (2014)) or managers' abilities and firm growth (see, among others, Queiro (2015)). This result strengthens the role of entrepreneurs in making decisions about their entrepreneurial technology. In this paper we focus on the role of entrepreneurs' skills on firm growth, and we show that higher growth rate of technological advances makes entrepreneurials' skills more easily depreciated with age. And, we obtain that employment-age profiles and TFP-age profiles can be flatter in less developed economies, but also in faster growing economies.

We build an occupational choice life-cycle model with exogenous growth. As in Guner et al. (2015), agents take their irreversible decision on whether to become entrepreneurs at the first period of their life-cycle. And, following Poschke (2015), we consider that managers' skills determine the entrepreneurial technology they produce with. In this economy, we assume that the exogenous technological process implies a higher degree of complexity in the production process. It will be assumed that managers with higher skill will be able to produce with a higher degree of complexity (a higher number of intermediate inputs). Managers can increase their life-cycle skills, however their relative skill with respect to the whole entrepreneurial's population will also depend on how fast the economy is growing.

Our simulations are consistent with the empirical observation that employment-age profile is steeper among more developed economies, but also among slow growing economies (for a given level of development ($\widetilde{A}$)).

5 References

Akcigit U., and H. Alp, 2014. "Lack of Selection and Imperfect Managerial Contracts: FIrm Dynamics in Developing Countries" mimeo.

Atkeson A., and P.J. Kehoe, 2005. "Modeling and Measuring Organization Capital." Journal of Political Economy, 113 (5), 1026-1052.

Bahttarachya D., Nezih Guner and G. Ventura, 2013. "Distortions, endogenous managerial skills and productivity differences." Review of Economic Dynamics.

Barba Navaretti G., D. Castellani and F. Pieri, 2014. "Age and firm growth: evidence from three European countries." Small Business Econonomics, 43: 823–837.

Bloom, Nicholas, Sadun, Raffaella and John Van Reenen, 2012. "Management as a Technology?" mimeo.

Caliendo, L., G. Mion, L.D: Opromolla and E. Rossi-Hansberg, 2015, "Productivity and Organization in Portuguese Firms", mimeo.

Caunedo, J. and E. Yurdagul, 2016. "Who Quits Next? Firm Growth in Growing Economies", mimeo Universidad Carlos III (Spain).

Comin, D. and B. Hobijn, 2010, "An Exploration of Technology Diffusion", American Economic Review 100: 2013:2059.

Dunne, T., Roberts, M., & Samuelson, L. (1989), "The growth and failure of U.S. manufacturing plants.", Quarterly Journal of Economics, 104(4), 671–698.

Evans, D. S. (1987a), "The relationship between firm growth, size and age: Estimates for 100 manufacturing industries.", Journal of Industrial Economics, 35, 567–581.

Evans, D. S. (1987b), "Tests of alternative theories of firm growth.", Journal of Political Economy, 95(4), 657–674.

Fariñas, J. C., & Moreno, L. (2000), "Firms' growth, size and age: A nonparametric approach.", Review of Industrial Organization, 17, 249–265.

Foster, Lucia, John Haltiwanger, and Chad Syverson, 2008, "Reallocation, Firm Turnover, and Efficiency: Selection on Productivity or Profitability?" American Economic Review, 98 (2008), 394–425.

García-Santana, Manuel, and Roberto Ramos, 2014, "Distortions and the Size Distribution of Plants: Evidence from Cross-Country Data" manuscript.

Garicano, L. and E. Rossi-Hansberg, 2015. "Knowledge-Based Hierarchies: Using Organizations to Understand Economy", Annual Review of Economics, 7:1-30.

Guner, N., A. Parkhomenko and G. Ventura, 2015, "Managers and Productivity Differences," IZA Discussion Paper No. 9586.

Hall , Robert E. and Charles I. Jones , 1999, "Why Do Some Countries Produce So Much More Output per Worker than Others?"Quarterly Journal of Economics 114(1):83–116.

Haltiwanger, J. C., Jarmin, R. S., & Miranda, J. (2013), "Who creates jobs? Small VS. large VS. Young.", The Review of Economics and Statistics, Vol. XCV(2): 347-361.

Hsieh, C. and P. Klenow, 2009, "Misallocation and Manufacturing TFP in China and India," Quarterly Journal of Economics, 124 (4), 1403–1448.

Hsieh, C. and P. Klenow, 2014, "The life-cycle of plants in India and Mexico," The Quarterly Journal of Economics, 129 (3), 1035–1084.

Klenow, Peter J. and Andrés Rodríguez-Clare, 1997, "The Neoclassical Revival in Growth Economics: Has it Gone Too Far?" In Macroeconomics Annual 1997. , ed. Ben S. Bernanke and Julio J. Rotemberg, 73–102. Cambridge:MIT Press.

Jovanovic, B. and Rousseau, 2008,'Specific capital and technological variety', Journal of Human Capital 2(2), 129–153.

López-García, P., & Puente, S. (2012), "What makes a high-growth firm in Spain? A dynamic probit analysis using firm-level data.", Small Business Economics, 39(2), 1029–1041.

Lucas, Robert E., 1978, "On the size distribution of business firms," The Bell Journal of Economics, 9, 508-523.

Lucas, R. and B. Moll, 2014, "Knowledge Growth and the Allocation of Time," Journal of Political Economy, 122 (1), 1–51.

Luttmer, E., 2007, "Selection, Growth, and the Size Distribution of Firms," Quarterly Journal of Economics, 122 (3), 1103–1144.

Luttmer, E., 2007, "Models of Growth and Firm Heterogeneity," Annual Reviews of Economics, 2010, pp. 547–76.

Poschke, M. (2015), "The firm size disribution across countries and skill-biased change in entrepreneurial technology", IZA Working Paper 799.

Queiro, F. 2015. "The Effect of Manager Education on Firm Growth", mimeo.

Van Reenen, J., 2015, "Solving the Growth Puzzle?", Slides, Conference IIEA, Dublin.