Empirical Estimation of Elasticities and Their Use

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January 2016

Abstract

Applied research in economics contains many papers that empirically estimate an elasticity (or a set of elasticities) and uses the estimate(s) for policy analyses. These estimates are typically based on an estimated relationship (such as a demand function). If the specific functional form used in the estimation yields a constant elasticity in the form of an estimated coefficient alone, that coefficient represents the elasticity. If, on the other hand, the formula for the elasticity involves other regressors and coefficients, empirical estimation of the elasticity is based on applying the calculus-based definition of elasticity to this expression. However, a differential change in a variable is only an approximation to the actual discrete change for small changes, and the approximation can may be quite poor when large changes are considered. This paper advocates using the actual percentage change in the predicted value of the dependent variable when the variable with respect to which the elasticity is estimated changes by one percent. The example provided shows that the difference can be substantial when elasticities are estimated this way.

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1 Introduction

This paper focuses on empirically estimating elasticities from an estimated econometric model. The elasticity of one variable (such as demand) with respect to another variable is a fundamental concept introduced in introductory courses in economics. Using price elasticity of demand as example, the elasticity of demand with respect to price is defined as the percentage change in demand when the price changes by one percent. This discrete (finite) change in demand is approximated by the partial derivative of the demand function with respect to price. Although this is just an approximation, almost all applied research in economics estimate the price elasticity by first estimating an econometric model and then using the calculus-based definition of elasticity. This typically results in an expression that involves estimated coefficients and regressors, which needs to be evaluated in order to obtain elasticities. It is typical to evaluate this expression either at the observed means of all the variables that enter the elasticity formula. Alternatively, the expression is evaluated for every observation in the sample and then their average is presented as the elasticity.

Why is it that we still rely on an approximation when we want to estimate an elasticity? This is the main point this paper makes. We have enough computing power these days to calculate the discrete change in demand when price changes by one percent. In fact, it is easy enough to do this for every observation in our sample however large our sample may be. As shown in the following sections, the differences between using the approximations and the actual discrete changes can be quite substantial. This is particularly important when the estimated elasticities are used for policy analyses that involve substantial price changes. It is not unusual to predict the change in demand when price changes more than 20%.[1]

This paper is organised as follows. Section 2 presents a more detailed discussion of how elasticities are estimated using a demand system as an example. Section 3 uses a data set from the literature to demonstrate the differences in obtained elasticity estimates when one uses approximations and actual discrete changes. Section 4 concludes the paper.

[1]Some recent examples of such analyses are Mhurchu et al. (2015), Tiffin et al. (2015), Briggs et al. (2013)).

2 Elasticities from Econometric Models

Although an elasticity is a general concept that can relate to any variable, we can focus on a specific case without any loss of generality as what follows can easily be modified to any other situation. Let's suppose that we want to estimate the own and cross price elasticities of n commodities. The standard approach starts with specifying and estimating an econometric model for a system of demand functions. Let the the demand functions for the population be denoted by

$$y_i = f_i(\mathbf{x}_i, \boldsymbol{\beta}_i) + u_i, \quad i = 1, \ldots, n, \tag{1}$$

where $\mathbf{x}$ and $\boldsymbol{\beta}$ are vectors of regressors and their coefficients, and u_i is the error term. The regressors contain the prices of the commodities, $p_1, \ldots, p_n$, and y_i denotes the demand for commodity i. Then the conditional expected value of y_i given $\mathbf{x}_i$ is the population regression function $E[y_i|\mathbf{x}_i] = f_i(\mathbf{x}_i, \boldsymbol{\beta}_i)$ under the usual zero conditional mean assumption $E[u_i|\mathbf{x}_i] = 0$. The estimated version of $E[y_i|\mathbf{x}_i]$ is then the sample regression function

$$\hat{y}_i = f_i(\mathbf{x}_i, \hat{\boldsymbol{\beta}}_i), \quad i = 1, \ldots, n, \tag{2}$$

which gives the predicted value of the demand for good i for each individual (or household) when evaluated at the given values of the regressors for those individuals.

The price elasticity of the demand for good i, y_i, with respect to the price of good j, p_j, is defined as the percentage change in y_i when p_j changes by 1%:

$$\varepsilon_{ij} = \frac{\%\Delta y_i}{\%\Delta p_j} = \frac{\Delta y_i}{\Delta p_j}\frac{p_j}{y_i}. \tag{3}$$

For infinitesimal changes, the elasticity can be expressed as

$$\varepsilon'_{ij} = \frac{dy_i}{dp_j}\frac{p_j}{y_i}, \tag{4}$$

which is an approximation to ε_{ij} for small changes in p_j:

$$\varepsilon'_{ij} \approx \varepsilon_{ij}. \tag{5}$$

The common approach in estimating elasticities is based on applying equation (4) to

(2). This results, in general, in an expression that involves the regressors and estimated coefficients:

$$\hat{\varepsilon}'_{ij} = g(\mathbf{x}_i, \hat{\boldsymbol{\beta}}_i), \quad i = 1, \ldots, n, . \tag{6}$$

Although $\hat{\boldsymbol{\beta}}_i$ is the same for all the households, the value of $\mathbf{x}_i$ varies. There are, therefore, two ways to estimate the price elasticity just as one calculates the marginal effects after estimating an econometric model in any other situation: the average of predicted elasticities of all households using the observed values of the regressors, and the predicted elasticity of the average household for whom the values of the regressors are the mean values in the sample. These can be expressed as

$$\hat{\varepsilon}'^{AE}_{ij} = \frac{\sum_k g(\mathbf{x}_i^k, \hat{\boldsymbol{\beta}}_i)}{N}, \quad \text{(Average elasticity)} \tag{7}$$

and

$$\hat{\varepsilon}'^{EA}_{ij} = g(\bar{\mathbf{x}}_i^k, \hat{\boldsymbol{\beta}}_i), \quad \text{(Elasticity at the average)} \tag{8}$$

where N is the number of households in the sample, and $\bar{\mathbf{x}}_i^k$ is the vector of the averages of the regressors. The usual recommendation is to use the first method for policy analysis since it represents the overall effect.[2]

An alternative but never employed approach is to apply equation (3) to (2). This finite-difference approach evaluates equation (2) by using $p'_j = 1.01p_j^0$ to calculate the change in predicted demand when p_j changes by 1%. The price elasticity is then simply the difference between the predicted demand at p'_j and at the initial observed price p^0:

$$\hat{\varepsilon}_{ij} = 100\frac{f(\mathbf{x}'_i, \hat{\boldsymbol{\beta}}_i) - f(\mathbf{x}_i^0, \hat{\boldsymbol{\beta}}_i)}{f(\mathbf{x}_i^0, \hat{\boldsymbol{\beta}}_i)}, \quad i = 1, \ldots, n, . \tag{9}$$

Then the elasticity can be estimated as

$$\hat{\varepsilon}^{ae}_{ij} = \frac{\sum_k f(\mathbf{x}'_i, \hat{\boldsymbol{\beta}}_i) - f(\mathbf{x}_i^0, \hat{\boldsymbol{\beta}}_i)}{N}, \tag{10}$$

or

$$\hat{\varepsilon}^{ea}_{ij} = f(\bar{\mathbf{x}}'_i, \hat{\boldsymbol{\beta}}_i) - f(\bar{\mathbf{x}}_i^0, \hat{\boldsymbol{\beta}}_i), \tag{11}$$

just as before.

[2] See the short discussion in Section 5.2.4 in Cameron and Trivedi (2005) about this.

It is surprising that researchers always use the calculus-based approach even though it is well-known that the calculus method is an approximation to the discrete change case and a 1% change in price is actually a very large change for which this approximation can be very poor.[3]

3 An Illustration

Here we illustrate the differences in the magnitudes of the estimated elasticities by replicating the estimation in Poi (2012) using the same econometric model and the same data set. Poi (2012) estimates a four-equation demand system using data from the 1987-1988 Nationwide Food Consumption Survey conducted by the United States Department of Agriculture. Demands for four categories of food are estimated: meats, fruits and vegetables, breads and cereals, and miscellaneous. The sample used consists of 4,048 households. The econometric model is the Quadratic Almost Ideal Demand System (QUAIDS) of Banks et al. (1997). The model implies that the predicted uncompensated calculus-based price elasticities are given by

$$\hat{\varepsilon}_{ij} = -\delta_{ij} + \frac{1}{w_i}\left(\hat{\gamma}_{ij} - \left[\hat{\beta}_i + \hat{\boldsymbol{\eta}}_j'\mathbf{z} + \frac{2\hat{\lambda}_i}{b(\mathbf{p})c(\mathbf{p},\mathbf{z})}\ln\left\{\frac{m}{\bar{m}_0(\mathbf{z})a(\mathbf{p})}\right\}\right] \times \right.$$
$$\left.\left(\hat{\alpha}_j + \sum_l \hat{\gamma}_{jl}\ln p_l\right) - \frac{\left(\hat{\beta}_j + \hat{\boldsymbol{\eta}}_j'\mathbf{z}\right)\hat{\lambda}_i}{b(\mathbf{p})c(\mathbf{p},\mathbf{z})}\left[\ln\left\{\frac{m}{\bar{m}_0(\mathbf{z})a(\mathbf{p})}\right\}\right]^2\right), \quad (12)$$

where δ_{ij} is the Kronecker delta equaling one when $i = j$ and zero otherwise, $\hat{\gamma}, \hat{\beta}, \hat{\eta}, \hat{\lambda}$ are some estimated coefficients, w is the budget share, p's are prices, m is total expenditure, $\mathbf{z}$ contains the number of children and a urban-rural dummy variable.[4]

Table 1 presents the own and cross price elasticities obtained by using this equation. The first column presents the average elasticities where equation (12) is evaluated for every household in the sample and then an average is taken. The third column presents the elasticities at the average where equation (12) is evaluated just at the average values of the regressors. Columns two and four calculate the elasticities from discrete changes in predicted demand when the relevant price changes by 1%.

[3]See, for example, Section 10.6 in Cameron and Trivedi (2010).

[4]See Poi (2012) for the definitions of the functions $a(\mathbf{p}), b(\mathbf{p})$, and $c(\mathbf{p},\mathbf{z})$.

Table 1: Predicted Elasticities

	Average Elasticity (AE)		Elasticity at Average (EA)	
	Calculus-based	Discrete	Calculus-based	Discrete
ε_{11}	-0.63	-0.69	-0.70	-0.69
ε_{12}	-0.18	-0.15	-0.15	-0.15
ε_{13}	-0.12	-0.09	-0.09	-0.10
ε_{14}	-0.12	-0.10	-0.10	-0.11
ε_{21}	-0.23	-0.18	-0.18	-0.18
ε_{22}	-0.61	-0.69	-0.70	-0.69
ε_{23}	0.01	0.00	0.00	0.01
ε_{24}	-0.02	-0.01	-0.01	-0.01
ε_{31}	-0.53	-0.37	-0.36	-0.35
ε_{32}	-0.02	-0.01	-0.01	-0.01
ε_{33}	-0.38	-0.56	-0.58	-0.59
ε_{34}	-0.09	-0.06	-0.06	-0.06
ε_{41}	-0.20	-0.15	-0.15	-0.15
ε_{42}	-0.06	-0.05	-0.04	-0.04
ε_{43}	-0.04	-0.03	-0.03	-0.02
ε_{44}	-0.74	-0.80	-0.81	-0.80

The differences between the estimated values of elasticities are presented in Table 2. The reported relative errors are calculated by subtracting the calculus-based measure from the discrete-change elasticity and dividing the difference by the discrete-change elasticity.

We next consider a policy that results in a 10% decrease in the price of good 2 (fruit and vegetables), and predict the changes caused by this on the consumption of all the commodities. We first use the elasticities presented in Table 1 in order to determine the percentage change in consumption. These percentage changes are reported in the first four columns of Table 3. The reported numbers are obtained by simply multiplying the elasticities in Table 1 by 10. The numbers in the last two columns are obtained not by using the elasticities, but by directly predicting the new demands when p_2 decreases by 10%. That is, finite changes in predicted demands are calculated and reported as percentage changes. These are actual changes in the values of the estimated demand functions when p_2 is lowered by 10%. Column five does this for every household and then takes the average. Column 6 does this only for the "average" household for whom the values of the regressors are set at their average values.

Table 2: Relative Errors

	Average Elasticity (AE)	Elasticity at Average (EA)
ε_{11}	-0.10	0.02
ε_{12}	0.22	-0.03
ε_{13}	0.23	-0.02
ε_{14}	0.22	-0.03
ε_{21}	0.29	-0.01
ε_{22}	-0.12	0.01
ε_{23}	0.17	-0.14
ε_{24}	-0.51	0.32
ε_{31}	0.42	0.03
ε_{32}	0.57	0.43
ε_{33}	-0.32	-0.01
ε_{34}	0.45	0.06
ε_{41}	0.34	0.02
ε_{42}	0.37	0.06
ε_{43}	0.36	0.04
ε_{44}	-0.07	0.01

Table 3: Percentage change in consumption when p_2 decreases by 10%

	Elasticity-based				Discrete Change	
	Calculus-based		Discrete			
	AE	EA	AE	EA	Average	At the Average
Good 1	1.79	1.48	1.46	1.53	1.54	1.61
Good 2	6.13	7.02	6.94	6.93	7.6	7.59
Good 3	0.20	0.09	0.13	0.06	0.14	0.07
Good 4	0.63	0.44	0.46	0.42	0.48	0.44

4 Discussion and Concluding Remarks

The results presented in the previous section show that the estimated elasticities can differ substantially between the calculus-based and finite-difference approaches, particularly when one uses average elasticities. The finite-difference approach simply uses the estimated demand function, which is the basis for the calculus-based approach. The discrete changes calculated by simply evaluating the demand functions before and after the price changes can therefore be taken as 'true' changes. The relative (relative to the 'true' changes) errors reported in Table 2 are quite large for average elasticities, ranging from 7% to 57%. This is quite alarming, given that average elasticities are the recommended ones for policy analysis.

Table 3 takes this further by considering the impact of a large discrete change in a price. It is typical in existing studies to use the estimated elasticities to determine the impact on demand of large changes in a price. If, for example, a price changes by 10% as considered here, the impact is usually determined by simply multiplying the estimated elasticity by 10. The first four columns in Table 3 do precisely that. It is found that the percentage change in the demand for good 2, the good whose price is lowered by 10%, is between 6 and 7% depending upon which elasticity measure is used. The actual change in demand is, however, 7.6%, given the estimated demand function. We see that the error in the predicted change in the demands for the four commodities is between 16 and 31% when calculus-based average elasticities are used. Even finite-change elasticities yield to predictions with errors that range between 4 and 8.8%.

It is so easy to avoid these 'approximation errors', since estimated demand functions already exist. There is no need to approximate a change when the exact change can easily be calculated. This is particularly relevant when the elasticities are used as inputs in determining the effect of certain policies that involve multiple large price changes.

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