MODELLING TERMS OF TRADE VOLATILITY IMPACT ON OUTPUT DYNAMICS IN RUSSIA

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Abstract

In this paper, we analyze terms of trade volatility impact on macroeconomic dynamics in the dynamic stochastic general equilibrium (DSGE) model for the Russian economy. The question of volatility impact on the Russian economy is still uncovered in modern literature. It hasn't been studied in the context of DSGE models yet. First, we build an aggregate index of terms of trade volatility which takes into account export components share as well as their dynamics through past years. This index is interpreted as uncertainty measure of external economic conditions for our country. We present methodology of constructing volatility index with a help of stochastic volatility model (SV model) and estimate it with MCMC. SV model shows that 2009 and 2014-2015 years experienced the highest volatility picks for the last fifteen years. Second, we explain why SV is suitable for DSGE modelling. Third, we integrate terms of trade volatility index in small open economy DSGE model. In order to estimate uncertainty impact we build two-sector DSGE model with exogenous terms of trade volatility shocks and four types of home economic agents: households, export-oriented firms, firms producing intermediate goods and firms producing final goods aggregating import and domestic intermediate goods. It is worth noting that volatility impact shocks consideration requires approximation of nonlinear DSGE model around steady state up to third order.

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1. Introduction

Russian economy is highly dependent on external economic conditions due to its export-orientation focus. Substantial variability in prices of exporting goods has an impact on income of many economic agents. Income changes in its turn have a direct effect on consumption, savings and investments, which are driving factors for total production dynamics. The question of straight resource influence on Russian economy has already been discussed in details; however, in academic literature much less attention has been paid to the impact of prices volatility or terms of trade volatility. Through this work, the term terms of trade stands for export and import prices ratio, but the key distortion role stands for export prices. In order to understand better what is meant by terms of trade volatility let us answer the question: is the period of oil price changing for the same value significant? Answering this question from households and firms point of view we can state: the quicker prices' changes are, the higher the dispersion of future prices is going to be expected. Such mechanism is conjugate with poor forecast and high uncertainty. Households have consumption habits that's why they prefer to consume relatively the same basket during life. A sudden drop in income pursues households to increase their savings today, by the way, their income can bounce back at the nearest future. Associated redistributions can lead to additional costs, moreover in savings increase periods investments have fallen short. Summing it all up, high uncertainty leads to ineffective economic functioning and output shrinkage. Through this paper, we will try to estimate terms of trade volatility impact on output dynamics taking into account economic agents interconnectedness. In order to measure volatility influence on output it is essential to measure the depth of uncertainty in crisis periods 2008-2009 and 2014-2015 years.

The structure of the paper is organized in the following way: chapter 1 is devoted to related literature, chapter 2 offers new index of uncertainty in export prices, chapter 3 is devoted to macroeconomic modelling of economic agents' interconnectedness with volatility shocks, chapter 4 provides empirical results.

2. Related literature

Through the last years, academic literature pays high attention to the impact of uncertainty on macroeconomic dynamics. The mostly common and widely spread indices of exogenous uncertainty are CBOE S&P 500 Volatility Index (VIX), CBOE S&P 100 Volatility Index (VXO) and Macro Risk Index (MRI). VIX, VXO are tickers of volatility indices of Chicago stock exchange. They are evaluated as 30-days implied volatility of call and put options with underlying assets S&P 500 and S&P 100 correspondingly. MRI is a Citibank index, which is aggregated on several components: government bonds spread of advanced economics and economics of transition, exchange rate volatility, stock exchange indices.

Bloom[11] presents a number of stylized facts concerning tendencies in volatility indices and uncertainty. Thus, VIX shows a vivid example of counter-cyclical movement to the USA recessions, which are classified by NBER. Implied volatility shows an average 58 % increase during declining periods. In its term, GDP and production volatility is 35 % higher in crisis periods in comparison to stable ones. The reason for volatility upsurge can be the increase of risk-aversive behavior of investors, which has an immediate effect on option prices and their volatility value.

There is a number of articles presenting quantitative impact of uncertainty on the macroeconomic variables dynamics. In his other work Bloom[10] , uses VAR model to estimate the volatility of securities prices impact on the economy. The main result shows that high uncertainty results in lower economic performance, decrease of investment activity and decline of production. Ramey and Ramey [26] run a panel model with fixed effects in order to evaluate GDP volatility impact on economic growth among 92 countries. Engle and Rangle[15] build on a spline-GARCH model for modelling GDP and inflation volatility. They get robust result that emerging economies show higher volatility in comparison to advanced economies. According to the most empirical works shown above there is a common opinion that financial markets volatility explains a substantial share of volatility in fundamental macroeconomic variables.

The role of uncertainty is touched upon in some empirical research concerning Russian economy. The most widespread measure of uncertainty in foreign markets is VIX. Lomivorotov[3] uses BVAR in order to show the financial markets volatility impact on key economic branches' output, investments and exchange rate. In his other work[2], VIX is used as control variable for monetary shocks identification. Pestova and Mamonov [5] use BVAR as well and get that an increase in VIX value has a statistical significant impact on output. Russian Central Bank in its report[1] examines VIX and MRI as indicators of exogenous uncertainty.

However, Russian economy experiences a high dependence on export; in particular, a number of works present output dependence on oil dynamics, export prices volatility can be a more appropriate measure of uncertainty. From the one hand, such measure evaluates the dispersion revenue of investment projects of export-oriented companies, high value of which can lead to the reduction of investment projects, from another hand, high uncertainty in export sector results in high uncertainty of aggregate output, which can result in consumption cut due to precautionary motive.

Summing up what has been said, we can infer that the role of uncertainty is crucial but no index clearly represents Russian economy specificity, therefore, the aim of current work is to develop an export prices volatility index (EPVI) that will contain information in Russian export structure and dynamics.

3. Construction of terms of trade volatility index

The current part of article is devoted to method of EPVI construction. It is crucial to mention that the aim of article to build a time-varying volatility measure, as a counter to a classical understanding of dispersion or volatility as a sum of square deviation from population mean.

In this current work, we will assume that all variety of exporting products can be combined into several N homogeneous common groups. Russian export is resource oriented, assuming Russia to be a small open economy, we suggest that Russia is a price taker on world market; vector of logarithmic prices growth can be characterized by the following expression:

$$\boldsymbol{u_t} = [log(\frac{p_t^1}{p_{t-1}^1})...log(\frac{p_t^i}{p_{i-1}^1})...log(\frac{p_t^N}{p_{t-1}^{N-1}})]' \sim N(\boldsymbol{\gamma_t}, \boldsymbol{V_t}) \tag{1}$$

where p_t^i —– price of a certain group of goods i in time t, $\boldsymbol{u_t}$ — normal vector with mathematical expectation equal to $\boldsymbol{\gamma_t}$ and covariance matrix equal to $\boldsymbol{V_t}$.

The growth rate of Laspeyres price index let us define by the following:

$$\frac{I_t}{I_{t-1}} = \frac{\sum_i p_t^i * q_{t-1}^i}{\sum_i p_{t-1}^i * q_{t-1}^i} = \sum_i \frac{p_{t-1}^i * q_{t-1}^i}{\sum_i p_{t-1}^i * q_{t-1}^i} * \frac{p_t^i}{p_{t-1}^i} = \sum_i w_{t-1}^i \frac{p_t^i}{p_{t-1}^i} \tag{2}$$

where I_t — aggregate price index on exporting goods, q_t^i— amount of each group iat period t, w_t^i — share of each group of goods i in period t in total export.

Using logarithmic approximation, we can get:

$$log(\frac{I_t}{I_{t-1}}) = \sum_i w_{t-1}^i log(\frac{p_t^i}{p_{t-1}^i}) \tag{3}$$

Therefore, the logarithmic index growth on export prices will have the following distribution:

$$log(\frac{I_t}{I_{t-1}}) \sim N(\boldsymbol{w_{t-1}'\gamma_t}, \boldsymbol{w_{t-1}' V_t w_{t-1}}) \tag{4}$$

In order to calculate EPVI we need to collect data on particular share of each export group of goods in total export and get the covariance matrix of stochastic process for prices' movements. The data on export share can be collected using Russian trade data, while the covariance matrix is unobserved, therefore, it is needed to be estimated by econometric modelling. In our empirical analysis we are leaning on stochastic volatility model presented in Taylor[29] work.

Multivariate stochastic volatility model implies several alternative specification. The simplest one assume that matrix $\boldsymbol{V_t}$ is diagonal, in other words, correlation between different export prices is zero. However, preliminary analysis 8.1 shows that that this correlation is non-zero in most cases. Extensions that are more complicated admit time-varying correlations among export commodity prices, for example, Mayer and Yu[30]. We will concentrate on intermediate model assuming constant correlation between prices, but time-varying dispersion:

$$\begin{cases} log(\frac{p_t^i}{p_{t-1}^i}) = \gamma_t^i + v_t^i e^{\frac{h_t^i}{2}} \\ h_t^i = \mu^i + \rho^i h_{t-1}^i + \theta_i w_t^i \\ \boldsymbol{v_t'} = [v_t^1 ... v_t^i ... v_t^N]' \sim N(\boldsymbol{0}, \boldsymbol{C}) \\ \boldsymbol{w_t'} = [w_t^1 ... w_t^i ... w_t^N]' \sim N(\boldsymbol{0}, \boldsymbol{D}) \end{cases} \tag{5}$$

Where $\boldsymbol{C}, \boldsymbol{D}$ — correlation matrixes, the main diagonal of which is unit.

For volatility EPVI construction we need to know matrix $\boldsymbol{C}$ and standard deviations of shocks $e^{\frac{h_t^i}{2}}$. Model could be estimated by maximum likelihood method, but h_t^i is integrated into first equation non-linearly that's why we cannot use standard procedures like Kalman[19] filter. Stochastic volatility model estimation requires sampling methods. However, lot's of parameters will be estimated, that's why we propose the next three-step procedure. Firstly, we estimate $log(\frac{p_t^i}{p_{t-1}^i})$, using simple ARMA (p, q) for each group of goods price in accordance with Box – Jenkins procedure, forecast values on ARMA (p, q) model give us $\widehat{log(\frac{p_t^i}{p_{t-1}^i})}$.

Secondly, using $\widehat{log(\frac{p_t^i}{p_{t-1}^i})}$, forecast we estimate volatility of each group $\hat{h_t^i}$. Finally, we normalize each price growth on its standard deviation estimate:

$$\hat{v_t^i} = \frac{log(\frac{p_t^i}{p_{t-1}^i}) - \widehat{log(\frac{p_t^i}{p_{t-1}^i})}}{e^{\frac{\hat{h_t^i}}{2}}} \tag{6}$$

Table 1. Methods of estimating stochastic volatility model

Method	Idea of method	Related literature
Gaussian approximation	Approximation of Chee-square distribution with Gaussian distribution	Ciplakov 2009, Kitagawa, Sato 2001
Sequential Monte-Carlo method	By Monte-Carlo simulations choose with replacement those particles that are most informative for SV process	Fernandez-Villaverde 2007, Fernandez-Villaverde 2011, Kitagawa, Sato 2001
Bayesian estimation with Metropolis-Hasting algorithm	Need to choose prior distribution for parameters μ, ρ, θ	Chib 1995, Katner 2014, Kastner 2016, Kim 1998

Source: author

estimation of correlation between innovations $\hat{v_t^i}$ gives us an estimation of matrix $\boldsymbol{C}$.

There several alternative methods for estimation of stochastic volatility model. The simplest method is based on Gaussian approximation of likelihood function and inference of Kalman filter. For example, Ciplakov [8] analysis accuracy of likelihood approximation of RTS index. Methods that are more complicated require simulations in order to estimate likelihood function. A short description of methods is presented at Table 1

In its work Kastner [21] applies Bayesian method using MCMC. Such approach requires specification of prior distributions of stochastic volatility model parameters μ, ρ, θ. It is shown to be much more accurate in empirical analysis, therefore, we will follow this method in subsequent analysis.

As Kim[22] mentions μ is usually assumed to be normally distributed with mean m_μ and standard deviation σ_μ. For stationarity reason $\rho \in (-1, 1)$. Kastner[21] proposes beta-distribution, where $\dfrac{\rho+1}{2} \sim \mathfrak{B}(a_0, b_0)$.Density function is the following:

$$p(\rho) = \frac{1}{2B(a_0, b_0)}\left(\frac{1+\rho}{2}\right)^{a_0-1}\left(\frac{1-\rho}{2}\right)^{b_0-1} \tag{7}$$

For θ^2 it is assumed in [19] Chi-square distribution $\theta^2 \sim b_\theta * \chi^2(1)$, where b_θ – hyperparameter. Degree of freedom equal to one in Chi-square distributions is a widely spread prior value for exchange rates and stock exchange market.

4. Construction of small open DSGE model with volatility shocks

This chapter is devoted to dynamic stochastic general equilibrium (DSGE) model construction. In order to estimate export prices volatility contribution we should specify model to make external shocks have a direct impact on economic conference. Uribe[31] proposes to introduce an export-oriented sector, where firms react to price shocks by adjusting production. Such specification assumes price shocks to be exogenous which is suitable for Russian economy. Export-oriented firms involve our economy to be open and introduce import-consuming firms. Following Polbin[4] it is reliable to assume economy to have a share of firms using only home-produced goods. Labor deliverers are households, who exchange their work on goods. Summing it all up, we have small open economy with four types of economic agents.

1) Households;

2) Export-oriented firms;

3) Firms producing intermediate goods;

4) Firms producing final goods with the usage of import good as well as final goods

Economic agents interconnectedness in DSGE model is presents at Pic. 1.

Moving on to model specification it is crucial to discuss EPVI influence channels. In open economy model uncertainty increase is presented by increase in export prices volatility. High volatility represents high unpredictable range of future prices. Moreover, the longer predicted period is the higher is uncertainty. For export-oriented firms resources are the main income, volatility increase will lead to revenue shrinkage. At the same time for Russian firms as well as for households the permanent income hypothesis is held. Economic agents are tend to consume relatively the same basket during the life, which leads to high sensibility from every deviation of consumption from habits. A sudden drop in income pursues households to increase their savings

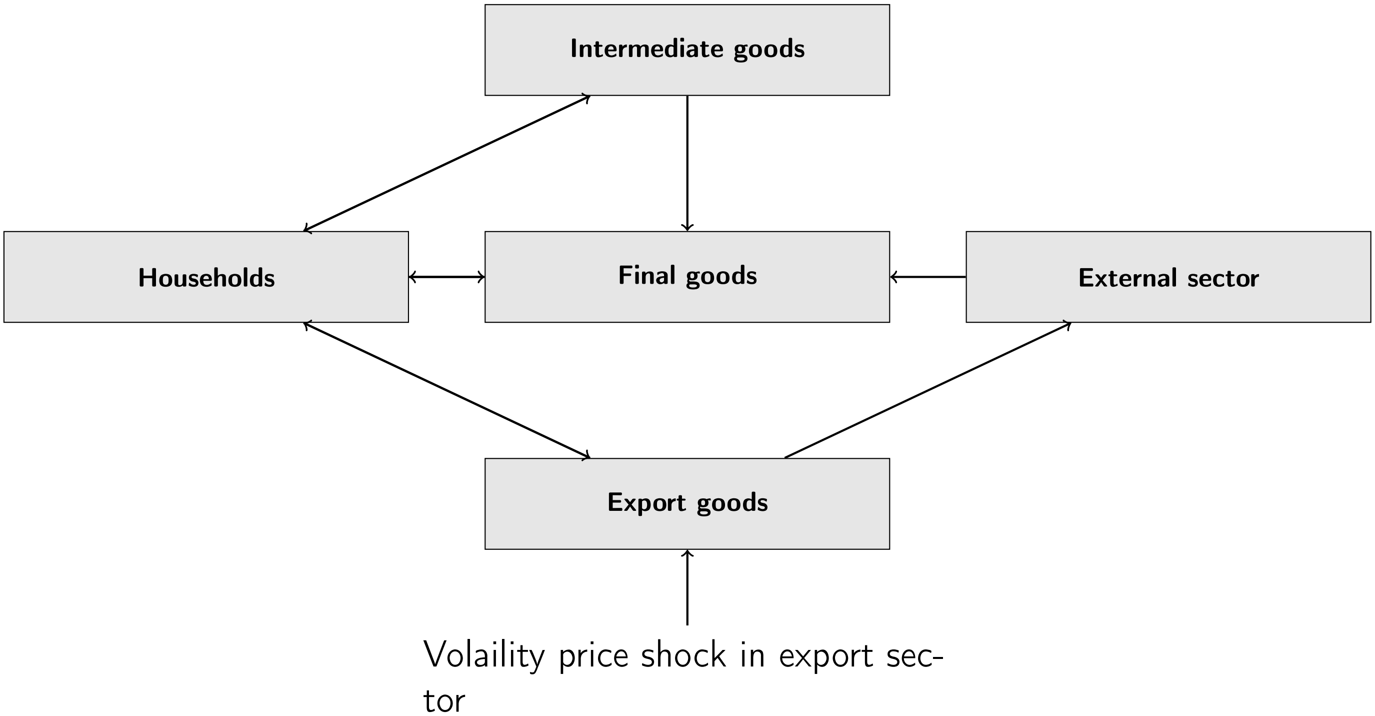

Figure 1. Economic agents interconnectedness in DSGE model. Source: author

today, by the way, their income can bounce back at the nearest future. Associated redistributions can lead to additional costs, moreover in savings increase periods investments have fallen short. Summing it all up, high uncertainty leads to ineffective economic functioning and output shrinkage.

The utility function with habits[25] in consumption has the following view:

$$U(c_t, l_t) = log(c_t - H_t) - \theta * \frac{l_t^{1+\psi}}{1+\psi} \tag{8}$$

where U(*) - households' utility function, which is positively dependent on consumption and negatively dependent on working hours, c_t - consumption, l_t - number of hours worked, ψ - reciprocal of labor supply elasticity of wage, θ - normalization parameter. H_t - consumption habits variable. Polbin[4] mentions that consumption habits are proportional to previous one $H_t = h * C_{t-1}$. The higher parameter h is the less invividual get utility from consumption. The next peculiarity of the model is to assume addition costs of issuing bonds. Such restriction is aimed to evade sharp increase of issuing bonds in response to increase of interests rates.

$$\frac{\phi}{2}(b_t - b_t^{ss})^2 \tag{9}$$

where b_t - number of issued bonds t, b_t^{ss} - number of bonds in steady state, ϕ - costs parameter.

We begin with the description of export-oriented sector. It is the sector, which contains exogenous export prices volatility shock. The classical introduction of export-oriented firms is a Cobb–Douglas function with two production factors: labor and capital in equation (10):

$$\begin{cases} y_t^{ex} = A_t^{ex}(k_t^{ex})^{\alpha_2}(l_t^{ex})^{1-\alpha_2} \\ \max\ p_t^{ex}y_t^{ex} - w_t l_t^{ex} - r_t^k k_t^{ex} \end{cases} \tag{10}$$

where y_t^{ex} - export-oriented firms output, A_t^{ex} - labor productivity, k_t^{ex} - capital, l_t^{ex} - labour, p_t^{ex} - exports' price. In our specification price p_t^{ex} is exogenous, and price volatility is introduced by SV model:

$$\begin{cases} log(\frac{p_t^i}{p_{t-1}^i}) = \gamma_t^i + v_t^i e^{\dfrac{h_t^i}{2}} \\ h_t^i = \mu^i + \rho^i h_{t-1}^i + \theta_i w_t^i \\ \boldsymbol{v_t'} = [v_t^1 ... v_t^i ... v_t^N]' \sim N(\boldsymbol{0}, \boldsymbol{C}) \\ \boldsymbol{w_t'} = [w_t^1 ... w_t^i ... w_t^N]' \sim N(\boldsymbol{0}, \boldsymbol{D}) \end{cases} \tag{11}$$

Domestic firms specialize on production of final and intermediate goods. Our paper assumes intermediate goods to be produced only by internal firms:

$$\begin{cases} y_t^d = A_t^d(k_t^d)^{\alpha_1}(l_t^d)^{1-\alpha_1} \\ \max\ p_t^d y_t^d - w_t l_t^d - r_t^k k_t^d \end{cases} \tag{12}$$

where y_t^d - firms output, A_t^d - labor productivity, k_t^d -capital, l_t^d - working hours, p_t^d - selling price, w_t - wage.

Our paper assumes final goods to be produced from import and internal products:

$$\begin{cases} y_t = \frac{(y_t^d)^{1-\omega}(y_t^{Im})^{\omega}}{\omega^{\omega}(1-\omega)^{1-\omega}} \\ \max\ p_t y_t - p_t^d y_t^d - p_t^{im} y_t^{im} \end{cases} \tag{13}$$

where y_t - firms output, w - share of import sector, therefore, 1-w - share of domestic firms. The next assumtion is the absolute mobility of production factors.

$$\begin{cases} k_t = k_t^{ex} + k_t^d \\ l_t = l_t^{ex} + l_t^d \end{cases} \tag{14}$$

Households have the aim to maximize their ulitily using budget constrain. Capital growth rate, which is received by households is equal to r_t^k.

$$\begin{cases} w_t l_t + r_t^k k_t + (1 + r_{t-1}) b_{t-1} = p_t c_t + p_t i_t + b_t + \frac{\phi}{2}(b_t - b_t^{ss})^2 \\ k_{t+1} = i_t + (1 - \delta) k_t \\ \max\ E_0 \sum_{t=0}^{\propto} \beta^t * U(c_t, l_t) \end{cases} \tag{15}$$

No-ponzy game condition has the following view:

$$\lim_{j->\propto} = E_t \frac{b_{t+j}}{\prod_{s=0}^{j}(1 + r_s)} \tag{16}$$

$$\beta * (1 + r) = 1 \tag{17}$$

Final goods balance is specified in the followng way:

$$y_t = c_t + i_t \tag{18}$$

Current account balance:

$$-(b_t - b_{t-1}) = p_t^{ex} y_t^{ex} - p_t^{im} y_t^{im} - r_{t-1} b_{t-1} \tag{19}$$

First order conditions for economic sectors.

Export-oriented sector:

$$\begin{cases} \max\ p_t^{ex} A_t^{ex} (k_t^{ex})^{\alpha_2} (l_t^{ex})^{1-\alpha_2} - w_t l_t^{ex} - r_t^k k_t^{ex} \text{ по } k_t^{ex}, l_t^{ex} \end{cases} \tag{20}$$

Partial derivatives allow us to get:

$$\begin{cases} \alpha_2 p_t^{ex} \frac{y_t^{ex}}{k_t^{ex}} = r_t^k \\ (1 - \alpha_2) p_t^{ex} \frac{y_t^{ex}}{l_t^{ex}} = w_t \end{cases} \tag{21}$$

Domestic firms production of intermediate goods is:

$$\begin{cases} \max\ p_t^d A_t^d (k_t^d)^{\alpha_1} (l_t^d)^{1-\alpha_1} - w_t l_t^d - r_t^k k_t^d \text{ по } k_t^d, l_t^d \end{cases} \tag{22}$$

Partial derivatives allow us to get:

$$\begin{cases} \alpha_1 p_t^d \frac{y_t^d}{k_t^d} = r_t^k \\ (1-\alpha_1) p_t^d \frac{y_t^d}{l_t^d} = w_t \end{cases} \tag{23}$$

Domestic firms production of final goods is:

$$\begin{cases} \max\ p_t \frac{(y_t^d)^{1-\omega} (y_t^{Im})^{\omega}}{\omega^{\omega}(1-\omega)^{1-\omega}} - p_t^d y_t^d - p_t^{im} y_t^{im} \text{ по } y_t^d, y_t^{Im} \end{cases} \tag{24}$$

Partial derivatives allow us to get:

$$\begin{cases} p_t (\frac{y_t^{Im}}{y_t^d})^{\omega} (\frac{1-\omega}{\omega})^{\omega} = p_t^d \\ p_t (\frac{y_t^d}{y_t^{Im}})^{1-\omega} (\frac{\omega}{1-\omega})^{1-\omega} = p_t^{Im} \end{cases} \tag{25}$$

Huseholds tasks are:

$$\begin{cases} \pounds = \\ E \sum_{t=0}^{\infty} \left[\beta^t U(c_t, l_t) + \lambda_t (p_t c_t + p_t (k_{t+1} + b_t - (1-\delta) k_t) + \frac{\phi}{2} (b_t - b_t^{ss})^2 - w_t l_t - r_t^k k_t - (1+r) b_{t-1}) \right] \\ Derivatives : c_t, l_t, b_t, k_{t+1} \end{cases} \tag{26}$$

Partial derivatives allow us to get:

$$\begin{cases} -\frac{U'_{c_t}(c_t,l_t)}{U'_{l_t}(c_t,l_t)} = \frac{p_t}{w_t} \\ E_t \frac{U'_{c_t}(c_t,l_t) * p_{t+1}}{U'_{c_{t+1}}(c_{t+1},l_{t+1})} = \frac{(1+r)*\beta*p_t}{\phi(b_t - b_t^{ss})+1} \\ E_t \frac{U'_{c_t}(c_t,l_t) * p_{t+1}}{U'_{c_{t+1}}(c_{t+1},l_{t+1})} = p_{t+1}(1-\delta) - r_{t+1}^k \end{cases} \tag{27}$$

Interconnectedness between economic agents in DSGE model is presented at system 28.

$$
\begin{cases}
\frac{1}{c_t \theta l_t^{\psi}} = \frac{p_t}{w_t} \\
E_t \frac{p_{t+1} c_{t+1}}{c_t} = \frac{(1+r)*\beta*p_t}{\phi(b_t - b^{ss})} \\
E_t \frac{p_{t+1} c_{t+1}}{c_t} = \beta(p_{t+1}(1-\delta) + r_{t+1}^k) \\
k_{t+1} = i_t + (1-\delta)k_t \\
\alpha_2 * p_t^{ex} \frac{y_t^{ex}}{k_t^{ex}} = r_t^k \\
(1-\alpha_2) * p_t^{ex} \frac{y_t^{ex}}{l_t^{ex}} = w_t \\
\alpha_1 * p_t^d \frac{y_t^d}{k_t^d} = r_t^k \\
(1-\alpha_1) * p_t^d \frac{y_t^d}{l_t^d} = w_t \\
p_t(\frac{y_t^{Im}}{y_t^d})^{\omega}(\frac{1-\omega}{\omega})^{\omega} = p_t^d \\
p_t(\frac{y_t^d}{y_t^{Im}})^{1-\omega}(\frac{\omega}{1-\omega})^{1-\omega} = p_t^{Im} \\
k_t = k_t^{ex} + k_t^d \\
l_t = l_t^{ex} + l_t^d \\
y_t = c_t + i_t \\
b_t - b_{t-1} = p_t^{ex} y_t^{ex} - p_t^{im} y_t^{im} + r b_{t-1} - \frac{\phi}{2}(b_t - b_t^{ss})^2 \\
y_t^{ex} = A_t^{ex}(k_t^{ex})^{\alpha_2}(l_t^{ex})^{1-\alpha_2} \\
y_t^d = A_t^d(k_t^d)^{\alpha_1}(l_t^d)^{1-\alpha_1} \\
y_t = \frac{(y_t^d)^{1-\omega}(y_t^{Im})^{\omega}}{\omega^{\omega}(1-\omega)^{1-\omega}} \\
log(\frac{p_t^i}{p_{t-1}^i}) = \gamma_t^i + v_t^i e^{\frac{h_t^i}{2}} \\
h_t^i = \mu^i + \rho^i h_{t-1}^i + \theta_i w_t^i \\
p_t^{im} = 1
\end{cases} \tag{28}
$$

5. Empirical estimation of EPVI impact on output dynamics

Empirical steps are organized in the following way. Firstly, we need to get time-series raw of export prices volatility index (EPVI). Secondly, we need to make simulations of DSGE model to get impulse response paths. In order to estimate EPVI we should choose estimation procedure between MCMC and Gaussian approximation and evaluate the scale of shock in 2008-2009 and 2014-2015 crisis. Simulation modelling is based on DSGE calibration and IRF construction of output response to EPVI shocks

In order to build EPVI it is necessary to describe prices for each export group. The data on Russian export to all other countries is presented on the FTS. Below you can see export structure of Russia in 2016 year (in decreasing order) Pic 2.

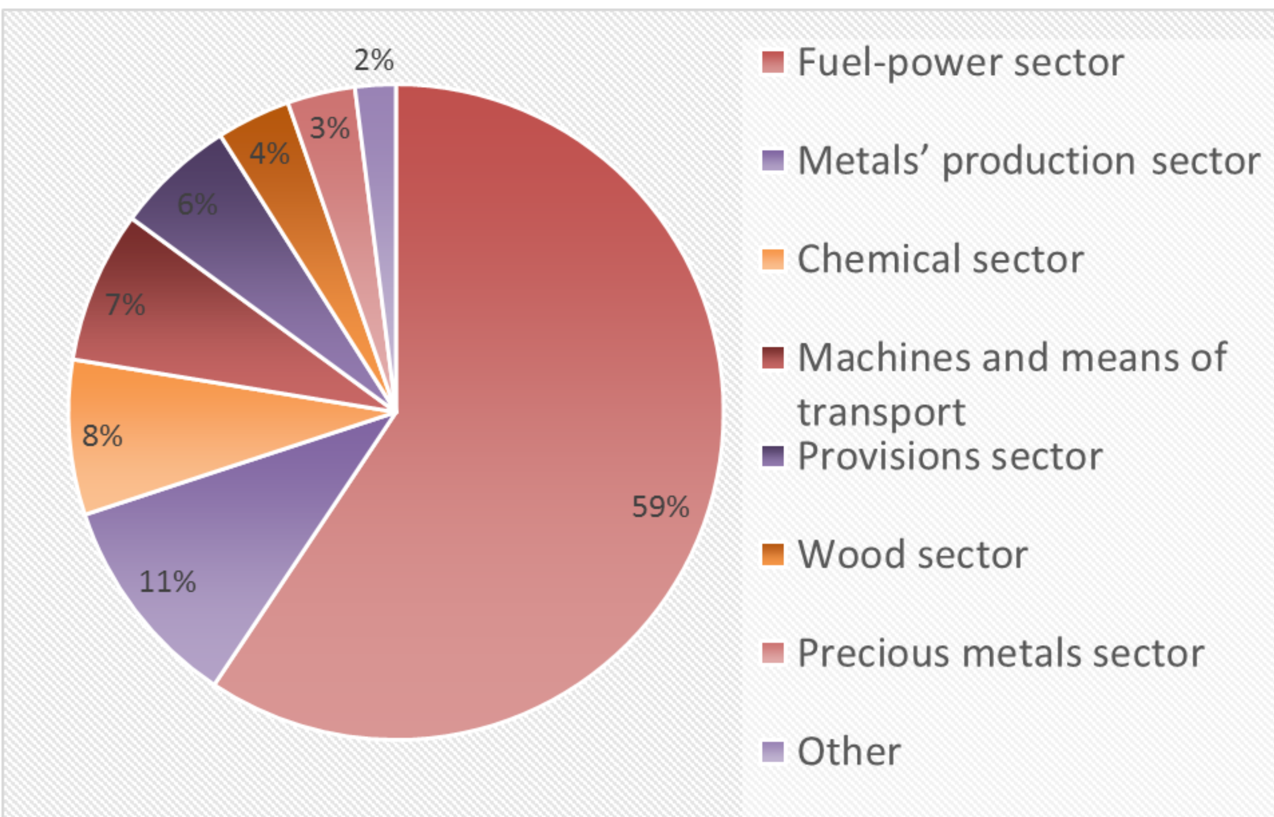

Figure 2. Russian export structure in 2016.
Source: author on Federal Custome Service data

According to 2016-year data 59% of all export stands for oil, oil products and gas, 25% of export stands for metals, chemical industry and machines, 10% stands for agriculture and wood, 3% - stands for precious metals. Categories mentioned above cover about 97% of all export in Russia. All export is divided to 9 categories or groups and for each group have selected proxy-variable which best describe particular category using FRED [33] data. FRED database contains the most list of trading products and the same time allows getting long and monthly frequent time series rows, which are crucial for estimation of stochastic volatility model. Proxy-variables for each group are presents in Table 2. As you can see from the table available period for each row is heterogeneous, but we prefer to use all data for each row in order to get the most accurate estimation of model parameters. After all, estimations for each group will be aggregated into one index starting from January 2001. Some critics can be inferred concerning the use of oil price Brent instead of Urals, Brent price in its term is quite higher than Urals, but rate of return difference of these two prices is insignificant.

Along with price dynamics, we use data on FTS, GS, Central bank about annual export structure. While constructing EPVI of month m year t we use share of particular group of commodities in previous period as a weight.

Moving on to empirical results Table 3 summarizes stochastic volatility model estimation for different export categories. Parameter μ is a constant in volatility equation, ρ is an autoregressive component and θ is a dispersion scale parameter. In brackets we present standard deviations of parameter estimations. In Picture 3 and 4 price volatility estimation for copper as well as posterior parameters density are presented. Denstity functions for other prices can be found in Appendix 8.3.

On the Figure 5 you can see the comparison between several volatility indices: EPVI, VIX and MRI. As can be drawn from the graph, 2008-2009 crisis shows an upsurge in both EPVI and VIX, therefore world crisis was supported by both financial and commodities term. During the 2014-2016 Russian crisis financial markets were rather stable, therefore, considering only VIX index could be concluded that uncertainty in external markets was low, but it is not the case for

Table 2. Relation between export groups and prices

Exporting sectors	Prices of exporting commodities	FRED data	Period
Fuel-power sector	Oil price	MCOILBRENTEU	1987m5-2016m12
Metals' production sector - aluminum	Aluminum price	PALUMUSDM	1980m1-2016m12
Metals' production sector - copper	Copper price	PCOPPUSDM	1980m1-2016m12
Metals' production sector - steel	Steel price	WPU101702	1990m1-2016m12
Chemical sector	Manurial price	PCU32533253	1985m1-2016m12
Machines and means of transport	Machines price	PCUOMFGOMFG	1986m1-2016m12
Provisions	Corn price	PMAIZMTUSDM	1980m1-2016m12
Wood and its derivatives	Wood price	WPU0911	1980m1-2016m12
Precious metals	Gold price	GOLDAMGBD228NLBM	1968m4-2016m12

Source: author

Table 3. Parameters estimation is SV model

Export products	μ	ρ	θ
Gold price	2,63 (0,27)	0,94 (0,02)	0,33 (0,06)
Oil price	4,05 (0,22)	0,88 (0,05)	0,38 (0,08)
Corn price	3,06 (0,12)	0,48 (0,12)	0,83 (0,12)
Aluminum price	3,13 (0,25)	0,94 (0,03)	0,24 (0,06)
Copper price	3,32 (0,19)	0,86 (0,07)	0,41 (0,1)
Wood price	1,43 (0,26)	0,78 (0,09)	0,62 (0,15)
Steel price	0,67 (0,44)	0,9 (0,04)	0,64 (0,12)
Machines price	-1,69 (0,4)	0,94 (0,03)	0,3 (0,07)
Manurial price	0,09 (0,35)	0,9 (0,04)	0,6 (0,11)

Source: author

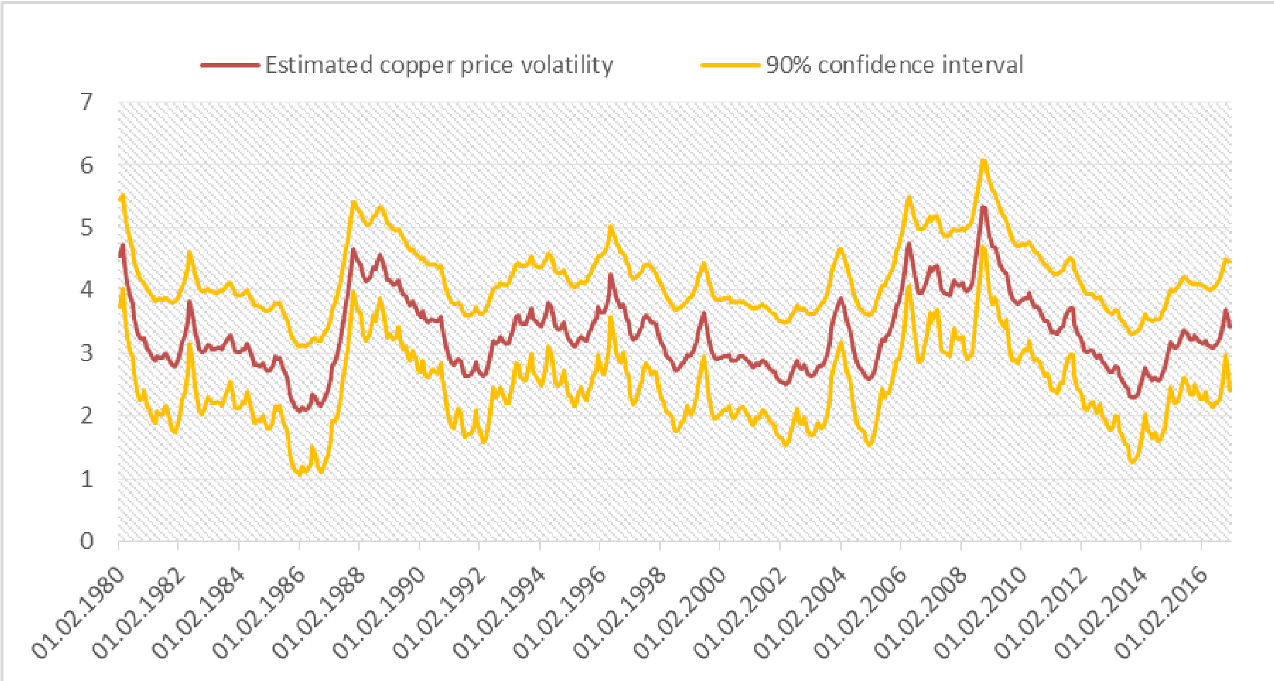

Figure 3. Estimated volatility with 90% confidence intervals. Source: author on FRED data

EPVI, which shows comparable upsurge in both crisis. The pic shows that MRI rise in uncertainty is in 2007, but at the same time a sudden drop in 2008. Considering only MRI index could result in earlier investigation of crisis peak of uncertainty. EPVI the most precisely reflects economic performance showing counter-cyclical growth on commodity markets in recent crisis. EPVI shows twofold jump in crysis periods in comparison to stable ones.

The current part is devoted to calibration of model parameters in DSGE, derivation of steady states, simulation of impulse response paths of macroeconomics variables on volatility shocks.

The model can be characterized by two types of parameters: the first group of parameters stands for dynamic properties of economic model. And should be calibrated to common in academic literature values, the second group should best of all reflect properties of Russian economy.

Parameter θ in utility function stands for time devoted to leisure and labor. It is normally in Russia 8 hours working day, about 1/3 of full day, therefore θ is equal to 1, in other words 2/3 of all day is spent to leisure. Parameters α_1, α_2 share of income from capital in total output, Kidland

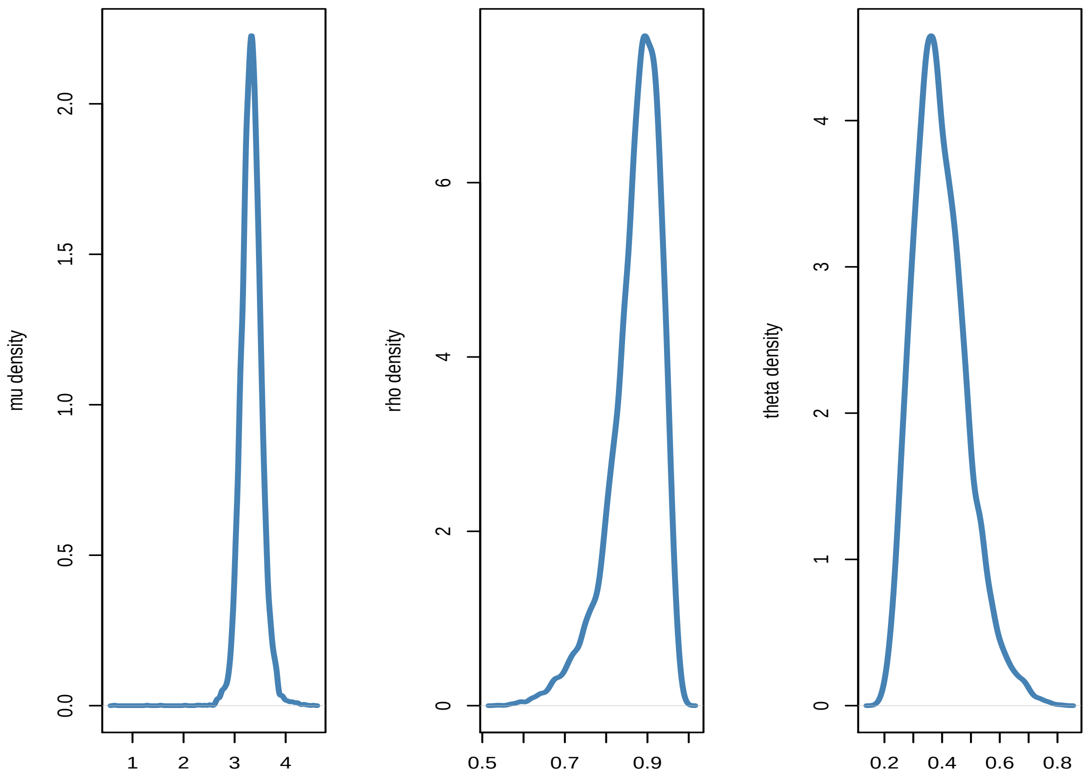

Figure 4. Posterior distibution on SV model parameters for copper. Estimator μ on the left, ρ on the right, θ in the middle.

Source: author

states these parameters be near 0.3. β - discounting parameter. This parameter discounts future cash flows to present ones. It can be evaluated as weighted-average on volume credits given less inflation. Average quarter real interest rest is 0.01, therefore β is equal to 0.99. Reciprocal of labor supply elasticity of wage is equal to 3.5. Depreciation rate is 7% per year. Habits parameter h is rather high 0.7. Share of import sector is about 0.26.

Non-linear model simulations need third-order Taylor approximation around steady state in order to capture direct effects from volatility shocks. This model can be estimated manually and Appendix 8.2 shows steady state values.

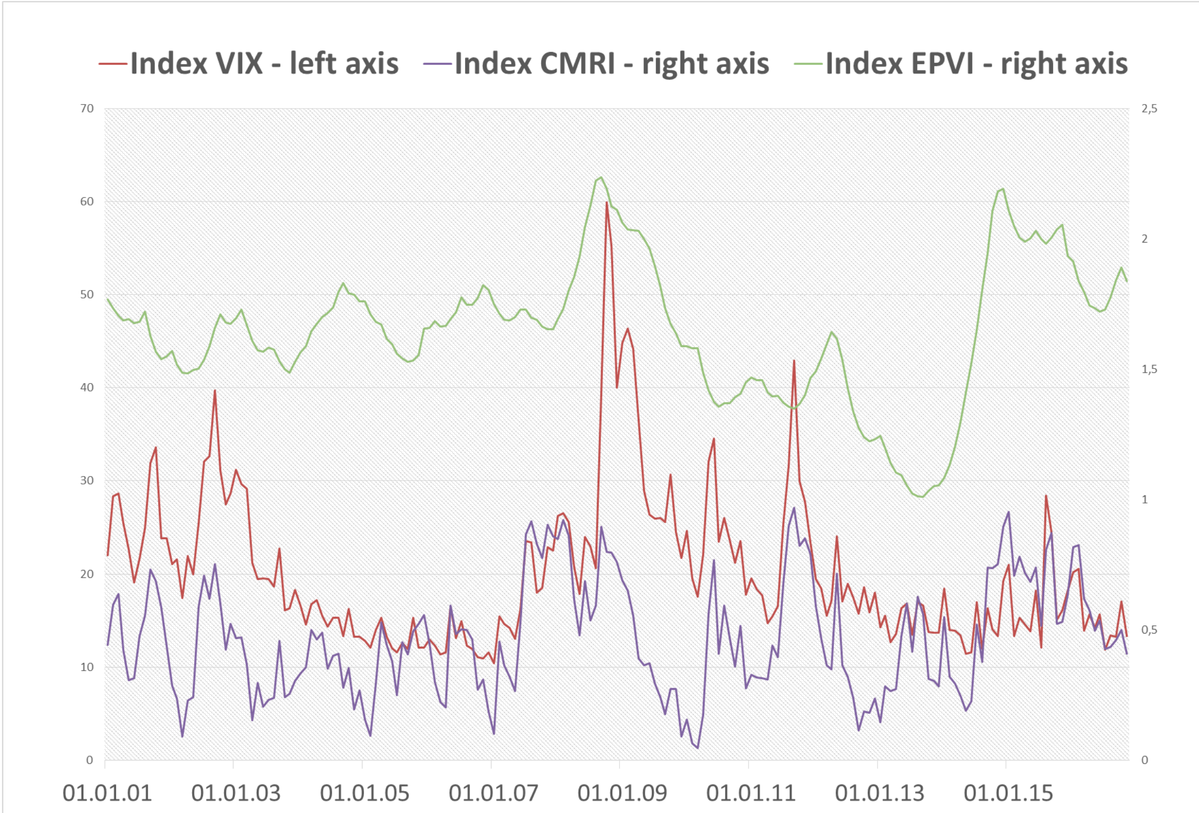

Figure 5. Comparison of alternative indices of uncertainty.

Source: author on Bloomberg[35] data and estimations (EPVI)

Picture 6 shows output, labor, investment and consumption response to EPVI shock comparable to recent 2008-2009 and 2014-2015 years crisis. The uncertainty impact hypothesis is proved/ Consumption habits leads to very small shrink of consumption today, but future consumption is on lower trajectory. Precautionary savings cause savings increase in response to volatility growth, but investments shrink on the contrary. The total decline is realized in output and labor declining. Summing it all up, twofold export prices volatility growth resulted in 1.5% output shrinkage.

Table 4. Parameters calibration

Parameter	θ	ψ	r	β	ϕ	δ	α_1	α_2	ω	A_t^{ex}	A_t^d
Value	1	3.5	0.01	0.99	0.00074	0.07	0.3	0.3	0.26	0.24	0.57

Source: author

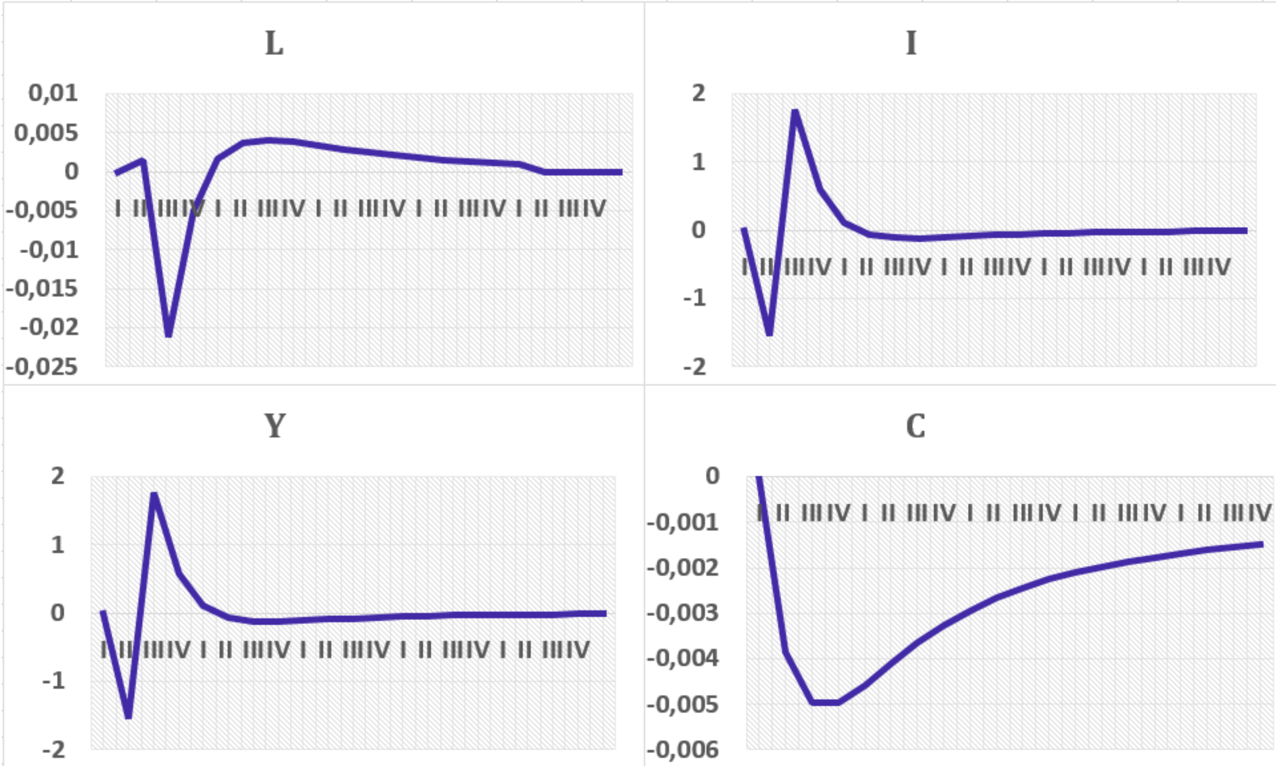

Figure 6. Impulse response functions to twofold EPVI shock. Source: author

6. Conclusion

This paper presents estimation of export prices volatility (EPVI) impact on short-turn output dynamics. We have built EPVI, which seems to be a relevant measure of uncertainty of external conditions in Russian economy. EPVI covers 97% of export groups and captures the effects of structure import changes and price movements. Our index shows comparable increase in uncertainty in 2008-2009 and 2014-2015 crisis. Terms of trade shocks have been integrated in small open DSGE model by stochastic volatility specification. The main influence channel stands for high uncertainty of future income of firms and households. Economic agents need to save extra money in order to smooth shocks, which results in shrinkage of investments. Twofold volatility increase causes 1.5% decrease in output.

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Table 5. Export prices correlation table

	Gold price	Oil price	Corn price	Alum price	Copper price	Wood price	Steel price	Mach price	Manur price
Gold price		0,14 (2,05)	0,15 (2,1)	0,26 (3,78)	0,28 (4,21)	-0,01 (-0,2)	0,09 (1,31)	0,14 (2,04)	0,01 (0,11)
Oil price	0,14 (2,05)		0,12 (1,76)	0,34 (5,1)	0,42 (6,54)	0,11 (1,65)	0,09 (1,31)	0,66 (12,65)	0,06 (0,86)
Corn price	0,15 (2,1)	0,12 (1,76)		0,18 (2,53)	0,12 (1,73)	0,14 (2,08)	0,12 (1,76)	0,11 (1,65)	0,16 (2,35)
Aluminum price	0,26 (3,78)	0,34 (5,1)	0,18 (2,53)		0,66 (12,61)	0,15 (2,23)	0,12 (1,69)	0,28 (4,14)	0,1 (1,37)
Copper price	0,28 (4,21)	0,42 (6,54)	0,12 (1,73)	0,66 (12,61)		0,11 (1,63)	0,09 (1,35)	0,29 (4,28)	0,08 (1,15)
Wood price	-0,01 (-0,2)	0,11 (1,65)	0,14 (2,08)	0,15 (2,23)	0,11 (1,63)		0,18 (2,55)	0,11 (1,63)	0,1 (1,43)
Steel price	0,09 (1,31)	0,09 (1,31)	0,12 (1,76)	0,12 (1,69)	0,09 (1,35)	0,18 (2,55)		0,18 (2,67)	0,15 (2,18)
Machines price	0,14 (2,04)	0,66 (12,65)	0,11 (1,65)	0,28 (4,14)	0,29 (4,28)	0,11 (1,63)	0,18 (2,67)		0,21 (3,11)
Manurial price	0,01 (0,11)	0,06 (0,86)	0,16 (2,35)	0,1 (1,37)	0,08 (1,15)	0,1 (1,43)	0,15 (2,18)	0,21 (3,11)	

Source: author

8. Appendix

8.1. Appendix 1. Export prices correlation table

Below you can see correlation table, and t-statictics for export price: $t = \frac{r}{(1-r^2)} * \sqrt{(n-2)} \sim t_{(n-2)}$ They are summarized in Table 5.

8.2. Appendix 2. Steady state values in DSGE

$$\begin{cases} c_t = 0.8 \\ l_t = 0.3 \\ p_t = 1 \\ w_t = 1 \\ b_t = 0 \\ r_t^k \\ k_t = 10 \\ i_t = 0.2 \\ y_t^{ex} = 0.056 \\ k_t^{ex} = 2.6 \\ l_t^{ex} = 0.08 \\ y_t^d = 0.154 \\ k_t^d = 7.4 \\ l_t^d = 0.22 \\ y_t^{im} = 0.056 \\ p_t^d = 1 \\ p_t^{ex} = 1 \\ p_t^{im} = 1 \\ y_t = 1 \end{cases} \tag{29}$$

8.3. Appendix 3. Posterior distributions

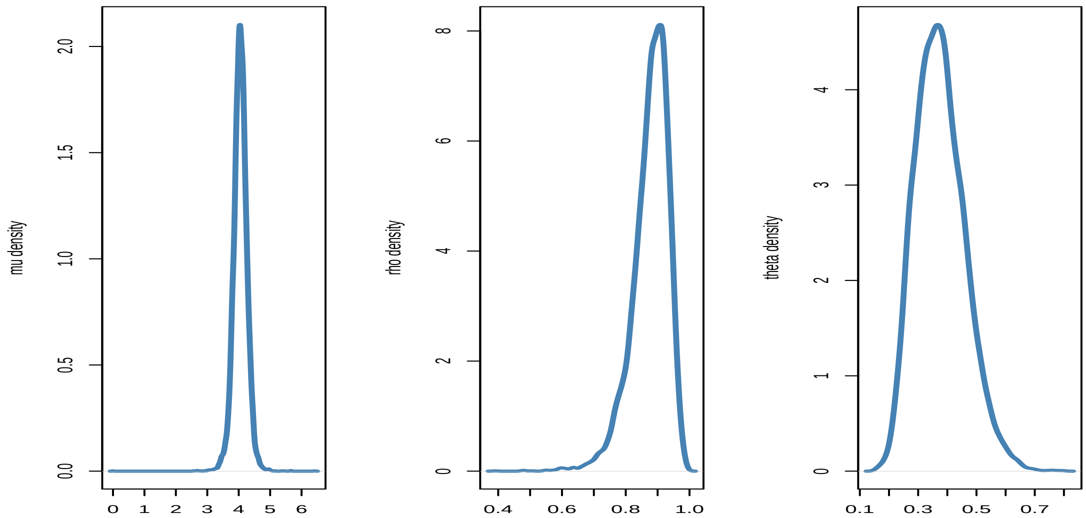

Figure 7. Posterior distibution on SV model parameters for oil. Estimator μ on the left, ρ on the right, θ in the middle.

Source: author

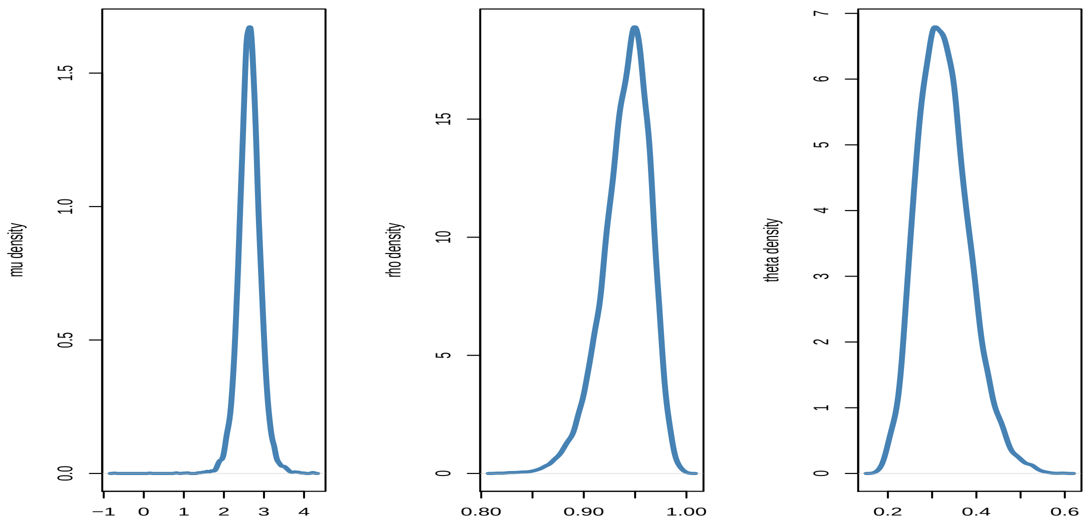

Figure 8. Posterior distibution on SV model parameters for gold. Estimator μ on the left, ρ on the right, θ in the middle.

Source: author

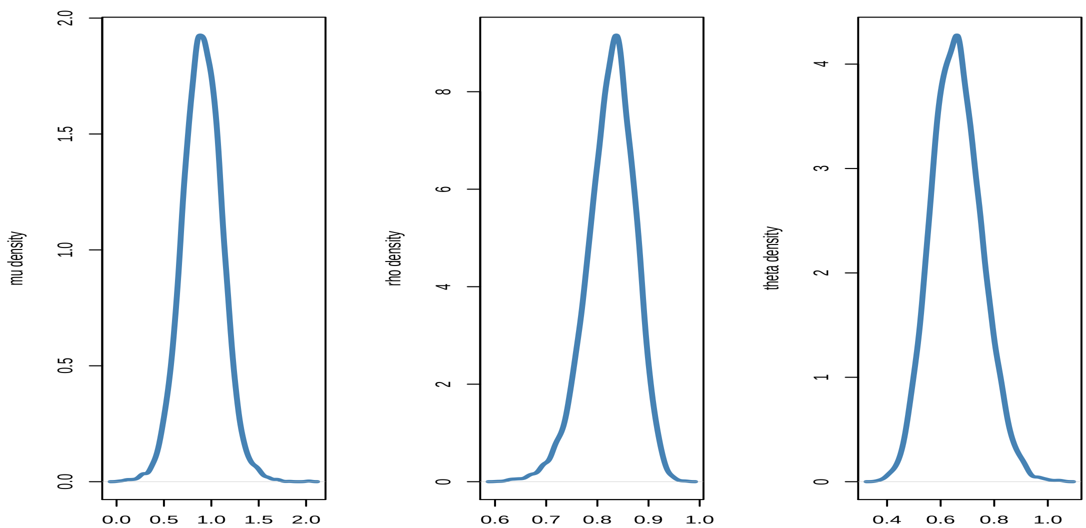

Figure 9. Posterior distibution on SV model parameters for wood. Estimator μ on the left, ρ on the right, θ in the middle.

Source: author

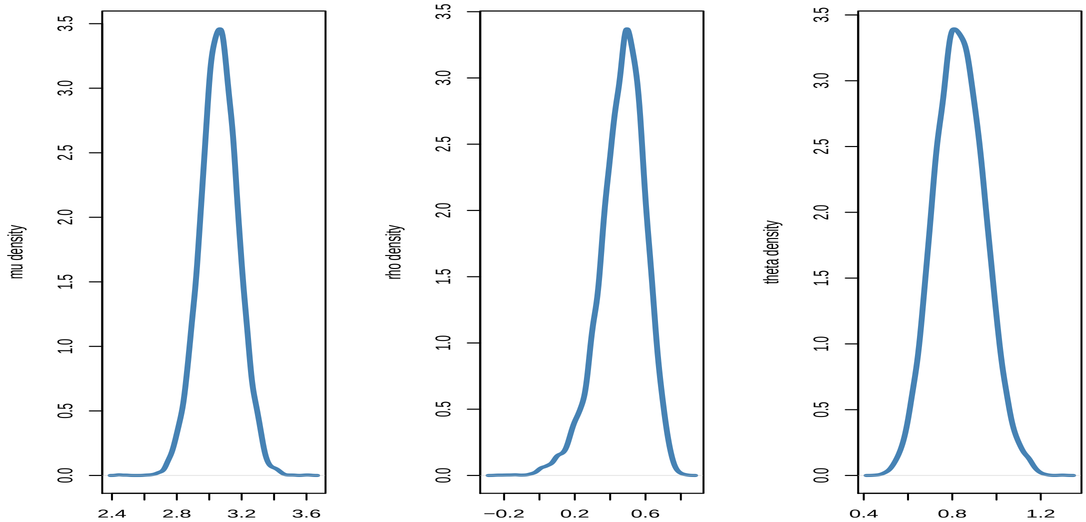

Figure 10. Posterior distibution on SV model parameters for corn. Estimator μ on the left, ρ on the right, θ in the middle.

Source: author

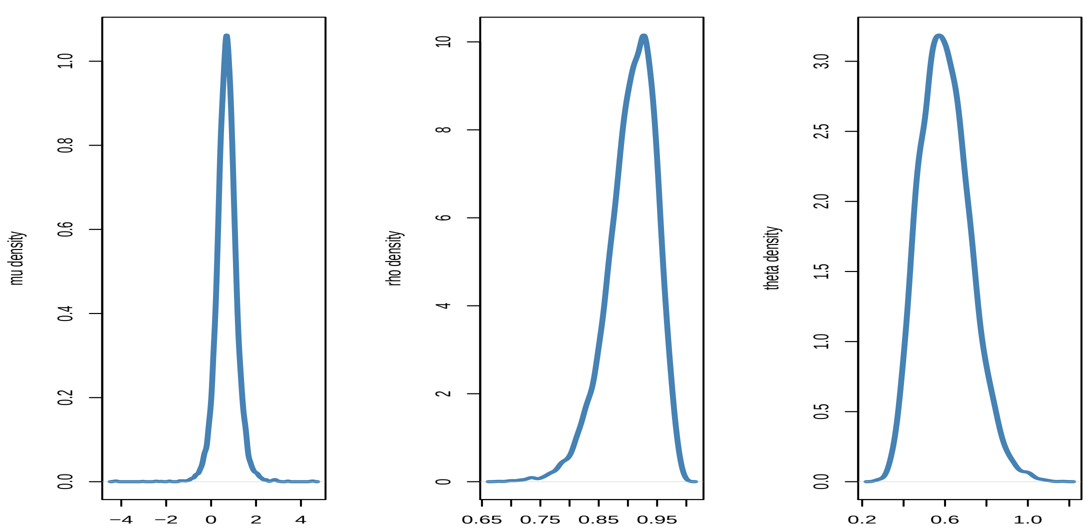

Figure 11. Posterior distibution on SV model parameters for steel. Estimator μ on the left, ρ on the right, θ in the middle.

Source: author

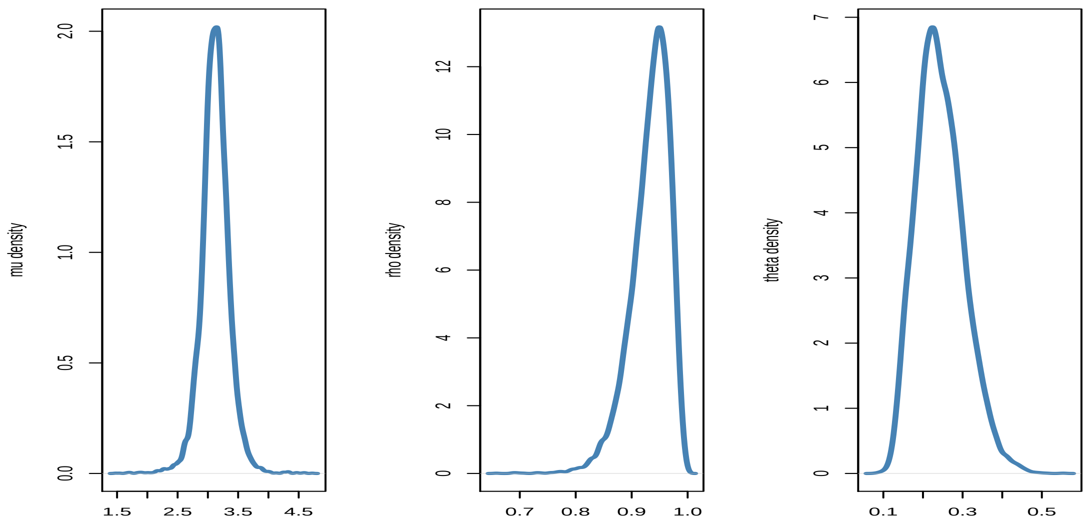

Figure 12. Posterior distibution on SV model parameters for aluminum. Estimator μ on the left, ρ on the right, θ in the middle.

Source: author

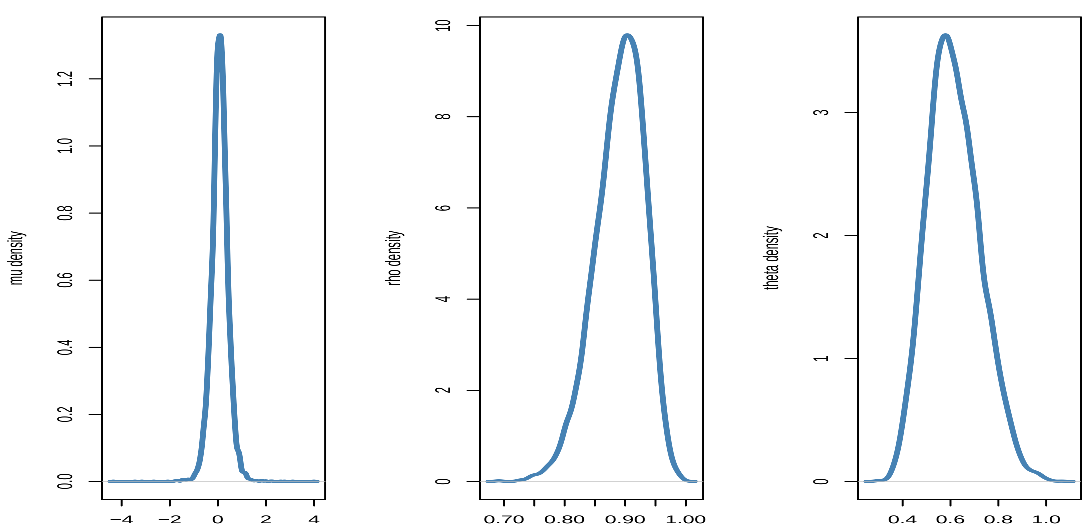

Figure 13. Posterior distibution on SV model parameters for manurials. Estimator μ on the left, ρ on the right, θ in the middle.

Source: author

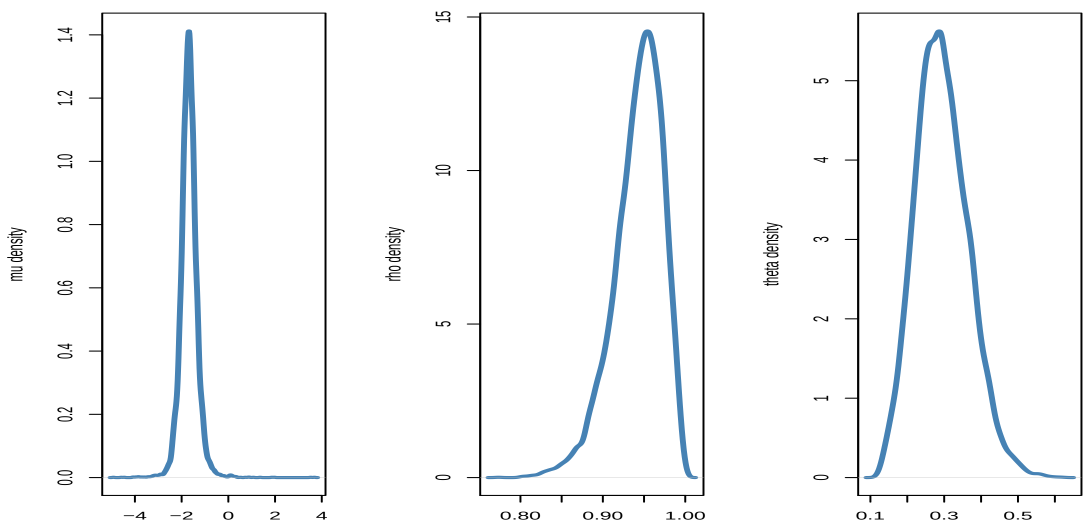

Figure 14. Posterior distibution on SV model parameters for machines. Estimator μ on the left, ρ on the right, θ in the middle.

Source: author