EMU's External Imbalances in a Two-Country Overlapping Generations Model

Abstract

The pronounced increase in external imbalances in the European Economic and Monetary Union (EMU) during the years running up to 2008 is traditionally explained by financial integration through the common currency. This paper examines in a one-good, two-country overlapping generations' model with production, capital accumulation and public debt the effects of financial integration on the net foreign asset positions of initially low-interest versus high-interest rate EMU countries. We find that a higher savings rate and a lower capital production share in the former ("North") than in the latter ("South") were transformed into the observed external imbalances when interest rates converged.

Keywords: External Imbalances, European Economic and Monetary Union, Overlapping Generations, Two-Country Model
JEL Code: F34, F36

Introduction and Motivation

The present sovereign debt crisis in the European Economic and Monetary Union (EMU) is partly attributable to the pronounced increase in external imbalances across northern (including center) and southern euro zone countries during the years running up to 2008 (Lane and Milesi-Ferretti 2011). The huge external deficits in the southern euro zone during the pre-crisis period are "traditionally explained" (Chen et al. 2012, p. 6) by financial integration after the inception of the common currency (e.g. Lane 2006, Coeurdacier and Martin 2009, Lane and Milessi-Ferretti 2008). The immediate convergence of nominal interest rates and the increase in net capital flows among the euro-area countries as manifestations of financial integration are caused by the elimination of country-specific currency risks and the reduction of transaction costs due to the harmonization of financial sector policies and goods market integration (Kamlemli-Ozcan et al. 2010). As a consequence and in line with basic neoclassic economic reasoning, financial capital flowed downhill from richer core to poorer periphery (converging) countries financing external deficits of the latter (Schmitz and von Hagen 2010). Since from a partial equilibrium perspective this explanation of periphery's external deficits seems to be intuitively obvious, it is rarely subjected to rigorous intertemporal general equilibrium analysis.[1] It is thus the objective of this paper to examine thoroughly in a simple one-good, two-country overlapping generations' model with production, capital accumulation and public debt the effects of financial integration on the net foreign asset positions (external balances) of initially low-interest rate ("northern") and high-interest rate ("southern") European countries before the launch of the common currency.

As is well-known, after the inception of the euro in 1999, northern and center euro countries (Austria, Belgium, Finland, Germany, Netherlands, France), in particular Germany, started to run current account surpluses while the southern and western periphery (= PIIGS: Portugal, Ireland, Italy, Greece and Spain) accumulated huge external deficits accompanied by a dramatic loss of international competitiveness due to striking increases in their wages and prices compared to the northern countries. Moreover, there was a significant divergence in the dynamics of private debt between northern and southern countries (Pisany-Ferry 2012, Figure 4): Up to the outburst of the global financial crisis southern debt boomed, mainly in order to finance housing investment while in the aftermath of the crisis government debt was substituted for private debt.

While the contribution of financial integration to the emergence of intra-EMU external imbalances is empirically largely undisputed[2], it remains an open theoretical question how divergent net foreign asset positions (external imbalances) can be related to financial integration in an intertemporal general equilibrium model of a heterogeneous currency area. Among the few who address this question are Fagan and Gaspar (2008) who use a two-good, two-country overlapping generations pure exchange model without public debt à la Yaari (1965) and Blanchard (1985) to compare the pre-euro financial autarky steady state to euro-related financial integration between southern and northern euro countries. Fagan and Gaspar

[1] Exceptions are Blanchard and Giavazzi (2002), Giavazzi and Spaventa (2010) and Campa and Gavilan (2011). However, Blanchard and Giavazzi (2002) focus in an infinitely-lived agent (ILA) context on declining savings in response to higher exogenous GDP growth expectations to explain larger current account deficits. Giavazzi and Spaventa (2010) consider also periphery's housing investment as driver of periphery's current account deficits but neglect the dynamics of capital accumulation in their two-period general equilibrium model. Campa and Gavilan (2011) use a small-open economy two-good ILA model to explain current account deficits mainly by goods and services trade imbalances. However, as shown by Chen et al. (2012), intra-EMU trade of goods and services is rather modest and the small-open economy assumption is not appropriate to the size (e.g. region-specific GDP as percent of euro area GDP) of either the core or the periphery.

[2] Chen et al. (2012, pp. 6-7) count among "traditional explanations for the rise of euro area imbalances" besides "financial integration" "expected growth", "over-optimism" and "excessive real appreciation". In addition, they point out that "new stylized facts" concerning the euro area imbalances and the rest of the world are also important to understand fully intra-EMU external imbalances.

(2008) find that the evolution of intra-EMU external imbalances can be traced back to North-South differences in time preference. The authors justify the neglect of production and capital accumulation in their model by emphasizing “the stylized fact that the creation of the monetary union has not been associated with changes in growth differentials” suggesting “that flows of capital and resulting changes in capital-output ratios have not been, to date, a major part of the story as far as the euro area is concerned.” (Fagan and Gaspar 2008, p. 13)

While this suggestion is true with respect to non-housing investment it is – as Fagan and Gaspar (2008) by themselves note – not true with respect to housing and residential investment. Since we know by the benefit of hindsight that housing investment in Ireland and Spain contributed significantly to the evolution of intra-EMU external imbalances the assumption of exogenously given and time-stationary capital endowment by Fagan and Gaspar (2008) remains questionable.[3] Even if the endowment-economy specification is empirically warranted with respect to overall investment and GDP growth it is of itself interesting to investigate whether the intra-EMU external imbalances can also be attributed to changes in capital (including housing)-output ratios through financial integration. In particular, it is interesting to know whether the core-periphery divergence in external imbalances is traceable to intra-EMU differences in fundamentals additionally to time preference. This is exactly the main research question which is addressed in the present paper.

In order to be able to deal with the effects of the housing capital-output dynamics on the external balances, a one-good, two-country overlapping generations’ (OLG) model with production, capital accumulation and public debt à la Diamond (1965) and Buiter (1981) is developed in this paper. The model specification features roughly main stylized macroeconomic facts of northern and southern euro zone countries before the advent and after the introduction of the common currency. Among them looms prominently the fact that the public debt-to-GDP ratios of several countries in southern euro area did not rise but declined. (Lane 2012, p. 51) Acknowledging moreover the fact that the debt to GDP ratios of northern euro zone countries increased only slightly, the modeling exercise assumes for simplicity that both northern and southern debt to GDP ratios stay constant over time. Introducing public debt into Buiter’s (1981) two-country OLG model is equivalent to extending Farmer’s (2006) closed-economy OLG model with government debt towards two countries connected through internationally mobile government bonds and private capital. As regards the notion of the real exchange rate we follow Ljungqvist (1988) and Lin (1994) in defining the real exchange rate as the ratio of southern to northern (real) wage rates.

[3] Giavazzi and Spaventa (2010, p. 7) argue similarly.

The most obvious manifestation of the creation of the euro zone was the convergence of high nominal (short- and long-term) interest rates in southern Europe towards the relatively low northern (German) rates. The main research question addressed by the following model analysis is whether and how the international macroeconomic divergence between northern and southern EMU countries can be attributed to the convergence of former different interest rates (financial autarky). We suggest that differences in economic fundamentals like saving rates and capital production shares existing between northern and southern European countries before the launch of the euro were transformed into the empirical observed macroeconomic divergences during the course of interest rate convergence.

The paper is organized as follows. In the next section, main stylized macroeconomic facts, existing both before financial integration and during the euro zone integration up to the outburst of the financial crisis in 2008 are assembled in order to motivate the model set-up. In the following section, the first-order conditions (FOC) for constrained intertemporal utility and temporal profit maxima and the market-clearing conditions are separately specified for pre-euro financial autarky and financial integration after euro inception. After deriving the respective equilibrium dynamics, the existence and dynamic stability of steady-state solutions before and after financial integration are investigated. With these results at hand, it is then shown how the southern net foreign debtor position through financial integration can be traced back to the lower savings rate and the larger capital production share in South. Concluding remarks in the final section summarize the modeling approach and its key results and provide suggestions for the set-up of a more realistic model.

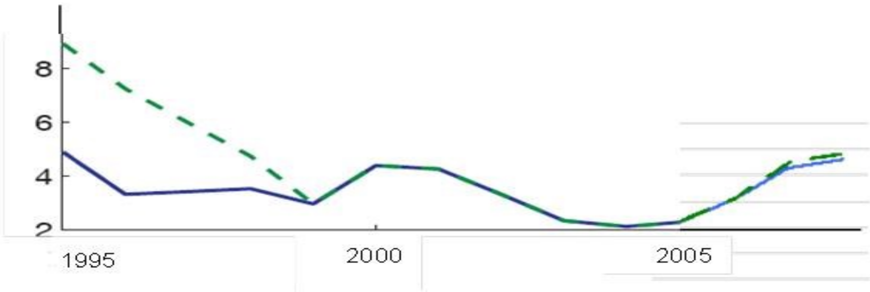
Stylized Macroeconomic Facts: Financial Autarky versus EMU's Financial Integration

In order to guide the design of the one-good, two-country OLG model, some stylized facts with respect to the macroeconomic performance of the EMU members before the launch of the euro in 1999 and up to 2008 are gathered in this section. Following Fagan and Gaspar (2008, p. 9), the euro zone countries are separated into two groups based on the criteria of relative nominal (short- and long-term) and real interest rates in the late 1990s, i.e. before the euro launch. The first group, usually denoted as the "core" countries, comprises low interest rate countries: Austria, Belgium, France, Germany and the Netherlands.[4] The second group

[4] Nowadays Finland is included within core countries. Fagan and Gaspar (2008) exclude Finland from core countries since in the 1990s the Finnish economy was distorted by special factors after the collapse of the Soviet Union. We follow Fagan and Gaspar (2008).

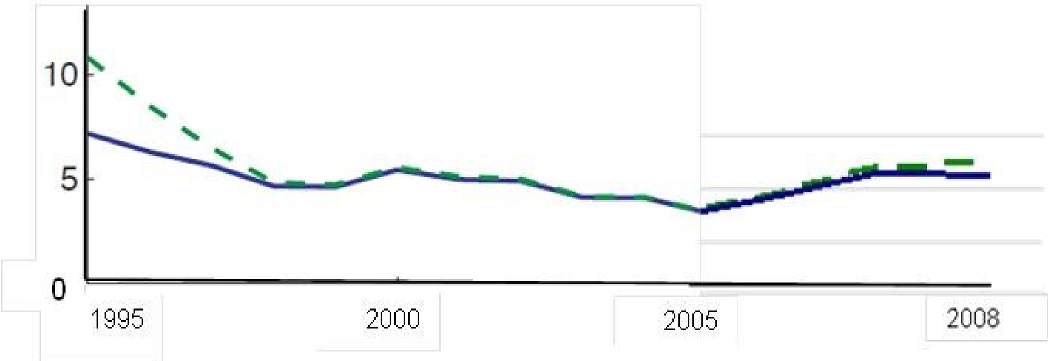
denoted as "periphery" or converging countries, consists of countries which had relatively high interest rates before the introduction of the euro (see figures 1-3).

Fig. 1 Nominal short-term interest rates 1995-2008



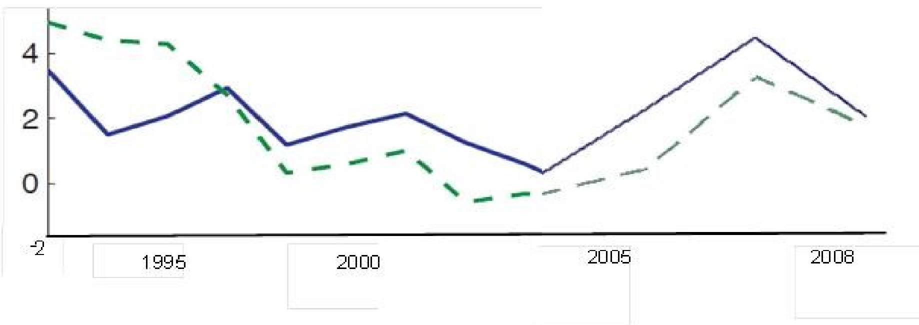
Legend: _ _ periphery, ___ core. Source: Fagan and Gaspar (2008, p. 34) and own calculations based on AMECO.

Fig. 2 Nominal long-term interest rates 1995-2008



Legend: _ _ _ periphery, ___ core. Source: Fagan and Gaspar (2008, p. 34) and own calculations based on AMECO.

Fig. 3 Real short-term interest rates 1995-2008



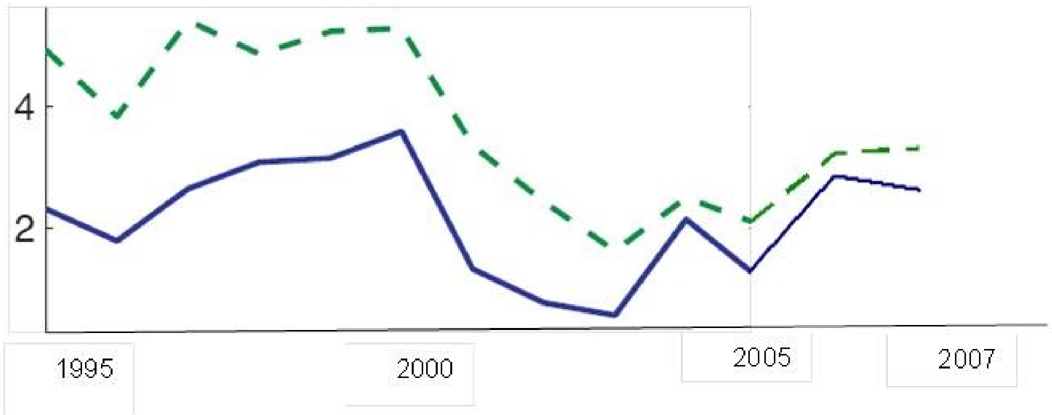
Legend: _ _ periphery, ___ core. Source: Fagan and Gaspar (2008, p. 34) and own calculations based on AMECO.

Figures 1-3 reveal the first macroeconomic fact regarding financial integration since euro inception in 1999:

Fact 1: In contrast to the pre-euro area situation (before 1999), there is a sizeable convergence of interest rates between euro core and periphery after the inception of the common currency (Fagan and Gaspar 2008, p. 10).

What is true with respect to interest rate convergence is not true regarding the differences in real GDP growth rates.

Fig. 4 GDP growth rates in euro core and periphery 1995-2007



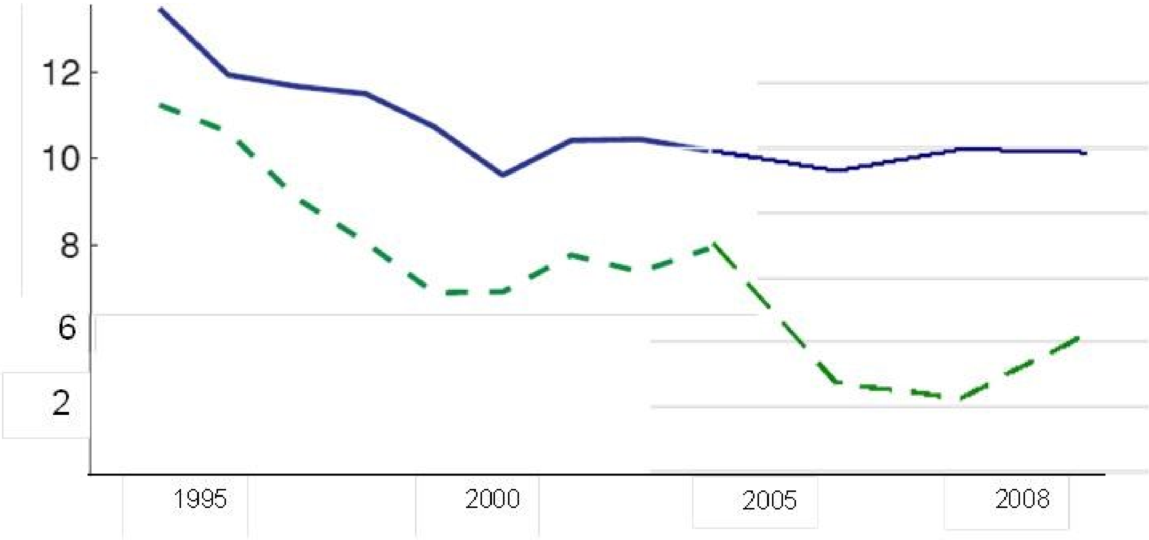
Legend: _ _ periphery, __ core. Source: Fagan and Gaspar (2008, p. 34) and IWF Database.

As figure 4 above demonstrates, the differences in the GDP growth do not change visibly through financial integration. In the words of Fagan and Gaspar (2008, p. 10): "… the process of interest convergence has had little impact on output growth differentials."

Figure 4 and the more precise econometric analysis of Fagan and Gaspar (2008) lead us to the next fact:

Fact 2: The interest rate convergence between euro core and periphery did not have significant impacts on GDP growth differentials.

Fig. 5 Personal savings ratios in euro core and periphery

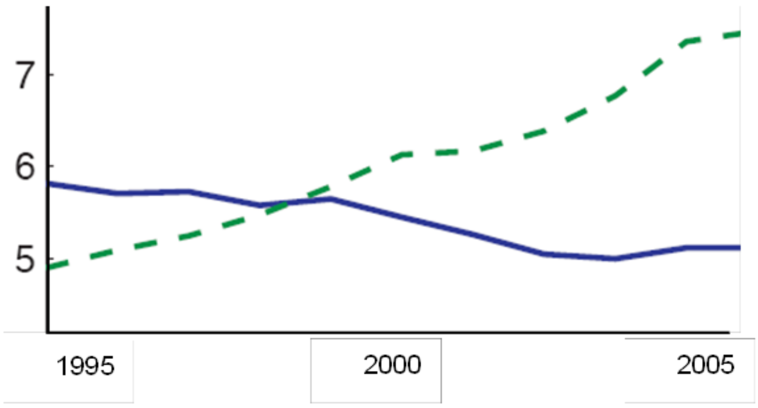


Legend: _ _ periphery, ___ core. Source: Fagan and Gaspar (2008, p. 34) and own calculations based on AMECO.

While there is no sizeable impact of financial integration of euro member countries on GDP growth differentials, the development of household expenditures differs markedly across the two groups of countries. As figure 5 above shows, the personal savings ratio is substantially lower in the periphery. In the 1990s and after 2005 it fell more rapidly in the periphery than in the core.

Moreover, housing investment (as percent of GDP) rose significantly in the periphery while it declined in the euro core, as can be seen in figure 6 below.

Fig. 6 Housing investment (as percent of GDP) in euro periphery and core 1995-2005



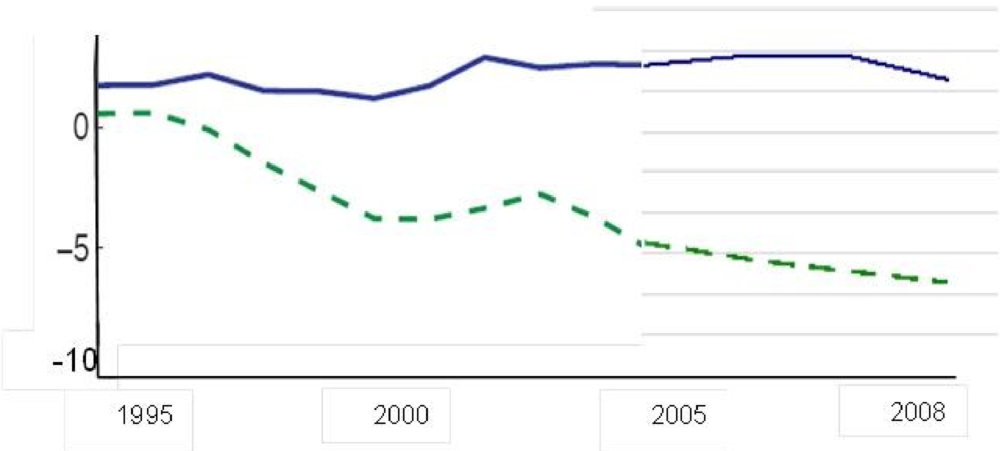
Legend: _ _ periphery, ___ core. Source: Fagan and Gaspar (2008, p. 34)

The message from figures 5-6 can be summarized as follows:

Fact 3: Starting from a significantly lower personal savings ratio in euro periphery relative to the core, housing investment expenditures in the periphery experienced a boom, while housing investment declined in the core countries. (Fagan and Gaspar 2008, p. 34)

In view of the sharp increase in private domestic expenditures in the periphery and the muted response of output, macroeconomic equilibrium had to be established through changes in the external balances of these countries. As figure 7 shows, this resulted in significant current account deficits in the periphery.[5]

Fig. 7 Current account balances (as percent of GDP) in euro periphery and core 1995-2008

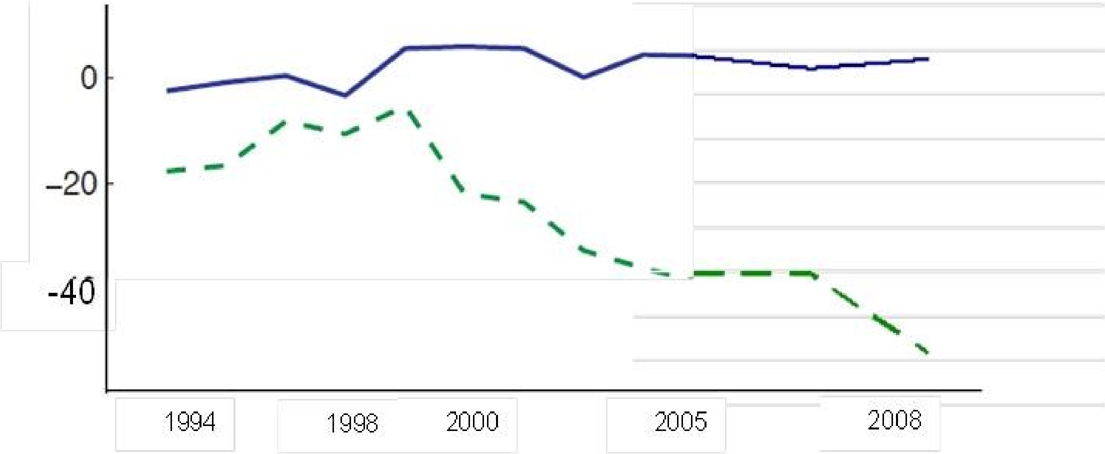


Legend: _ _ periphery, __ core. Source: Fagan and Gaspar (2008, p. 35) and IMF World Economic Outlook Database 2008.

Not surprisingly, periphery's current account deficits led to the accumulation of a significant net foreign debtor position as shown in figure 8 below.

Fig. 8 Net foreign assets (as percent of GDP) in euro core and periphery 1994-2008

[5] Chen et al. (2012) point out that periphery's current account deficits after euro launch and before 2008 while mainly financed by euro core are due to rising imports from rest of the world and not from euro core. Moreover, core's (in particular Germany's) exports went mainly to emerging countries like China. As a consequence of these non-traditional explanations of intra-EMU external imbalances the model exercise below assumes that euro core and periphery produce the same good (albeit using different technologies) and there is no trade in goods and services among core and periphery.



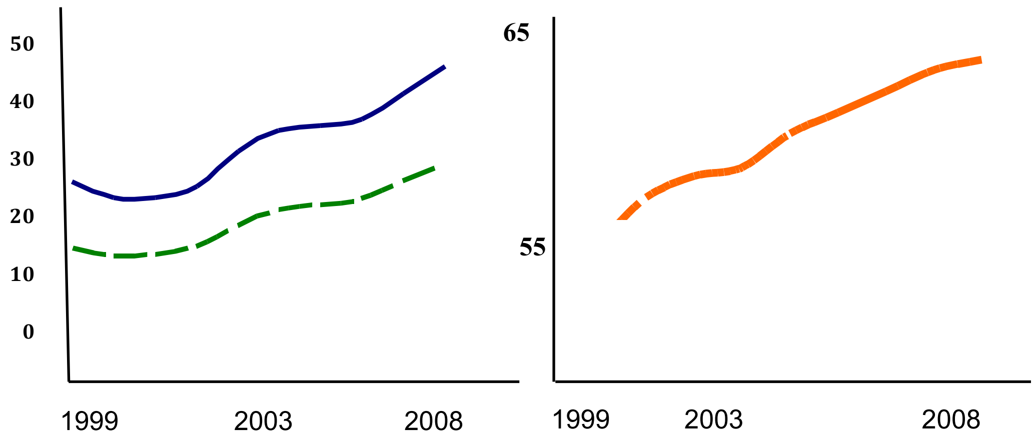
Legend: _ _ periphery, __ core. Source: Fagan and Gaspar (2008, p. 35) and IMF Principal Global Indicators.

Figures 7 and 8 thus suggest fact 4.

Fact 4: While euro core countries experienced a broadly stable position concerning both current account balance and net foreign asset position (creditor position) over the period 1994 to 2008, the periphery current account balance deteriorated sharply and foreign debt accumulated significantly after 1999.

The final fact concerns the shrinking differences in hourly labor compensation[6] between northern and southern euro zone member countries.

Fig. 9 Hourly labor compensation costs in manufacturing in U.S. dollars (left diagram) and ratio of southern to northern labor compensation costs in % (right diagram) 1999-2008



Legend: _ _ periphery, __ core. Source: U.S. Bureau of Labor Statistics, 2012.

Fact 5: The southern hourly labor compensation in manufacturing rose quicker than northern hourly compensation or more precisely: the ratio of southern to northern hourly labor compensation increased from 55% in 1998 towards 63.5% in 2008.

[6] Compensation costs include direct pay, social insurance expenditures and labor-related taxes.

Basic Model

Consider an infinite-horizon economy consisting of two countries, named South, representing southern euro zone countries, and North (indexed by *), representing northern and center euro zone countries. In each country one commodity, representing the aggregate of thousands of goods and services is produced. This can be used for the purpose of consumption as well as for investment. Perfectly competitive firms in South (North) employ in every period $t = 1,2,...$ labor services N_t (N_t^*) and capital services K_t (K_t^*) using the Cobb-Douglas (CD) production function $M(a_t N_t)^{1-\alpha}(K_t)^{\alpha}$ $(M^*(a_t N_t^*)^{1-\alpha^*}(K_t^*)^{\alpha^*})$ to produce southern (northern) aggregate output Y_t (Y_t^*) where $M > 0$ $(M^* > 0)$ denotes total factor productivity in South (North), a_t is the common labor productivity and α (α^*) with $0 < \alpha < 1$ $(0 < \alpha^* < 1)$ is the capital production share in South (North).

One-period profit maximization by firms in South (North) implies the following FOCs:

$$w_t = (1-\alpha) M a_t \left(\frac{K_t}{a_t N_t} \right)^{\alpha}, \; (w_t^* = (1-\alpha^*) M^* a_t \left(\frac{K_t^*}{a_t N_t^*} \right)^{\alpha^*}) \tag{1}$$

$$q_t = \alpha M \left(\frac{K_t}{a_t N_t} \right)^{\alpha-1}, \; (q_t^* = \alpha^* M^* \left(\frac{K_t^*}{a_t N_t^*} \right)^{\alpha^*-1}) \tag{2}$$

whereby w_t (w_t^*) denotes the real wage rate in South (North) and q_t (q_t^*) denotes real unit capital user costs in South (North).[7]

As usual in a Diamond (1965) type OLG framework, two generations of homogeneous individuals overlap in each period t. At date t, a new generation of size L_t enters the economy in South, and in North the new generation is of size L_t^*. For simplicity we assume that $L_t = L_t^*$ for all $t = 1,2,...$ and that the growth factors of northern and southern populations are identical and equal to G^L. In view of the empirically rather similar GDP growth rates in southern and northern euro zone countries (see figure 4 above) we assume moreover that the respective growth factors of labor productivities G^a and G^{a*} are equal in South and North, which also implies that the natural growth factor $G^n = G^a G^L$ is the same in both countries.

[7] In view of stylized facts presented in previous section a purely real model is clearly unable to explain all relevant empirical facts regarding EMU imbalances.

Each generation lives for two periods, working during the first when young, and retiring in the second when old. The choice variables of each generation, when young, are denoted by superscript 1, and, when old, they are denoted by superscript 2. Each member of the generation entering the economy in period t supplies one unit of labor in-elastically to firms since households attribute no value to leisure.

In order to describe the optimization problems of households more specifically the institutional framework regarding international transactions across both countries is now addressed. To mimic the introduction of the common currency in 1999 within the present highly abstract intertemporal general equilibrium model we follow Gourinchas and Jeanne (2006) as well as Fagan and Gaspar (2008), and assume that before 1999 South and North were financially autarkic while after the launch of the common currency South and North became financially integrated. Not surprisingly, financial integration impacts on the choice set and constraints of younger households as well as on market clearing conditions. In order to work out the consequences of financial integration as clearly as possible, the optimization problems of (younger) households and firms as well as the market clearing conditions are now described separately for the two cases of financial autarky and financial integration through EMU.

Financial Autarky

In order to facilitate the modeling of the pre-euro situation as financial autarky, we first recall that large real interest rate differences existed between the euro zone core (North) and the euro zone periphery (South) before the creation of the currency union. As figure 3 shows, southern real interest rates were sizeable larger than the corresponding northern rates. Second, in contrast to the later financial integration of the euro zone, in the 1990s the European South did not run large current account deficits (as percent of GDP). On the contrary, in the mid-1990s Greece for example achieved a current account surplus, and at the end of the decade the Greek net foreign asset position was only moderately negative (Krugman 2012). Hence, when modeling the period before the euro start it is not unrealistic to assume that both the current account and the net foreign asset position of South and North were zero. Third, in contrast to the current post-crisis situation where huge differences in government debt to GDP ratios exist between euro-zone periphery and core, in the late 1990s the un-weighted average debt to GDP ratio of the euro periphery was not that different from the corresponding euro zone core value. Moreover, the North-South debt to GDP differences did not widen until the outburst of the global financial crisis. Since the objective of the modeling is to explain the intra euro zone development before the financial crisis it is appropriate to assume that the government debt to GDP ratios in South and North remain constant over time. Additionally, as figure 5 above shows the personal savings ratio in South was persistently lower than in euro zone core countries (North). Finally, in view of the differential development of labor compensations costs across euro-zone core and periphery, it is natural to assume[8] corresponding differences in southern and northern production technologies.

Against this empirical background of stylized facts the intertemporal utility maximization problem in South (North) reads as follows:

$$max \rightarrow ln\, x_t^1 + \beta\, ln\, x_{t+1}^2 \quad (max \rightarrow ln\, x_t^{*,1} + \beta^*\, ln\, x_{t+1}^{*,2}\ )$$

s. t.:

$$\text{(i)}\ x_t^1 + s_t = w_t\left(1-\tau_t\right),\ s_t = \frac{K_{t+1}}{L_t} + \frac{B_{t+1}^d}{L_t},\ (x_t^{*,1} + s_t^{\ *} = w_t^*\left(1-\tau_t^{\ *}\right),\ s_t^{\ *} = \frac{K_{t+1}^*}{L_t} + \frac{B_{t+1}^{*,d}}{L_t}) \tag{3}$$

$$\text{(ii)}\ x_{t+1}^2 = \left(1+i_{t+1}\right)s_t,\ \ (x_{t+1}^{*,2} = \left(1+i_{t+1}^*\right)s_t^{\ *}) \tag{4}$$

where $0<\beta\leq 1$ $(0<\beta^*\leq 1)$ denotes the time discount factor of southern (northern) younger generation, x_t^1 ($x_t^{*,1}$) is the consumption per capita of the commodity produced in South (North), s_t (s_t^*) is southern (northern) per-capita savings, τ_t (τ_t^*) denotes the southern (northern) lump-sum tax rate[9], x_{t+1}^2 ($x_{t+1}^{*,2}$) is old-age consumption per capita of the commodity produced in South (North), i_{t+1} (i_{t+1}^*) denotes the real interest rate on southern (northern) government bonds and B_{t+1}^d ($B_{t+1}^{*,d}$) stands for the aggregate southern (northern) government bonds the southern younger household wants to hold at the beginning of its retirement period. Constraint (i) depicts the working period budget constraint while constraint (ii) represents the retirement period budget constraint. The southern (and similarly the northern) government taxes labor income and uses the proceeds from additional borrowing to finance the interest

[8] This is tantamount to assume that the labor compensation cost differentials are not solely due to differences in output prices and national fiscal instruments.

[9] The assumption of flat wage taxes clearly clashes with European tax code reality. However, since this paper does not focus on taxation for the sake of analytical simplicity a constant wage tax rate is assumed.

costs on existing government debt and government expenditures. The budget constraint of southern (northern) government reads as follows:

$$B_{t+1} - B_t + \tau_t w_t L_t = i_t B_t + \Gamma_t \,,\ (B^*_{t+1} - B^*_t + \tau^*_t w^*_t L_t = i^*_t B^*_t + \Gamma^*_t) \tag{5}$$

where Γ_t (Γ^*_t) denote southern (northern) real government expenditures and B_t (B^*_t) is the level of real southern (northern) government debt at the beginning of period t. In line with Diamond (1965) we assume that government expenditures are unproductive. The assumption of unproductive government expenditures accords rather well with southern euro zone reality, but it is more at odds with northern reality.

In addition to the restrictions imposed by agent and firm optimization and by the above government budget constraint, markets for labor and for financial assets have to clear in all periods (the market for the output of production is cleared by means of Walras' Law).

$$N_t = L_t \quad (N^*_t = L_t) \tag{6}$$

$$L_t s_t = K_{t+1} + B_{t+1} \ (L_t s^*_t = K^*_{t+1} + B^*_{t+1}) \tag{7}$$

Since the market for financial assets is competitive, transaction and adjustment costs do not occur, no risk (aversion) prevails, the following no-arbitrage condition (= national Fisher equation) holds:

$$1 + i_{t+1} = q_{t+1} + 1 - \delta \,,\ (1 + i^*_{t+1} = q^*_{t+1} + 1 - \delta) \tag{8}$$

whereby $0 < \delta \leq 1$ depicts the fixed depreciation rate of private capital (period by period) in South and equally in North.

Moreover, in order to model the fact of time-stationarity of both southern and northern public debt to GDP ratios between 1999 and 2008 we transform total outstanding government debt in southern (northern) government's budget constraint into debt to GDP ratios. This is achieved by dividing both sides of (5) by Y_t (Y^*_t) and by defining the debt to GDP ratios as $b_t = B_t / Y_t$ ($b^*_t = B^*_t / Y^*_t$), and we obtain for South (North):

$$G^Y_t b_{t+1} = (1 + i_t) b_t + \gamma_t - \tau_t (1 - \alpha), \text{ with } G^Y_t \equiv \frac{Y_{t+1}}{Y_t},\ \gamma_t \equiv \frac{\Gamma_t}{Y_t},\ \frac{w_t L_t}{Y_t} = 1 - \alpha \tag{9}$$

$$(G_t^{Y,*} b_{t+1}^* = \left(1+i_t^*\right) b_t^* + \gamma_t^* - \tau_t^* \left(1-\alpha^*\right), \text{ with } G_t^{Y,*} \equiv \frac{Y_{t+1}^*}{Y_t^*}, \gamma_t^* \equiv \frac{\Gamma_t^*}{Y_t^*}, \frac{w_t^* L_t}{Y_t^*} = 1-\alpha^*)$$

Dividing the market clearing condition (7) on both sides by Y_t (Y_t^*) and using the definition of the capital output ratio $v_t \equiv K_t / Y_t$ ($v_t^* \equiv K_t^* / Y_t^*$), (7) can be rewritten as follows:

$$G_t^Y b_{t+1} + G_t^Y v_{t+1} = \frac{L_t s_t^1}{Y_t} = \frac{\beta}{(1+\beta)} \frac{(1-\tau_t) w_t L_t}{Y_t} = \frac{\beta}{(1+\beta)} (1-\tau_t)(1-\alpha) = \sigma(1-\alpha)(1-\tau_t),$$
$$\sigma \equiv \frac{\beta}{(1+\beta)}. \tag{10}$$

$$(G_t^{Y,*} b_{t+1}^* + G_t^{Y,*} v_{t+1}^* = \frac{L_t s_t^{1,*}}{Y_t^*} = \frac{\beta^*}{\left(1+\beta^*\right)} \frac{\left(1-\tau_t^*\right) w_t^* L_t}{Y_t^*} = \frac{\beta^*}{\left(1+\beta^*\right)} \left(1-\tau_t^*\right)\left(1-\alpha^*\right)$$
$$= \sigma^* \left(1-\alpha^*\right)\left(1-\tau_t^*\right), \sigma^* \equiv \beta^* / (1+\beta^*))$$

In view of the C-D production function and acknowledging $G_t^Y = (K_{t+1})^\alpha (a_{t+1} L_{t+1})^{1-\alpha} / (K_t)^\alpha (a_t L_t)^{1-\alpha} = (a_{t+1} L_{t+1}) / (a_t L_t) (K_{t+1} / a_{t+1} L_{t+1})^\alpha / (K_t / a_t L_t)^\alpha$, it turns out that $G_t^Y = G^n \left(v_{t+1} / v_t\right)^{\alpha/(1-\alpha)}$ ($G_t^{Y,*} = G^n \left(v_{t+1}^* / v_t^*\right)^{\alpha/(1-\alpha)}$).

Acknowledging the empirical fact that the pre-crisis public debt to GDP ratios in northern and southern euro zone countries remained roughly constant over time we assume a time-stationary public debt to GDP ratio:

$$\frac{B_t}{Y_t} = \frac{B_{t+1}}{Y_{t+1}} = b, \forall t,\ b > 0 \ \left(\frac{B_t^*}{Y_t^*} = \frac{B_{t+1}^*}{Y_{t+1}^*} = b^*, \forall t,\ b^* > 0\right). \tag{11}$$

Moreover, we assume time-stationary government expenditure shares:

$$\gamma_t = \gamma_{t+1} = \gamma, \forall t, 0 < \gamma < 1 \ (\gamma_t^* = \gamma_{t+1}^* = \gamma^*, \forall t, 0 < \gamma^* < 1). \tag{12}$$

The government budget constraint (9) together with (11) and (12) yields $1-\tau_t$ ($1-\tau_t^*$) as follows:

$$1-\tau_t = \frac{1-\alpha-\gamma}{1-\alpha} + \frac{b}{1-\alpha}\left[G_t^Y - (1+i_t)\right] \ (1-\tau_t^* = \frac{1-\alpha^*-\gamma^*}{1-\alpha^*} + \frac{b^*}{1-\alpha^*}\left[G_t^{Y,*} - \left(1+i_t^*\right)\right]). \tag{13}$$

Using the Cobb-Douglas production function it is easily seen that $K_t/Y_t \equiv v_t = (1/M)[K_t/(a_tN_t)]^{1-\alpha}$ $(K_t^*/Y_t^* \equiv v_t^* = (1/M^*)[K_t^*/(a_tN_t^*)]^{1-\alpha^*})$. Thus, the FOC for profit maximization (2) can be equivalently written as follows:

$$\frac{\alpha}{v_t} = q_t = i_t + \delta \;(\frac{\alpha^*}{v_t^*} = q_t^* = i_t^* + \delta). \tag{14}$$

In order to simplify the algebra, we assume $\delta = 1$. Then, acknowledging (14) in (13) and taking into account $G_t^Y = G^n\left(v_{t+1}/v_t\right)^{\frac{\alpha}{1-\alpha}}$ $(G_t^{Y,*} = G^n\left(v_{t+1}^*/v_t^*\right)^{\frac{\alpha^*}{1-\alpha^*}})$ yields:

$$1-\tau_t = \frac{1-\alpha-\gamma}{1-\alpha} + \frac{b}{1-\alpha}\left[G^n\left(v_{t+1}/v_t\right)^{\frac{\alpha}{1-\alpha}} - \frac{\alpha}{v_t}\right] = \frac{1-\alpha-\gamma}{1-\alpha} + \frac{bG^n\left(v_{t+1}/v_t\right)^{\frac{\alpha}{1-\alpha}}}{1-\alpha} - \frac{\alpha b}{\left(1-\alpha\right)v_t}. \tag{15}$$

$$(1-\tau_t^* = \frac{1-\alpha^*-\gamma^*}{1-\alpha^*} + \frac{b^*}{1-\alpha^*}\left[G^n\left(v_{t+1}^*/v_t^*\right)^{\frac{\alpha^*}{1-\alpha^*}} - \frac{\alpha^*}{v_t^*}\right] = \frac{1-\alpha^*-\gamma^*}{1-\alpha^*} + \frac{b^*G^n\left(v_{t+1}^*/v_t^*\right)^{\frac{\alpha^*}{1-\alpha^*}}}{1-\alpha^*} - \frac{\alpha^*b^*}{\left(1-\alpha^*\right)v_t^*})$$

The intertemporal equilibrium dynamics of the capital-output ratio in South (North) is obtained by inserting (15) into (10):

$$G^n\left(v_{t+1}/v_t\right)^{\frac{\alpha}{1-\alpha}}\left(v_{t+1}+b\right) = \sigma\left[1-\alpha-\gamma+bG^n\left(v_{t+1}/v_t\right)^{\frac{\alpha}{1-\alpha}} - \frac{\alpha b}{v_t}\right],$$
$$(G^n\left(v_{t+1}^*/v_t^*\right)^{\frac{\alpha^*}{1-\alpha^*}}\left(v_{t+1}^*+b^*\right) = \sigma^*[1-\alpha^*-\gamma^*+b^*G^n\left(v_{t+1}^*/v_t^*\right)^{\frac{\alpha^*}{1-\alpha^*}} - \frac{\alpha^*b^*}{v_t^*}])$$

or:

$$\left(v_{t+1}\right)^{\frac{1}{1-\alpha}} + b\left[1-\sigma\right]\left(v_{t+1}\right)^{\frac{\alpha}{1-\alpha}} = \frac{\sigma}{G^n}\left[1-\alpha-\gamma-\frac{\alpha b}{v_t}\right]\left(v_t\right)^{\frac{\alpha}{1-\alpha}}.$$
$$(\left(v_{t+1}^*\right)^{\frac{1}{1-\alpha^*}} + b^*\left[1-\sigma^*\right]\left(v_{t+1}^*\right)^{\frac{\alpha^*}{1-\alpha^*}} = \frac{\sigma^*}{G^n}\left[1-\alpha^*-\gamma^*-\frac{\alpha^*b^*}{v_t^*}\right]\left(v_t^*\right)^{\frac{\alpha^*}{1-\alpha^*}}) \tag{16}$$

As usual, a steady-state intertemporal equilibrium is defined as a fixed point of the difference equation in (16): $v_{t+1} = v_t = v$ $(v_{t+1}^* = v_t^* = v^*)$. Evaluating (16) at a steady state yields:

$$v^2+\frac{G^n b-\beta(1-\alpha-\gamma)}{G^n(1+\beta)}v+\frac{\alpha\beta b}{G^n(1+\beta)}=0 \; ((v^*)^2+\frac{G^n b^*-\beta^*(1-\alpha^*-\gamma^*)}{G^n(1+\beta^*)}v^*+\frac{\alpha^*\beta^* b^*}{G^n(1+\beta^*)}=0). \quad (17)$$

Proposition 1 (Existence of steady solutions in South[10])

Suppose that $0<b\le\bar{b}<\beta(1-\alpha-\gamma)/G^n$ while $\bar{b}$ solves $\beta(1-\alpha-\gamma)-G^n\bar{b}$ $=2\sqrt{\alpha\beta(1+\beta)G^n\bar{b}}$. Then, there are exactly two strictly positive steady state solutions as follows:

$$\begin{aligned} v_1 &= \frac{\beta(1-\alpha-\gamma)-bG^n-\sqrt{\left[\beta(1-\alpha-\gamma)-bG^n\right]^2-4\alpha\beta(1+\beta)bG^n}}{2(1+\beta)G^n}, \\ v_2 &= \frac{\beta(1-\alpha-\gamma)-bG^n+\sqrt{\left[\beta(1-\alpha-\gamma)-bG^n\right]^2-4\alpha\beta(1+\beta)bG^n}}{2(1+\beta)G^n}. \end{aligned} \quad (18)$$

Proof. Since $b<\beta(1-a-\gamma)/G^n$, the expression in front of the square root is strictly larger than zero. Moreover, since $b\le\bar{b}$, the square root is non-negative. Hence, v_2 is real and strictly larger than zero. This is also true for v_1 since the term in front of the square root is larger than zero and the square root is certainly smaller than the term in front of the square root. Obviously, $v_2>v_1$. ■

Since there are two steady-state solutions (local) dynamic stability needs to be investigated which is done in proposition 2.

Proposition 2 (Dynamic stability of steady solutions in South[11])

Suppose that $0<b<\bar{b}$. Then, the steady-state solution v_1 in (18) is asymptotically unstable while the steady-state solution v_2 in (18) is asymptotically stable.

Proof. Calculating dv_{t+1}/dv_t from (16) and evaluating the result in any steady-state solution yields: $dv_{t+1}/dv_t=\{\alpha(\sigma/G^n)[1-\alpha-\gamma-(\alpha b)/v]+(1-\alpha)(\sigma/G^n)(\alpha b)/v\}/[v+b(1-\sigma)\alpha]$. From (16) we know: $v+b(1-\sigma)=(\sigma/G^n)[1-\alpha-\gamma-(\alpha b)/v]$. Hence, dv_{t+1}/dv_t simplifies towards: $dv_{t+1}/dv_t=\{\alpha[v+b(1-\sigma)]+(1-\alpha)(\sigma/G^n)(\alpha b)/v\}/[v+b(1-\sigma)\alpha]$. Obviously, $dv_{t+1}/dv_t > 1 \Leftrightarrow(\sigma/G^n)(\alpha b)/v^2>1$ and $dv_{t+1}/dv_t < 1 \Leftrightarrow(\sigma/G^n)(\alpha b)/v^2<1$. From

[10] To save on space the analysis of existence and dynamic stability of steady states is performed only for South. Northern existence and stability analysis runs analogously.

[11] See previous footnote.

proposition 1 we know that there are two solutions $v_1 < v_2$ which solve the steady-state equation $v + b(1-\sigma) = (\sigma/G^n)[1-\alpha-\gamma-(\alpha b)/v]$. Obviously, the slope of the left-hand side of this equation equals unity while the slope of the right-hand side equals $(\sigma/G^n)(\alpha b)/v^2$. $v_1 < v_2$ implies $(\sigma/G^n)(\alpha b)/v_1^{\ 2} > (\sigma/G^n)(\alpha b)/v_2^{\ 2}$. Hence, $dv_{t+1}/dv_t > 1$ occurs in the neighborhood of v_1 while $dv_{t+1}/dv_t < 1$ evaluated at v_2. Thus, the smaller steady-solution v_1 is asymptotically unstable while the larger steady-state solution is asymptotically stable. ■

Knowing that the larger steady state in (18) is asymptotically stable we are entitled to inquire whether the empirically observed pre-euro North-South differences with respect to the real interest rates (and real wage rates) can be attributed to North-South differences regarding fundamentals including private savings rates, governments' expenditure ratios and capital production shares. In order to proceed two steps are necessary: First, we have to inspect the larger steady-state solution in (18) to know how the fundamental parameters impact the steady-state value of the capital-output ratio under financial autarky. Second, we need information about the relative magnitudes of the savings rates and capital production shares in pre-euro North and South.

Doing the first step, it is helpful to re-write the larger steady-state solution in (18) by using the definition of the savings rates $\sigma \equiv \beta/(1+\beta)$ ($\sigma^* \equiv \beta^*/(1+\beta^*)$) as follows:

$$v_2 = (2G^n)^{-1}\{(1-\alpha-\gamma)\sigma-(1-\sigma)bG^n + \sqrt{[(1-\alpha-\gamma)\sigma-(1-\sigma)bG^n]^2 - 4\alpha bG^n\sigma}\}, \tag{19a}$$

$$v_2^* = (2G^n)^{-1}\{(1-\alpha^*-\gamma^*)\sigma^*-(1-\sigma^*)b^*G^n + \sqrt{[(1-\alpha^*-\gamma^*)\sigma^*-(1-\sigma^*)b^*G^n]^2 - 4\alpha^* b^* G^n\sigma^*}\}. \tag{19b}$$

Comparing the right-hand side of (19a) to that of (19b) we are led to the following proposition 3.

Proposition 3. Suppose for simplicity that $b^* = b$. Moreover, assume that $b < \bar{b}$. If $\alpha > \alpha^*$, $\gamma > \gamma^*$ and $\sigma < \sigma^*$, then $v_2 < v_2^*$ implying $i > i^*$ and $w < w^*$.

Proof. Since by assumption $\alpha > \alpha^*$, $\gamma > \gamma^*$ and $\sigma < \sigma^*$ while $b^* = b$ it is easy to see that the first term (in front of the square root) in curled brackets on the right-hand side of (19b) is larger than the corresponding term in (19a). Since $b < \bar{b}$ implies that both square roots are strictly larger than zero the square root in (19b) is larger than that in (19a). Hence, $v_2^* > v_2$. Equation (14) implies $i > i^*$. Equation (1) and the definition of v yield $w < w^*$. ■

The second step is to ensure that the assumptions of proposition 3 are empirically warranted with respect to northern and southern candidate countries for EMU in the late 1990s. The simplifying assumption $b^* = b$ is not warranted (Lane 2012, p. 51), however, the better fitting assumption $b > b^*$ would only enforce the claim in proposition 3 as can be numerically verified. $\alpha > \alpha^*$ is empirically warranted since the southern European countries were (are) less developed (lower GDP per capita) than the northern European countries and there are prominent empirical examples for the fact that the capital production share is higher in catching-up than in advanced countries (see Bai and Quian (2010) for the high Chinese capital production share of nearly 50% and Caselli and Feyrer (2007) for the much lower US capital production share of 30%). The opposite holds with respect to the government expenditure quota: less developed countries exhibit lesser expenditure quotas than highly developed countries. Since, however, large-economy Italy belongs to the southern bloc $\gamma^* = \gamma$ is rather close to reality which implies that proposition 3 remains relevant. Finally, in view of the empirical evidence provided by figure 5 above it is natural to assume that $\sigma < \sigma^*$, i.e. the savings rate of the southern European countries is less than that of northern countries.

Proposition 3 says that the relatively high capital production share and the low savings rate in South imply under financial autarky that the steady-state capital output ratio in South is lower than in North, and is associated with a higher real interest and a relatively low real wage rate. This claim is intuitively plausible. A low savings rate implies for a given capital output ratio low savings thus driving the capital output ratio down to ensure asset market clearing. The capital output ratio is also depressed by a relatively high capital income share since this implies a relatively low labor income share associated with low per capita savings. Due to decreasing marginal productivity of capital the lower capital output ratio is associated with a higher interest rate and a lower real wage rate.

Not surprisingly, under financial autarky the southern (northern) net foreign asset position $\Phi_{t+1} \equiv L_t s_t - a_{t+1} L_{t+1} M^{1/(1-\alpha)} v_{t+1}^{\alpha/(1-\alpha)} (v_{t+1} + b)$ ($\Phi^*_{t+1} \equiv L_t s_t^* - a_{t+1} L_{t+1} (M^*)^{1/(1-\alpha^*)} (v^*_{t+1})^{\alpha^*/(1-\alpha^*)} (v^*_{t+1} + b^*)$) is zero, i.e. no international borrowing and lending takes place in spite of the interest rate differential across countries. Obviously, the costs associated with shifting capital from low-yielding North to profitable South are prohibitively large. When modeling the advent of the common currency we assume that these international capital mobility costs are completely removed over night while the structural parameters of both economies remain as assumed in proposition 3. The consequences of completely removing international capital transaction costs are investigated in the next section.

International Equilibrium under Financial Integration

To mimic financial integration arising through the set-up of the EMU we assume in line with Buiter (1981) and Lin (1994) that both physical capital[12] and government bonds can be freely traded across both countries without incurring any transaction costs. In view of the higher interest rate in South, northern younger households will use their savings to invest in southern physical capital and buy the bonds emitted by the southern government until the southern real interest rate declines as much as there is no longer an incentive to shift northern savings towards the South.

Since the same composite commodity is produced in North and South, financial integration does not induce any commodity trade.[13] Thus, while younger households in South cannot choose between consumption of the domestic commodity and of the foreign commodity, they can choose between investing their savings in domestic or foreign real capital and domestic and foreign government bonds. The budget constraint (in real and per-capita terms) of the household living in South, when young is:

$$x_t^1 + s_t = w_t - \tau_t, \text{ with } s_t \equiv \frac{K_{t+1}^S}{L_t} + \frac{K_{t+1}^{*S}}{L_t} + \frac{B_{t+1}^S}{L_t} + \frac{B_{t+1}^{*S}}{L_t}. \tag{20}$$

Here K_{t+1}^S/L_t and K_{t+1}^{*S}/L_t denote the stocks of domestic and of foreign physical capital respectively, and B_{t+1}^S/L_t and B_{t+1}^{*S}/L_t denote the stocks of domestic and of foreign government bonds respectively, which the household in South plans to hold at the beginning of period $t+1$. Now, domestic and foreign real capital as well as domestic bonds and foreign bonds are perfect substitutes from the perspective of South's younger household.

When old the budget constraint of period-t young household in South is:

$$x_{t+1}^2 = q_{t+1}\left(\frac{K_{t+1}^S}{L_t}\right) + \left(1+i_{t+1}\right)\left(\frac{B_{t+1}^S}{L_t}\right) + q_{t+1}^*\left(\frac{K_{t+1}^{*S}}{L_t}\right) + \left(1+i_{t+1}^*\right)\left(\frac{B_{t+1}^{*S}}{L_t}\right), \tag{21}$$

whereby q_t^* denotes northern real unit capital cost and i_t^* denotes the real interest rate on northern government bonds.

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12 To mimic the facts presented in Figure 6 above we assume that in physical capital housing investment is included.

13 See the argumentation in footnote 5 above.

Southern households' preferences are represented by the same intertemporal log-linear utility function as under autarky. Each younger household in South maximizes its utility function $ln\, x_t^1 + \beta\, ln\, x_{t+1}^2$ subject to the budget constraints defined by equations (20) and (21).

Analogously, the corresponding budget constraints for the typical northern household read as follows:

$$x_t^{*,1} + s_t^* = w_t^* - \tau_t^*, \text{ with } s_t^* \equiv \frac{K_{t+1}^N}{L_t} + \frac{K_{t+1}^{*N}}{L_t} + \frac{B_{t+1}^N}{L_t} + \frac{B_{t+1}^{*N}}{L_t}, \tag{20*}$$

$$x_{t+1}^{*,2} = q_{t+1}^* \left(\frac{K_{t+1}^N}{L_t} \right) + \left(1 + i_{t+1}^*\right) \left(\frac{B_{t+1}^N}{L_t} \right) + q_{t+1} \left(\frac{K_{t+1}^{*N}}{L_t} \right) + \left(1 + i_{t+1}\right) \left(\frac{B_{t+1}^{*N}}{L_t} \right). \tag{21*}$$

Here, K_{t+1}^N / L_t (K_{t+1}^{*N} / L_t) and B_{t+1}^N / L_t (B_{t+1}^{*N} / L_t) denote the respective stocks of domestic (foreign) real capital and government bonds which northern households want to hold at the beginning of period $t+1$.

The typical northern household maximizes $ln\, x_t^{*,1} + \beta^* \, ln\, x_{t+1}^{*,2}$ subject to (20*) and (21*).

Since physical capital and government bonds of each country are perfectly substitutable, and since within the monetary union both assets can be assumed to be perfectly mobile across South and North, the following international Fisher equation (= real international interest parity condition) holds in addition to the national Fisher equations:

$$1 + i_{t+1} = 1 + i_{t+1}^*. \tag{22}$$

As under financial autarky both governments pursue a "constant stock" budget policy which implies that lump-sum taxes in both countries become endogenous and are determined by equations (13).

Market clearing of the national labor markets in South and North requires:

$$N_t = L_t, \; N_t^* = L_t, \; t = 0,1,2... \tag{23}$$

The union-wide market for southern (northern) real capital clears according to:

$$K_{t+1} = K_{t+1}^S + K_{t+1}^{*N}, \; (K_{t+1}^* = K_{t+1}^N + K_{t+1}^{*S}), \; t = 0,1,2.... \tag{24}$$

The union-wide market for southern (northern) bonds clears according to:

$$B_{t+1} = B_{t+1}^{S} + B_{t+1}^{*N}, \; (B_{t+1}^{*} = B_{t+1}^{N} + B_{t+1}^{*S}), \; t = 0,1,2.... \tag{25}$$

The union-wide asset market clearing condition requires that the total amount of savings in the union equals the total world supply of assets from South and North:

$$L_t s_t + L_t s_t^{*} = K_{t+1} + K_{t+1}^{*} + B_{t+1} + B_{t+1}^{*}. \tag{26}$$

Having described the optimization problems of households and firms as well as the market clearing conditions, the intertemporal equilibrium dynamics can now be derived.

From (14) and from the international Fisher equation (22), the following relationship between northern and southern capital output ratios results:

$$v_{t+1}^{*} = \frac{\alpha^{*}}{\alpha} v_{t+1}. \tag{27}$$

Dividing (26) on both sides by Y_t and introducing the definitions of the capital output ratios as well as the debt output ratio, the asset market clearing condition (26) can be rewritten as follows:

$$\frac{L_t s_t}{Y_t} + \frac{L_t s_t^{*}}{Y_t^{*}} \frac{Y_t^{*}}{Y_t} = G_t^{Y} v_{t+1} + G_t^{Y,*} v_{t+1}^{*} \frac{Y_t^{*}}{Y_t} + G_t^{Y} b + G_t^{Y,*} b^{*} \frac{Y_t^{*}}{Y_t}. \tag{28}$$

Using southern and northern production functions, the ratio of northern to southern GDP turns out to be as follows:

$$\frac{Y_t^{*}}{Y_t} = \mu v_t^{\chi}, \; \mu \equiv \frac{(M^{*})^{1/(1-\alpha^{*})}}{M^{1/(1-\alpha)}} \left(\frac{\alpha^{*}}{\alpha} \right)^{\alpha^{*}/(1-\alpha^{*})}, \; \chi \equiv \frac{\alpha^{*} - \alpha}{(1-\alpha^{*})(1-\alpha)}. \tag{29}$$

Acknowledging (29), the definitions of the GDP growth rates and the optimal savings functions in (28) yield:

$$\begin{aligned} v_{t+1} \left(\frac{v_{t+1}}{v_t} \right)^{\alpha/(1-\alpha)} + \mu v_{t+1}^{*} \left(\frac{v_{t+1}^{*}}{v_t^{*}} \right)^{\alpha^{*}/(1-\alpha^{*})} v_t^{\chi} + b \left(\frac{v_{t+1}}{v_t} \right)^{\alpha/(1-\alpha)} + b^{*} \mu v_{t+1}^{*} \left(\frac{v_{t+1}^{*}}{v_t^{*}} \right)^{\alpha^{*}/(1-\alpha^{*})} v_t^{\chi} \\ = \frac{\sigma}{G^{n}} (1-\alpha)(1-\tau_t) + \frac{\sigma^{*}}{G^{n}} (1-\alpha^{*})(1-\tau_t^{*}) \mu v_t^{\chi}. \end{aligned} \tag{30}$$

Considering equations (15) and (27) in equation (30) and multiplying the result on both sides by $v_t^{\alpha/(1-\alpha)}$ eventually yields the following first-order difference equation with respect to v_t describing the law of motion for the international asset market:

$$
\begin{aligned}
&{v_{t+1}}^{1/(1-\alpha)} + \left(\frac{\alpha^*}{\alpha}\right)\mu {v_{t+1}}^{1/(1-\alpha^*)} + b(1-\sigma){v_{t+1}}^{\alpha/(1-\alpha)} + b^*(1-\sigma^*)\mu(v_{t+1})^{\alpha^*/(1-\alpha^*)} \\
&= \frac{\sigma}{G^n}[(1-\alpha-\gamma)-\frac{\alpha b}{v_t}](v_t)^{\alpha/(1-\alpha)} + \frac{\sigma^*}{G^n}[(1-\alpha^*-\gamma^*)-\frac{\alpha b^*}{v_t}]\mu {v_t}^{\alpha^*/(1-\alpha^*)}.
\end{aligned}
\tag{31}
$$

Equation (31) represents the intertemporal equilibrium capital-output ratio dynamics of the one-good, two-country OLG model under financial integration.

With the dynamics of the capital output ratio are associated the intertemporal equilibrium path of the union-wide real interest rate, southern and northern real wage rates and the real exchange rate between South and North, e_t. The real exchange rate is defined in line with Lundquist (1988) and Lin (1994) as $e_t = w_t / w_t^*$.[14]

$$
\begin{aligned}
&i_{t+1} = \frac{\alpha}{v_{t+1}}\text{-}1,\ w_{t+1} = (1-\alpha)a_{t+1}M^{1/(1-\alpha)}(v_{t+1})^{\alpha/(1-\alpha)} \\
&w_{t+1}^* = (1-\alpha^*)a_{t+1}(M^*)^{1/(1-\alpha^*)}\left(\frac{\alpha^*}{\alpha}\right)^{\alpha^*/(1-\alpha^*)}(v_{t+1})^{\alpha^*/(1-\alpha^*)}
\end{aligned}
\tag{32}
$$

$$
\begin{aligned}
e_{t+1} &= \frac{(1-\alpha)a_{t+1}M^{1/(1-\alpha)}(v_{t+1})^{\alpha/(1-\alpha)}}{(1-\alpha^*)a_{t+1}(M^*)^{1/(1-\alpha^*)}\left(\frac{\alpha^*}{\alpha}\right)^{\alpha^*/(1-\alpha^*)}(v_{t+1})^{\alpha^*/(1-\alpha^*)}} \\
&= \frac{(1-\alpha)M^{1/(1-\alpha)}}{(1-\alpha^*)(M^*)^{1/(1-\alpha^*)}\left(\frac{\alpha^*}{\alpha}\right)^{\alpha^*/(1-\alpha^*)}}(v_{t+1})^{\frac{\alpha-\alpha^*}{(1-\alpha^*)(1-\alpha)}}
\end{aligned}
\tag{33}
$$

As expected, the common real interest rate declines with rising capital output ratio in South, while both the southern and the northern real wage rates increase with rising capital output ratio in South (North). Moreover, the real exchange rate of South increases with rising southern capital output ratio when $\alpha > \alpha^*$ (as we assumed).

[14] Lin (1994, p. 97) shows that this definition of the real exchange rate is consistent with those in standard textbooks as e.g. in Dornbusch and Fischer (1990, pp. 184-185) where the real exchange rate e is defined as: $e = EP/P^*$ with E standing for the nominal exchange rate (price of domestic currency in terms of foreign currency), and P and P^* being the domestic and foreign price level, respectively.

Financially Integrated versus Financially Autarkic Steady State

A steady state of the equilibrium dynamics under financial integration is defined as $v_{t+1}=v_t=v,\quad v^*_{t+1}=v^*_t=v^*,\quad e_{t+1}=e_t=e,\quad i_{t+1}=i_t=i,\quad w_{t+1}=w_t=w,\quad w^*_{t+1}=w^*_t=w^*$. Evaluating (27) and (31) at a non-trivial steady state yields:

$$v^*=\frac{\alpha^*}{\alpha}v, \tag{34}$$

$$\begin{aligned}&\frac{\sigma}{G^n}\left(1-\alpha-\gamma-\frac{\alpha b}{v}\right)-\left[v+b(1-\sigma)\right]=\\&\qquad -\left\{\left[\frac{\sigma^*}{G^n}\left(1-\alpha^*-\gamma*-\frac{\alpha b^*}{v}\right)\right]-\left[\left(\frac{\alpha^*}{\alpha}\right)v+b^*\left(1-\sigma^*\right)\right]\right\}\mu v^{\chi},\end{aligned} \tag{35}$$

or

$$\phi=-\frac{Y^*}{Y}\phi^*=-\mu v^{\chi}\phi^*, \tag{36}$$

where $\phi_{t+1}\equiv\Phi_{t+1}/Y_{t+1}$ ($\phi^*_{t+1}\equiv\Phi^*_{t+1}/Y^*_{t+1}$) denote the net foreign asset position per GDP of South (North) in period $t+1$. Thus, in steady state the southern net foreign asset position is exactly mirrored by a negative position of northern net foreign assets multiplied by the northern to southern GDP ratio.

Before addressing the main question of whether, starting from financial autarky with both net foreign asset positions equal to zero and the real interest rate in South higher than in North, financial integration implies a negative southern and a positive northern net foreign asset position, we investigate the more technical questions concerning existence, uniqueness and dynamic stability of steady states under financial integration.

Regarding the existence of steady-state solutions one has to explore whether equation (35) exhibits a strictly positive solution for the southern capital output ratio. Casual inspection of equation (35) immediately reveals that in general an explicit solution is infeasible. To facilitate the existence analysis we proceed in two steps: First, we investigate the existence of non-trivial steady-state solutions for the special case of “similar” production technologies across North and South. Production technologies are meant to be similar if total factor productivities differ across North and South ($M\neq M^*$) but southern and northern capital production shares are identical. Second, the more technical analysis of the general case internationally differing capital production shares is delegated towards the appendix.

In the case of similar technologies in North and South $\chi = 0$, and thus equation (35) collapses onto the following second-order polynomial:

$$\left(1+\mu\right)v^2 + \left[b\left(1-\sigma\right)+b^*\left(1-\sigma^*\right)\mu - \frac{\sigma}{G^n}\left(1-\alpha-\gamma\right) - \frac{\sigma^*}{G^n}\left(1-\alpha-\gamma^*\right)\mu\right]v + \frac{\alpha}{G^n}\left(\sigma b+\sigma^* b^* \mu\right) = 0. \quad (37)$$

Proposition 4 (Existence of non-trivial steady-state solutions under similar technologies)
Let $\alpha = \alpha^*$. If $b(1-\sigma)+b^*(1-\sigma)\mu \le \bar{b}(1-\sigma)+\bar{b}^*(1-\sigma)\mu < [\sigma(1-\alpha-\gamma)+\sigma^*\mu(1-\alpha-\gamma^*)]/G^n$ while $(\bar{b},\bar{b}^*) \in \mathrm{B} = \{(\bar{b},\bar{b}^*) \,|\sigma(1-\alpha-\gamma)-\bar{b}(1-\sigma)G^n + \sigma^*(1-\alpha-\gamma^*)\mu - \bar{b}^*(1-\sigma^*)\mu G^n$ $= 2\sqrt{\alpha G^n(1+\mu)(\sigma\bar{b}+\sigma^*\bar{b}^*\mu)}\}$, then there are exactly two non-trivial steady-state solutions as follows:

$$v_1 = \frac{\sigma(1-\alpha-\gamma)-\bar{b}(1-\sigma)G^n+\sigma^*(1-\alpha-\gamma^*)\mu-\bar{b}^*(1-\sigma^*)\mu G^n}{2(1+\mu)G^n}$$
$$\frac{-\sqrt{[\sigma(1-\alpha-\gamma)-\bar{b}(1-\sigma)G^n+\sigma^*(1-\alpha-\gamma^*)\mu-\bar{b}^*(1-\sigma^*)\mu G^n]^2-4\alpha G^n(1+\mu)(\sigma b+\sigma^* b^*\mu)}}{2(1+\mu)G^n},$$

$$v_2 = \frac{\sigma(1-\alpha-\gamma)-\bar{b}(1-\sigma)G^n+\sigma^*(1-\alpha-\gamma^*)\mu-\bar{b}^*(1-\sigma^*)\mu G^n}{2(1+\mu)G^n}$$
$$\frac{+\sqrt{[\sigma(1-\alpha-\gamma)-\bar{b}(1-\sigma)G^n+\sigma^*(1-\alpha-\gamma^*)\mu-\bar{b}^*(1-\sigma^*)\mu G^n]^2-4\alpha G^n(1+\mu)(\sigma b+\sigma^* b^*\mu)}}{2(1+\mu)G^n}. \quad (38)$$

Proof. The proof is similar to that of proposition 1. ■

Since there are again two non-trivial steady states an analysis of dynamic stability of both solutions is in order. The result is exhibited in proposition 5.

Proposition 5 (Dynamic stability of steady states under similar technologies)

Let $\alpha = \alpha^*$. Suppose that the assumptions of proposition 4 for the existence of the two non-trivial steady-state solutions (38) hold. Then, the steady state v_1 in (38) is asymptotically unstable while the steady state v_2 in (38) is asymptotically stable.

Proof. Calculating dv_{t+1}/dv_t from (31) and evaluating the result in any steady-state solution yields: $dv_{t+1}/dv_t = \{v^{\alpha/(1-\alpha)-1}\{\alpha/(1-\alpha)\sigma/G^n[1-\alpha-\gamma-\alpha b/v]+\sigma\alpha b/v\} + v^{\alpha^*/(1-\alpha^*)-1}$

$\{\alpha^*/(1-\alpha^*)\,\mu\sigma/G^n\,[1-\alpha^*-\gamma^*-\alpha b^*/v]+\sigma\alpha b/v\}\}\,/\,\{v^{\alpha/(1-\alpha)-1}\,/\,(1-\alpha)[v+\alpha b(1-\sigma)]$ $+v^{\alpha^*/(1-\alpha^*)-1}\,/\,(1-\alpha^*)[\alpha^*/\alpha\,\mu v+\alpha^* b^*(1-\sigma^*)\mu]\}$. Acknowledging $\alpha=\alpha^*$ and the equation $(1+\mu)v+b(1-\sigma)+b^*(1-\sigma^*)\mu=\sigma/G^n\,[1-\alpha-\gamma-\alpha b/v]+\sigma^*/G^n\,[1-\alpha-\gamma^*-\alpha b^*/v]\,\mu$, $dv_{t+1}/dv_t=\{\alpha[(1+\mu)v+b(1-\sigma)+b^*(1-\sigma^*)\mu]+\alpha(1-\alpha)[\sigma b+\sigma^* b^*]/G^n v\}\,/\{(1+\mu)v$ $\alpha b(1-\sigma)+\alpha b^*(1-\sigma^*)\mu\}$. It is easily seen that $dv_{t+1}/dv_t<1(>1)\Leftrightarrow\alpha(\sigma b+\sigma^* b^*)/G^n v^2$ $<1+\mu\ (>1+\mu)$. Proceeding similarly as in the proof of proposition 2 we can see that $dv_{t+1}/dv_t<1\Leftrightarrow\alpha(\sigma b+\sigma^* b^*)/G^n(v_2)^2<1+\mu$ and $dv_{t+1}/dv_t>1\Leftrightarrow\alpha(\sigma b+\sigma^* b^*)/G^n(v_1)^2>$ $1+\mu$. ■

On knowing from the appendix that also for $\alpha\neq\alpha^*$ the larger steady-state solution v_2 under financial integration is unique and dynamically stable, proposition 6 below provides an answer to the main question raised above.

Proposition 6. Suppose that the assumptions of proposition 3 hold, i.e. the southern financial autarky interest rate, i, is larger than the northern financial autarky interest rate, i^*. Then, after financial integration, the ratio of the net foreign asset position to GDP of South (North) is negative (positive), i.e. $\phi<0\ (\phi^*>0)$.

Proof. By assumption, $i>i^*$. Thus, $1+i=\alpha/v>1+i^*=\alpha^*/v^*$. Financial integration means that the positive differential between southern and northern autarky interest rates diminishes as the southern interest rate declines and the northern interest rate rises. Due to decreasing marginal productivity of capital the decline in southern interest rate is associated with a rise in southern capital output ratio and vice versa in North. Remembering the definition of the southern net foreign asset position in steady state as $\phi(v)\equiv\sigma/G^n\,(1-\alpha-\gamma-\alpha b/v)-[v+b(1-\sigma)]$, differentiation of ϕ with respect to v yields $\phi'(v)=\alpha\sigma b/G^n\,v^2-1$. From proposition 2 we know that there is a small neighborhood of the southern autarky steady state with the larger capital intensity in which $\phi'(v_2)<0$ holds. Hence, the southern net foreign asset position deteriorates with rising southern capital output ratio. Since at the autarky value of v_2 the southern net foreign asset position is zero, and since the southern net foreign asset position declines with rising capital output ratio, at v_2 thus providing a solution for the international asset market equilibrium condition (35), $\phi(v_2)<0$. Due to equation (36), the northern net foreign asset position after financial integration is positive. ■

On evaluating equations (32) for the non-trivial steady state, it becomes immediately apparent that the common real interest rate is lower than southern autarkic real interest rate and that the southern real wage rate is higher and the northern real wage rate is lower than the corresponding autarkic value. However, it still remains unclear whether the ratio of southern to northern real wage rate or the southern real exchange rate is higher or lower than in financial autarky. This question is clarified by corollary 1.

Corollary 1. Suppose that southern production elasticity of capital (= capital production share) is strictly larger than northern capital production share, i.e. $\alpha > \alpha^*$. Then, southern real exchange rate rises in the course of financial integration.

Proof. On evaluating equation (33) in the steady state, we obtain a positive relationship between the real exchange rate and southern capital output ratio when the southern capital production share is larger than the corresponding northern share. Since under financial integration southern capital output ratio rises from its pre-integration value, the real exchange rate defined as the ratio of southern to northern real wage rate rises too. ■

A Numerical Ilustration

Before concluding a numerical specification of the basic model is used to illustrate both its explanatory power and its limitations. The parameter values are chosen such that the model is able to reproduce roughly main stylized facts listed above both under financial autarky and under financial integration as well as to accord with assumptions in the propositions above. The structural and policy parameters are as follows: $\alpha = 0.2,\ \beta = 0.45,\ \gamma = 0.3,$ $M = 4,\ b = 0.028,\ G^n = 1.8,\ \alpha^* = 0.16,\ \beta^* = 0.6,\ \gamma^* = 0.4,\ M^* = 6,\ b^* = 0.024$.

Table 1. Main endogenous variables in South and North (calculated on a yearly basis) under financial autarky

Financial Autarky

South		*North*	
v	1.14571	v^*	1.60517
i	0.0607078	i^*	0.0371976
w	2.093866	w^*	4.2026445
τ	0.4615783	τ^*	0.4994076
S / Y	0.13367712	S^* / Y^*	0.1576866
x^1	0.77750549	$x^{*,1}$	1.31488239
x^2	1.2550364	$x^{*,2}$	2.8299483
u	0.86118967	u^*	2.45444897
ϕ	0	ϕ^*	0

Note that yearly southern (northern) interest rate exhibited in table 1 is not too far from the real interest rates in late 1990s portrayed in figure 3. Similarly, household's savings ratio for South respectively for North shown in Table 1 corresponds roughly to the personal savings ratios in euro core and periphery exhibited in Figure 5 above. Finally, zero net foreign asset positions in South and in North shown in Table 1 accord approximately with small net foreign debt in South and small northern net foreign credit in late 1990s portrayed in Figure 8.

Table 2. Main endogenous variables in South and North (calculated on a yearly basis) under financial integration

Financial Integration

South		*North*	
v	1.66309	v^*	1.33047
i	0.045014	i^*	0.045014
w	2.298312	w^*	4.055045
τ	0.41722581	τ^*	0.5106605
S/Y	0.1446888	S^*/Y^*	0.1541419
x^1	0.92372186	$x^{*,1}$	1.24018349
x^2	1.24970634	$x^{*,2}$	2.2371311
u	1.0211859	u^*	2.0104849
ϕ	-0.353524	ϕ^*	0.210388

Before comparing the results in Table 2 to those in Table 1 note that the southern ratio of net foreign assets to GDP in Table 2 corresponds to the value reported in Fig. 8 for 2007 but the northern net foreign credit to GDP ratio exhibited in Table 2 is a little bit too large compared to the close-to-zero value in Fig. 8. Here, the limitations of the two-country approach (lacking rest of the world) become apparent.

The comparison of the results in Table 2 to those in Table 1 reveals three interesting insights. First, through financial integration (equalization of southern and northern real interest rates) periphery's capital-output ratio (including housing and residential objects) increases while core's capital-output ratio decreases mimicking the development of housing investment relative to GDP in the periphery versus the core portrayed in Fig. 6. Second, southern wage tax rate decreases markedly while the northern wage tax rate increases to lesser extent. This is due to the pronounced decrease of southern real interest rate and due to the much less pronounced increase of northern interest rate. Third, through financial integration southern (northern) working-period consumption increases (decreases) at the expense (in favor) of old-age consumption. Southern (northern) working-period consumption increases (decreases) since the southern wage rate rises (falls) through financial integration while southern old-age consumption decreases (increases) due to the lower (higher) real interest rate. Fourth, in contrast to Gaspar and Fagan (2008) who report utility increases through financial integration both in periphery and in core, life-time utility in our model increases in South but decreases in North. Fifth, while the savings ratios under financial autarky generated by the model are roughly in accordance with empirical facts exhibited in Fig. 5, the model predicts in contrast to stylized fact 3 an increase in periphery's savings ratio. Moreover, the increase of southern real exchange rate from 0.5543447 towards 0.56677833 (not shown in Table 2) is only one tenth of the actual rise documented by stylized fact 5. Here, another limitation of our model set-up becomes obvious: the exclusion of over-optimistic convergence expectations which were according to Lane and Pels (2012) another important driver of external imbalances in the euro area.

Concluding Remarks

This paper explores, within a simple one-good, two-country OLG model with production, capital accumulation and public debt, the emergence of external imbalances among northern (euro zone core) and southern (euro zone periphery) EMU member countries after the inception of the common currency and before the outburst of the global financial crisis in 2008. It models the pre-euro situation as financial autarky and the EMU before the global financial crisis as financial integration characterized by complete convergence of real interest rates. After assuring the existence and dynamic stability of financial autarky steady states, a lower savings rate and a higher capital production share in South were shown to imply the empirically observed southern high real interest rate and low real wage rate associated with no external imbalances between South and North before the advent of the common currency. Symmetrically, the higher savings rate and the lower capital production share in North implied a northern low real interest rate and a high real wage. After the inception of the common currency, free capital mobility among South and North induced immediate international real interest parity leading to a quick fall in southern interest rates and to rising northern interest rates. Simultaneously, initial relatively high southern interest rates lead the northern core to invest their wealth in southern housing and residential objects. Thus, both northern and southern external balances widen: the euro zone periphery turn into a net foreign debtor position while the euro zone core become a net foreign creditor. Moreover, with the faster rising southern real wage rate southern real exchange rate appreciates.

We may thus conclude that the one-good, two-country OLG model is capable of reproducing qualitatively main stylized facts presented above: declining real interest rate for southern euro zone countries, both rising southern external debt and northern external credit, and a rising real exchange rate which diminished the international competitiveness of South. Proposition 3, 6 and corollary 1 corroborate the claim that the emergence of external imbalances between northern and southern euro zone countries after financial integration can be traced back to fundamental North-South differences in savings rates and capital production shares.

While the OLG model enables us to attribute the intra-EMU external imbalances to South-North differences in such fundamentals, the genesis of a "debt trap" (Baumgarten and Klodt 2010) for the euro zone periphery (and the euro zone as a whole) could not properly be explained by the basic model. One reason is that nominal variables like price levels, nominal interest rates and the quantity of money do not play any role in the basic model. Neither monetary nor fiscal policy is present. Moreover, there is only one commodity produced in the model economy. Another limitation is that agents in this model do not form 'bubbly' price expectations, i.e. prices leading to cumulative divergences from fundamentals. Indeed, Lane and Pels (2012) find that exaggerated growth income expectations in southern euro zone countries have contributed to the huge external imbalances.

How to extend the analysis of this paper in future research is obvious. These necessary extensions may be e.g. a second non-tradable good as in Fagan and Gaspar (2008) and bubbly expectations regarding asset prices in line with Carvalho et al. (2012). In order to differentiate properly between un-productive housing investment and productive non-housing investment, a three-period overlapping generations' model in line with Japelli and Pagano (1994) and Artige and Cavenaile (2012) would be in order. If EMU external imbalances during and after the crisis are also subject to scrutiny, then non-Walrasion underemployment equilibria à la Rankin (1987) or involuntary unemployment à la Farmer (2012) would also have to be introduced into the basic model.

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Appendix

Proposition 4A (Existence of steady states under dissimilar production technologies)

Suppose that $\alpha > \alpha^*$, $\boldsymbol{\omega} \equiv (\alpha, \alpha^*, \beta, \beta^*, G^n, M, M^*, b, b^*)$ be the parameter vector and $\Omega = [0,1]^4 \times R_+^5$ be the parameter space. If for any admissible parameter combination, $\boldsymbol{\omega} \in \Omega$, there exist some $\bar{b} \in (0,\infty)$ and $\bar{b}^* \in (0,\infty)$ such that $b \in (0,\bar{b})$ and $b^* \in (0,\bar{b}^*)$, then there exist two non-trivial steady-state solutions v_1 and v_2, with $v_1 < v_2$. For $b = b^* = 0$, there is only one non-trivial steady state.

Proof. For the proof it is useful to rewrite (35) as follows: $v + (\alpha^*/\alpha)\mu v^{1+v}$ $+\alpha/G^n(\sigma b/v + \sigma^* b^* \mu/v^{1-v}) = (\sigma/G^n)(1-\alpha-\gamma) - b(1-\sigma) + [(\sigma^*/G^n)(1-\alpha^*-\gamma^*) - b^*(1-\sigma^*)]\mu v^v$. Denote the left-hand side of this steady-state equation as $LHS(v)$ and the right-hand side as $RHS(v)$. Note that $LHS(v)$ consists of two parts: the monotonically increasing part $v + (\alpha^*/\alpha)\mu v^{1+v}$ and the monotonically decreasing part $\alpha\mu(\sigma b + \sigma^* b^*)/G^n v^{1-v}$. For small capital output ratios the second part dominates the first part. Thus, for relatively small v $LHS(v)$ is a monotonically decreasing function, for relatively large v the first increasing part dominates $LHS(v)$. Hence, $LHS(v)$ is for small capital output ratios decreasing and for larger capital output ratios increasing. On the other hand, $RHS(v)$ is monotonically decreasing for all admissible capital output ratios. Moreover, $LHS(v)$ shifts upwards with larger debt output ratios b respective b^* while $RHS(v)$ shifts downwards with higher debt outputs ratios b respective b^*. Thus, there must exist $b \in (0,\bar{b})$ and $b^* \in (0,\bar{b}^*)$ such that $LHS(v)$ and $RHS(v)$ intersect exactly twice. For $b = 0 = b^*$ $LHS(v)$ is overall monotonically increasing and hence there is exactly one intersection point. ■

Proposition 5A. Suppose that the assumptions of proposition 4A hold. Then, v_1 represents an asymptotically unstable steady state while v_2 is asymptotically stable.

Proof. Analogously to the proof of proposition 5 in main text, it is up to show that $dv_{t+1}/dv_t > 1$ in the neighborhood of v_1 and $dv_{t+1}/dv_t < 1$ in the neighborhood of v_2. From the proof of proposition 5 we know that irrespective of the steady-state solution it holds: $dv_{t+1}/dv_t = \{v^{\alpha/(1-\alpha)-1}\{\alpha/(1-\alpha)\sigma/G^n[1-\alpha-\gamma-\alpha b/v] + \sigma\alpha b/v\} + v^{\alpha^*/(1-\alpha^*)-1}\{\alpha^*/(1-\alpha^*)\mu\sigma/G^n$ $[1-\alpha^*-\gamma^*-\alpha b^*/v] + \sigma\alpha b/v\}\} / \{v^{\alpha/(1-\alpha)-1}/(1-\alpha)[v+\alpha b(1-\sigma)] + v^{\alpha^*/(1-\alpha^*)-1}/(1-\alpha^*)[\alpha^*/\alpha\,\mu v$ $+\alpha^* b^*(1-\sigma^*)\mu]\}$. In order to evaluate this expression we use the following steady-state version of equation (31):

$v + (\alpha^*/\alpha)\mu v^{1+v} + b(1-\sigma) + b^*(1-\sigma^*)\mu v^v = (\sigma/G^n)(1-\alpha-\gamma-\alpha b/v) +$ $(\sigma^*/G^n)(1-\alpha^*-\gamma^*-\alpha b^*/v)\mu v^v$. The slope of the left-hand side of this equation $LHS(v) \equiv$ $v + (\alpha^*/\alpha)\mu v^{1+v} + b(1-\sigma) + b^*(1-\sigma^*)\mu v^v$ reads as follows: $LHS'(v) \equiv 1 + (1+v)(\alpha^*/\alpha)\mu v^v$ $+ v b^*(1-\sigma^*)\mu v^{v-1}$. Moreover, the slope of the right-hand side $RHS(v) \equiv$

$(\sigma/G^n)(1-\alpha-\gamma-\alpha b/v)+(\sigma^*/G^n)(1-\alpha^*-\gamma^*-\alpha b^*/v)\mu v^\nu$ yields: $RHS'(v)=(\sigma/G^n)\alpha b/v^2$ $+(\sigma^*/G^n)\alpha b^*\mu/v^{2-\nu}+(\sigma^*/G^n)\nu(1-\alpha^*-\gamma^* \ -\alpha b^*/v)\mu v^{\nu-1}$. Proposition 4A tells us that the steady-state equation $LHS(v)=RHS(v)$ exhibits the two solutions $v_1<v_2$. Inspection of $LHS'(v_1)$ respective $RHS'(v_1)$ reveals immediately that $RHS'(v_1)>LHS'(v_1)$. On the other hand, $RHS'(v_2)<LHS'(v_2)$. Simple calculation shows that the above dv_{t+1}/dv_t can be equivalently written as follows:

$dv_{t+1}/dv_t=\{(1-\alpha^*)(\sigma/G^n)[\alpha(1-\alpha-\gamma-\alpha b/v)+(1-\alpha)\alpha b/v]+$ $(1-\alpha)(\sigma^*/G^n)[\alpha^*(1-\alpha^*-\gamma^*-\alpha b^*/v)+(1-\alpha^*)\alpha b^*/v]\mu v^\nu\}/\{(1-\alpha^*)[v+\alpha b(1-\sigma)]+(1-\alpha)$ $[(\alpha^*/\alpha)v+\alpha^* b^*(1-\sigma^*)]\mu v^\nu\}$. $dv_{t+1}/dv_{t\,|v=v_1}>1\Leftrightarrow$ $\{(1-\alpha^*)(\sigma/G^n)[\alpha(1-\alpha-\gamma-\alpha b/v_1)+$ $(1-\alpha)\alpha b/v_1]+$ $(1-\alpha)(\sigma^*/G^n)[\alpha^*(1-\alpha^*-\gamma^*-\alpha b^*/v_1)+(1-\alpha^*)\alpha b^*/v_1]\mu v_1{}^\nu\}$ $>\{(1-\alpha^*)[v_1$ $+\alpha b(1-\sigma)]+(1-\alpha)$ $[(\alpha^*/\alpha)v_1+\alpha^* b^*(1-\sigma^*)]\mu v_1{}^\nu\}$. This inequality is obviously equal to $RHS'(v_1)>LHS'(v_1)$ multiplied on both sides by v_1. Analogous procedure yields: $dv_{t+1}/dv_{t\,|v=v_2}<1\Leftrightarrow RHS'(v_2)<LHS'(v_2)$. ■