

# **Dividends of Environmental Tax with Endogenized Time and Medical Expenditures<sup>1</sup>**

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## **ABSTRACT**

Health effects of medical expenditures deserve consideration in the literature addressing the dividends of environmental taxation, since illness not only influences utility, but also affects leisure and working time. We for the first time differentiate the health effects and tax deductibility between medical treatment expenditure and illness prevention expenditure, and redefine the marginal social damage (MSD) of dirty goods consumption that was found incorrectly measured before.

After modifying the health production function and redefining MSD, we use the traditional decomposition approach to derive a new source of dividends, named “prevention-based tax-interaction effect”, that, however, is negative in sign and weakens the second dividend.

As an alternative approach, a social planning model is presented and simulation implemented. With tax neutrality, revenue raised from environmental tax is used to reduce income tax rate. The results, while confirming the first dividend, indicate that the tax reform increases neither the monetary value of utility nor labor employment. Nevertheless, an optimal bundle of income tax and environmental tax might exist that minimizes the potential welfare loss.

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## INTRODUCTION

Various factors had been considered in the literature addressing the existence of the dividends of environmental taxes through proper revenue disposal. Major determinants include product market structure (Barnett, 1980), tax-interaction effect (Bovenberg and Mooij, 1994*a*, 1994*b*; Goulder, 1995*b*; Parry, 1995; Kahn and Farmer, 1999), tax rates (Parry, 1995; Bovenberg and Goulder, 1996; Fullerton, 1997), tax base (Bovenberg and Mooij, 1994*b*; Goulder, 1995*b*; Parry, 1995), consumer's preference (Kahn and Farmer, 1999; Schwartz and Repetto, 2000; Williams, 2002, 2003; Pang and Shaw, 2007), labor market imperfection (Carraro, Galeotti and Gallo, 1996; Bosello and Carraro, 2001), role perception about the environmental tax (Goulder, Parry and Burtraw, 1997; Bovenberg, 1999; Parry and Bento, 2000; Schleiniger, 2001; Conrad and Löschel, 2005; Bento and Jacobsen, 2007), the properties of the health production function (Koç, 2007), and intertemporal welfare concern (Chiroleu-Assouline and Fodha, 2005, 2006).

Recently, health effect of medical expenditures was taken into account in this agenda, because illness not only influences utility, but also affects leisure and working time. Although the health production function is not a new concept (Grossman, 1972), the ways of its specification led to different conclusions about the components and existence of the dividends (Schwartz and Repetto, 2000; Williams, 2002, 2003; Pang and Shaw, 2007; Koç, 2007). In addition to the well-known effects such as Pigouvian effect (*PE*), revenue-recycling effect (*RE*), tax-interaction effect (*IE*), and benefit-side tax-interaction effect (*IE<sup>B</sup>*), a new component named mitigation-based tax-interaction

effect ( $IE^M$ ) was derived when the health effect of medical expenditure was taken into account. Although not theoretically proved,  $IE^M$  was shown positive and large enough to offset  $IE$  through simulation (Pang and Shaw 2007) and, therefore, lend much support to the double dividends hypothesis.

In light of the literature, one could identify at least three critical flaws. Firstly, medical expenditure, one of the key determinants of the health production function, was usually used as a surrogate without clear functional differentiation between medical treatment and illness prevention. A distinction between the two is warranted for two reasons: (a) Medical treatment expenditure (MTE) and illness prevention expenditure (SPE) may generate different health effects. (b) While MTE is tax deductible, SPE is not in general. Secondly, the marginal social damage (MSD) of dirty goods consumption defined in many cases is limited only to the marginal disutility of illness, leaving the leisure effect unattended even though leisure time is obviously affected by illness. Some studies did consider the utility loss of leisure time (see, for example, Williams, 2003; Pang and Shaw, 2007), however, it was improperly valued. Such an incorrect measurement of MSD tends to overestimate the second dividend. Finally, marginal utility of income was assumed constant. In fact, it may change as environmental tax increased and revenue disposed.

This paper modifies the health production function by incorporating both MTE and SPE and differentiating their tax deductibility, and redefines MSD. Following similar decomposition approach, a new source of dividends with environmental tax on dirty goods, named “prevention-based tax-interaction effect” ( $IE^A$ ), is derived, that, however, is negative in sign and weakens the overall second dividend. As an alternative approach, a social planning model is presented here and simulation implemented. With tax neutrality, revenue raised from environmental tax is used to

reduce income tax rate. The results, while confirming the first dividend, indicate that the tax reform increases neither the monetary value of utility nor labor employment, but utility levels and leisure. Nevertheless, an optimal bundle of income tax and environmental tax might exist that minimizes the potential welfare loss.

The paper is organized as follows. An individual optimization model is presented in the second section to examine the properties of demands for clean and dirty commodities, leisure, MTE and SPE. The marginal social damage of dirty goods consumption is redefined in the third section. The fourth section demonstrates the decomposition of the dividends of environmental tax. The fifth reports a social planning model with numerical simulation results, followed by concluding remarks.

### **Household's Optimization**

Previous literature revealed that the ways of specifying the health production function had something to do with the second dividend. Despite of its merits, the Grossman's (1972) specification was rarely adopted simply because static models are more commonly developed to examine the second dividend. In light of Schwartz and Repetto (2000) and Williams (2002), Williams (2003) created two functions to capture the health effects: health production function and illness-time function. The former, depending on medical expenditure ( $M$ ) and environmental quality ( $Q$ ), affects utility, while the later, depending solely on environmental quality, affects the time available for leisure and work. Pang and Shaw (2007) suggested that both functions be integrated into the illness-time function represented by  $S = S(Q, M)$ , where  $S_Q < 0$  and  $S_M < 0$ , so that  $M$  affects utility as well as time availability. Meanwhile, the utility function,  $U = U(v(X, Y, I), G, S)$ , is assumed weakly separable, where  $v$  is a concave subutility function of dirty good ( $X$ ), clean good ( $Y$ ) and leisure time ( $I$ );  $G$  is

the government expenditure on public goods, financed by labor income tax and environmental tax; and  $U$  is additive in  $v$ ,  $G$  and  $S$ .

Since the second dividend is linked to the government's disposal of tax revenues, here we consider three outlets for government spending:  $G_a$ ,  $G_b$  and  $G_c$ , representing, respectively, public goods, environmental protection, and direct transfer payment to households. In addition, both MTE and SPE are incorporated in a illness-time function,  $S = S(Q, M, A)^2$ , where  $M$  and  $A$  represents, respectively, MTE and SPE such that  $S_i < 0$ ,  $S_{ii} > 0 \quad \forall i = Q, M$  and  $A$ . The environmental quality is a function of  $G_b$  and aggregate pollutant emission ( $E$ ), i.e.,  $Q = Q(E, G_b)$ , characterized by  $Q_E < 0$ , and  $Q_{G_b} > 0$ .

Given the budget constraint (Eq. (1)) and time constraint (Eq. (2)), the household is assumed to solve the following optimization problem:

$$\underset{\{X, Y, l, M, A\}}{\text{Max}} \quad U = U(v(X, Y, l), S(Q, M, A), G_a)$$

$$\text{Subject to} \quad (1+t_x)X + Y + M + (1-s)A = (1-t_L)L + G_c + t_L \cdot M, \quad (1)$$

$$T = l + S(Q, M, A) + L \quad (2)$$

where  $T$  and  $L$  are, respectively, time endowment and work time;  $t_x$  and  $t_L$  represent, respectively, environmental tax rate on dirty goods and tax rate on labor income;  $s$  is the subsidy rate on SPE; and MTE is fully deducted (amount to  $t_L M$ )<sup>3</sup>.

Following conventional assumptions, prices of all commodities and labor are normalized in Eq. (2) and identical to one. Here environmental quality as well as

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<sup>2</sup> The health effect of  $M$  and  $A$  may differ not only in quantity but also in degree of uncertainty. Uncertainty is not addressed here.

<sup>3</sup> Including the subsidy for SPE may sound peculiar, but provides insightful implications with respect to the second dividend, as shown later.

policy parameters is considered exogenous. Solving the above problem leads to the household demand functions of commodities, leisure time,  $M$  and  $A$ , all depending on policy parameters and government expenditures. Although it is expected that an increase in tax rates tends to reduce  $X$  and increase  $Y$ , the comparative static analysis provides no deterministic signs. The marginal effects of policy parameters on  $l$ ,  $M$  and  $A$  are ambiguous as well since changes in policy parameters cause the budget line to shift in a unparallel manner.

### MARGINAL SOCIAL DAMAGE

The marginal social damage ( $MSD$ ) is typically defined as the marginal monetary loss of utility due to the consumption of dirty goods. Bovenberg and Mooij (1994a), for example, define  $MSD$  as:

$$MSD^B = - \left[ \frac{1}{\lambda} N \frac{\partial U}{\partial Q} \frac{\partial Q}{\partial(NX)} \right], \quad (3)$$

where  $N$  is the total number of households and  $\lambda$  the marginal utility of income.

Williams (2003) and Pang and Shaw (2007) define  $MSD$  as equations (4) and (5), respectively.

$$MSD^W = \frac{1}{\lambda} \frac{\partial U}{\partial H} \frac{\partial H}{\partial Q} - \frac{\partial S}{\partial Q}, \quad (4)$$

$$MSD^P = - \left( \frac{1}{\lambda} \frac{\partial U}{\partial S} - 1 \right) \frac{\partial S}{\partial Q} \frac{\partial Q}{\partial X}, \quad (5)$$

where  $H$  represents health condition, one of the utility determinants;  $Q = Q(X)$  ( $= \bar{Q} - X$  in Williams 2003) and  $Q_X < 0$ .

Note that the first term in equations (4) and (5) represents the monetary utility loss due to illness, and the second term the opportunity cost of sickness. The common problems with equations (4) and (5) are two fold: (a) The second term is not derived

directly from the associated utility function. (b) The second term implicitly assume that illness of one hour will lead to a loss of one hour available for either work or leisure (i.e.,  $\partial L / \partial S = \partial l / \partial S = -1$ ) and the normalized wage rate (= \$1) is used to value the loss of time for leisure and work due to illness. In general, this is not right since the monetary value that an individual places on leisure time may differ from that on work time<sup>4</sup>.

Accordingly, an accurate measurement of *MSD*, based on the utility function mentioned above, is expressed as follows:

$$MSD = -\frac{1}{\lambda} \left( \frac{\partial U}{\partial S} + U_{v_l} \frac{dl}{dS} \right) \frac{\partial S}{\partial Q} \frac{\partial Q}{\partial X}, \quad (6)$$

where  $dl / dS = -(1 + dL / dl) > 0$ , implying  $dL / dl < -1$ .<sup>5</sup>

Equation (6) implies that *MSD*, expected to be positive in practice, will be less than those highlighted by Equations (4) and (5). The dividends will, therefore, be weakened to some extent. Furthermore, sickness time tends to change in the same direction as leisure time, but in opposite way with work time. Pang and Shaw's (2007) simulation results revealed a side-by-side increase in *l* and *L* with increasing environmental tax, that obviously violate the above conditions.

## DIVIDEND DECOMPOSITION

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<sup>4</sup> Given the utility function  $U = U(v(X, Y, l), S, G)$ , the marginal rate of substitution between illness and leisure is  $MRS_{Sl} \equiv dl / dS = -U_S / U_{v_l} > 0$ . Using the time constraint,  $T = L + l + S$ , one obtains  $dl / dS = -(1 + dL / dl) > 0$ , implying  $dL / dl < -1$ . This implies the individual's marginal valuation of time for different purposes is different from one to the other.

<sup>5</sup> Totally differentiating the utility function  $U = U(v(X, Y, l), S, G_a)$  and then dividing both sides by  $dX$  and  $\lambda$  will lead to equation (6).



To decompose the dividends, additional information other than  $MSD$  is required, including mainly production technology and government budget constraint. An *ad hoc* production function exhibiting constant returns to scale (see equation (7)) is imposed here, while the government budget constraint is given by equation (8).

$$\sum_i L_i = X + Y + M + A + G \quad i = X, Y, M, A, G \quad (7)$$

$$G = t_x X + t_L \left( \sum_i L_i \right) - sA \quad (8)$$

where  $G = G_a + G_b + G_c$ . Note that tax revenue neutrality requires  $dG = 0$  while allowing all policy parameters to change<sup>6</sup>.

Following the traditional approach, the dividends of environmental tax could be decomposed into several components. It can be shown that the monetary value of the marginal welfare of environmental tax is as follows (see Appendix for the proof):

$$\frac{1}{\lambda} \frac{dU}{d\tau_x} = (-\partial X / \partial t_x) \left( MSD - t_x + [1 + (1 - t_L) \frac{\partial l}{\partial S}] \frac{\partial S}{\partial Q} \frac{\partial Q}{\partial E} \frac{\partial E}{\partial X} \right) \quad (9)$$

$$+ MWC \cdot (X + t_x \cdot \partial X / \partial t_x) \quad (10)$$

$$+ [-(MWC + 1)t_L (\partial l / \partial t_x)] \quad (11)$$

$$- [(MWC \cdot ((\partial l / \partial Q) + S_Q) + (\partial l / \partial Q)] t_L Q_E E_X (\partial X / \partial t_x) \quad (12)$$

$$+ [-t_L \cdot (1 + S_M (1 + MWC)) \cdot (\partial M / \partial t_x + M_Q Q_E E_X (\partial X / \partial t_x))] \quad (13)$$

$$+ [-(t_L S_A + s)(1 + MWC)(\partial A / \partial t_x + \partial A_Q Q_E E_X \cdot (\partial X / \partial t_x))], \quad (14)$$

where  $MWC$ , given by equation (15), represents the marginal welfare cost (or the excess burden) of labor income tax.

$$MWC = \Psi / \Omega > 0, \quad (15)$$

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<sup>6</sup> It is important to distinguish ex-ante neutrality from ex-post neutrality, particularly when implementing simulations. Based on ex-ante neutrality, Pang and Shaw's (2007) simulation results did not guarantee  $dG = 0$ .

where<sup>7</sup>

$$\begin{aligned} \Psi = & t_L \cdot \left( \frac{\partial l}{\partial t_L} + \frac{\partial l}{\partial Q} Q_E E_X \frac{\partial X}{\partial t_L} \right) + \left( MSD - t_x + [1 + (1 - t_L) \frac{\partial l}{\partial S}] \frac{\partial S}{\partial Q} \frac{\partial Q}{\partial E} \frac{\partial E}{\partial X} \right) \frac{\partial X}{\partial t_L} \\ & + [t_L \cdot (S_M + 1)] \cdot \left( \frac{\partial M}{\partial t_L} + M_Q Q_E E_X \frac{\partial X}{\partial t_L} \right) + (t_L S_A + s) \left( \frac{\partial A}{\partial t_L} + \frac{\partial A}{\partial Q} Q_E E_X \frac{\partial X}{\partial t_L} \right); \end{aligned} \quad (16)$$

and

$$\begin{aligned} \Omega = & (T - l - S) + t_x \frac{\partial X}{\partial t_L} - t_L \frac{\partial l}{\partial Q} Q_E E_X \frac{\partial X}{\partial t_L} - t_L \frac{\partial l}{\partial t_L} - t_L S_Q Q_E E_X \frac{\partial X}{\partial t_L} \\ & - (t_L S_A + s) \left( \frac{\partial A}{\partial t_L} + \frac{\partial A}{\partial Q} Q_E E_X \frac{\partial X}{\partial t_L} \right) - t_L S_M \left( \frac{\partial M}{\partial t_L} + M_Q Q_E E_X \frac{\partial X}{\partial t_L} \right). \end{aligned} \quad (17)$$

The RHS of equation (9) is known as the Pigouvian effect (*PE*). The component structure is different from earlier cases. Note that *PE* will be positive only when *MSD* is sufficiently large. The implication is that the first dividend might fail in case *MSD* is sufficiently small.<sup>8</sup>

The revenue-recycling effect (*RE*) is represented by equation (10), that is positive as predicted by previous studies, provided  $\partial X / \partial t_x < 0$ .

The tax-interaction effect (*IE*) is represented by equation (11). Bovenberg and Mooij (1994) pointed out that *IE* could be negative if  $\partial l / \partial t_x > 0$ . This is supported by the simulation results reported in Pang and Shaw (2007) and this paper, to be shown later.

The benefit-side tax-interaction effect (*IE<sup>B</sup>*) is given by equation (12). According to Williams (2002, 2003), this term is in principle indeterminate in sign, depending, as

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<sup>7</sup>  $\Psi$  and  $\Omega$  represents, respectively, the marginal welfare cost and marginal tax revenue of labor income tax rate.

<sup>8</sup> Although not show in this paper, examples of this kind are available from the authors.

shown by equation (12), on  $\text{sign}\{(\partial l / \partial Q) + S_e\}$ . In general,  $\partial l / \partial Q < |S_e|^9$ , implying a positive  $IE^B$ . On the contrary, Pang and Shaw's (2007) reported a negative value in their simulation.

The mitigation-based tax-interaction effect ( $IE^M$ ), given by equation (13), differs slightly from that originally identified by Pang and Shaw (2007)<sup>10</sup>. Although they reported a positive value that is large enough to offset  $IE$ ,  $IE^M$  is in general indeterminate in sign. It is positive only if  $|S_M(1 + MWC)| < 1$  and  $\partial M / \partial t_x < 0$ . In contrast,  $IE^M$  might become negative if the marginal productivity of the medical treatment expenditure (i.e.,  $|S_M|$ ) is sufficiently large.

The prevention-based tax-interaction effect ( $IE^A$ ) is newly obtained here and given by equation (14). The effect is unambiguously negative if  $\partial A / \partial t_x < 0$  and  $s = 0$ . Furthermore,  $IE^A$  is increasing in absolute value with the marginal productivity of the prevention expenditure (i.e.,  $|\partial S / \partial A|$ ). In other words, the existence of the prevention expenditure tends to reduce the overall second dividend since, just as the medical treatment expenditure could reduce sickness, so does the prevention expenditure ( $\because \partial S / \partial A < 0$ ). Nevertheless, the subsidy to the prevention expenditure could mitigate the negative effects since the subsidy induces more  $A$  and, therefore, reduces  $|\partial S / \partial A|$ .

## AN ALTERNATIVE APPROACH AND SIMULATION RESULTS

To facilitate simulation, a social planning approach is adopted here, in which environmental quality is endogenized through a newly specified environmental

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<sup>9</sup> This is implied by the time constraint.

<sup>10</sup> The original  $IE^M$  derived by Pang and Shaw (2007) is identical to  $[-t_L S_M(1 + MWC) \cdot (\partial M / \partial t_x + M_Q Q_E E_X (\partial X / \partial t_x))]$ , that could be positive if and only if  $\partial M / \partial t_x > 0$  and sufficiently large. Nevertheless, this is rather unlikely.

quality function and an emission function, and specific functional form is specified for all relevant functions. Hence, the social planner is assumed to maximize Equation (18) subject to Equations (19) ~ (25).

$$(a) \text{ Utility function: } U = v - S + 10 \log(G),^{11} \quad (18)$$

where  $v = [\gamma C^{-\rho} + (1-\gamma)l^{-\rho}]^{-\frac{1}{\rho}}$ ,  $C = [\beta X^{-\zeta} + (1-\beta)Y^{-\zeta}]^{-\frac{1}{\zeta}}$ , with parameters given as  $\gamma = 0.836$ ,  $\rho = -0.167$ ,  $\beta = 0.667$  and  $\zeta = -0.5$ .

$$(b) \text{ Health production: } S = \frac{24}{1 + e^{1+0.08Q+2.7M+3.0A}} \quad (19)$$

$$(c) \text{ Environmental quality function: } Q = 12 - E \quad (20)$$

$$(d) \text{ Emission function: } E = X \quad (21)$$

$$(e) \text{ Production function: } L = X + Y + M + A + G \quad (22)$$

$$(f) \text{ Time constraint: } T = l + L + S \quad (23)$$

$$(g) \text{ Tax revenues: } G = t_X X + t_L L \quad (24)$$

(h) Household budget:

$$(1-t_L)(T-l-S) + t_L M = P_X(1+t_X)X + P_Y Y + M + (1-s)A \quad (25)$$

Simulations are conducted for the base case as well as the scenarios:

**Base case:**  $t_L = 0.4$ ,  $t_X = 0$ ,  $s = 0$ ,  $G_b = G_c = 0$ , and  $G = G_a = 9.178$ .

**Scenarios:**  $t_X$  increases by an interval of 0.019 until  $t_X = 0.551$ , while  $t_L$  decreases at an interval of 0.005 until  $t_L = 0.255$ . Tax revenues, remained the same as that in the base case throughout all scenarios, are used solely for public goods.

The differences of the scenario from the base case are reported in Figure 1 for commodities, Figure 2 for time allocation and environmental quality, and Figure 3 for

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<sup>11</sup> Adopted from Pang and Shaw (2007). Different functional forms were also considered and simulated, but not reported here.

the divergence of welfare from the base case. The findings are summarized as follows:

- (a) As expected, dirty goods consumption is depressed by increasing environmental tax, while clean goods consumption decreases initially, mainly due to income effect, and eventually increases.
- (b) Leisure time increases with environmental tax, while labor employment declines, consistent with the conditions associated with Equation (6). Illness time increases insignificantly, also consistent with the above expectations.
- (c) Environmental quality is improved, confirming the first dividend.
- (d) The difference of welfare between the base case and the scenarios is positive and increases initially with environmental tax, but begins to decline and eventually turns out to be negative when the tax rate is sufficiently high. Figure 3 indicates that there exists an optimal tax bundle  $(t_L^*, t_X^*)$  such that the welfare is maximized in the scenarios. This implies that the validity of the double dividends hypothesis to some extent depends on the tax bundle selected by the authorities.
- (e) Note that the marginal utility of income ( $\lambda$ ) decreases with environmental tax. Nevertheless, the conventional dividend decomposition approach usually assumes constant for  $\lambda$ . Consequently, this approach may generate estimate bias for the dividend components.

## **CONCLUDING REMARKS**

Both medical treatment expenditure and illness prevention expenditure are pervasively observed in the real world. Their health effects are different to some extent and so is their income tax deductibility. Failure to incorporate both expenditures in addressing the dividends of environmental tax tends to be incomplete.

The marginal social damage of consuming dirty goods was neither accurately defined nor correctly measured in literature since it ignores the fact that the consumer may value time for alternative uses differently. Assuming identical valuation of time allocation may also end up with biased estimates of the second dividend.

We modify the health production function by incorporating both medical treatment expenditure and illness prevention expenditure and tax deductibility, and redefine the marginal social damage of consuming dirty goods by incorporating both the disutility of illness and utility of leisure time. Following similar decomposition approach, a new source of dividends with environmental tax on dirty goods, named “prevention-based tax-interaction effect” ( $IE^A$ ), is derived, that, however, is negative in sign and weakens overall second dividend. The mitigation-based tax-interaction effect ( $IE^M$ ) might be negative as well, provided that the medical treatment expenditure being normal. In sum, whether or not the medical treatment expenditure and illness-prevention expenditure are normal goods play crucial role in signing  $IE^A$  as well as  $IE^M$ . Our model contends that both are more likely to be negative, and thus tend to weaken the overall dividends of the environmental tax on dirty goods.

Simulation results indicate that the difference of welfare between the base case and the scenarios is positive and increases initially with environmental tax, but begins to decline and eventually turns out to be negative when the tax rate is sufficiently high. Furthermore, there exists an optimal tax bundle  $(t_L^*, t_X^*)$  such that the welfare is maximized in the scenarios. This implies that the validity of the double dividends hypothesis to some extent depends on the tax bundle selected by the authorities.

While Logistic function is typically used to model the health production function (or the illness time function), it is not a good candidate when both medical treatment expenditure and illness prevention expenditure are taken into account, mainly because

the marginal rate of technical substitution between the two will be constant under Logistic specification and, therefore, corner solution is inevitable. Alternative functional form deserves consideration for future studies.

## Appendix: Decomposition of the dividends of environmental tax

1. Firstly, substitute the production constraint ( $L \equiv \sum_i L_i = X + Y + M + A + G_a$ ) into

time constraint ( $T = l + S + \sum_i L_i$ ,  $i = X, Y, M, A, G$ ). Equilibrium requires

$T^* = l^* + X^* + Y^* + M^* + A^* + G_a^* + S^*$ . Totally differentiating  $T^*$  and assuming

$dG_a = ds = 0$ , one obtains:

$$\begin{aligned}
 dT^* &= \frac{\partial l}{\partial t_x} dt_x + \frac{\partial l}{\partial t_L} dt_L + \frac{\partial l}{\partial Q} \frac{\partial Q}{\partial E} \frac{\partial E}{\partial X} \frac{\partial X}{\partial t_x} dt_x + \frac{\partial l}{\partial Q} \frac{\partial Q}{\partial E} \frac{\partial E}{\partial X} \frac{\partial X}{\partial t_L} dt_L + \frac{\partial S}{\partial M} \frac{\partial M}{\partial t_x} dt_x + \\
 &\frac{\partial S}{\partial M} \frac{\partial M}{\partial t_L} dt_L + \frac{\partial S}{\partial M} \frac{\partial M}{\partial Q} \frac{\partial Q}{\partial E} \frac{\partial E}{\partial X} \frac{\partial X}{\partial t_x} dt_x + \frac{\partial S}{\partial M} \frac{\partial M}{\partial Q} \frac{\partial Q}{\partial E} \frac{\partial E}{\partial X} \frac{\partial X}{\partial t_L} dt_L + \frac{\partial S}{\partial A} \frac{\partial A}{\partial t_x} dt_x + \frac{\partial S}{\partial A} \frac{\partial A}{\partial t_L} dt_L \\
 &+ \frac{\partial S}{\partial A} \frac{\partial A}{\partial Q} \frac{\partial Q}{\partial E} \frac{\partial E}{\partial X} \frac{\partial X}{\partial t_x} dt_x + \frac{\partial S}{\partial A} \frac{\partial A}{\partial Q} \frac{\partial Q}{\partial E} \frac{\partial E}{\partial X} \frac{\partial X}{\partial t_L} dt_L + \frac{\partial S}{\partial Q} \frac{\partial Q}{\partial E} \frac{\partial E}{\partial X} \frac{\partial X}{\partial t_x} dt_x + \frac{\partial S}{\partial Q} \frac{\partial Q}{\partial E} \frac{\partial E}{\partial X} \frac{\partial X}{\partial t_L} dt_L \\
 &+ \frac{\partial X}{\partial t_x} dt_x + \frac{\partial X}{\partial t_L} dt_L + \frac{\partial Y}{\partial t_x} dt_x + \frac{\partial Y}{\partial t_L} dt_L + \frac{\partial Y}{\partial Q} \frac{\partial Q}{\partial E} \frac{\partial E}{\partial X} \frac{\partial X}{\partial t_x} dt_x + \frac{\partial Y}{\partial Q} \frac{\partial Q}{\partial E} \frac{\partial E}{\partial X} \frac{\partial X}{\partial t_L} dt_L \\
 &+ \frac{\partial M}{\partial t_x} dt_x + \frac{\partial M}{\partial t_L} dt_L + \frac{\partial M}{\partial Q} \frac{\partial Q}{\partial E} \frac{\partial E}{\partial X} \frac{\partial X}{\partial t_x} dt_x + \frac{\partial M}{\partial Q} \frac{\partial Q}{\partial E} \frac{\partial E}{\partial X} \frac{\partial X}{\partial t_L} dt_L + \frac{\partial A}{\partial t_x} dt_x \\
 &+ \frac{\partial A}{\partial t_L} dt_L + \frac{\partial A}{\partial Q} \frac{\partial Q}{\partial E} \frac{\partial E}{\partial X} \frac{\partial X}{\partial t_x} dt_x + \frac{\partial A}{\partial Q} \frac{\partial Q}{\partial E} \frac{\partial E}{\partial X} \frac{\partial X}{\partial t_L} dt_L = 0
 \end{aligned}$$

Rearranging the above equation and dividing both sides by  $dt_x$  leads to the

following:

$$\begin{aligned}
 &\left[ \frac{\partial l}{\partial t_x} + \frac{\partial l}{\partial Q} \frac{\partial Q}{\partial E} \frac{\partial E}{\partial X} \frac{\partial X}{\partial t_x} + \frac{\partial S}{\partial M} \frac{\partial M}{\partial t_x} + \frac{\partial S}{\partial M} \frac{\partial M}{\partial Q} \frac{\partial Q}{\partial E} \frac{\partial E}{\partial X} \frac{\partial X}{\partial t_x} + \frac{\partial S}{\partial A} \frac{\partial A}{\partial t_x} \right. \\
 &+ \frac{\partial S}{\partial A} \frac{\partial A}{\partial Q} \frac{\partial Q}{\partial E} \frac{\partial E}{\partial X} \frac{\partial X}{\partial t_x} + \frac{\partial S}{\partial Q} \frac{\partial Q}{\partial E} \frac{\partial E}{\partial X} \frac{\partial X}{\partial t_x} + \frac{\partial X}{\partial t_x} + \frac{\partial Y}{\partial t_x} + \frac{\partial Y}{\partial Q} \frac{\partial Q}{\partial E} \frac{\partial E}{\partial X} \frac{\partial X}{\partial t_x} + \frac{\partial M}{\partial t_x} \\
 &+ \frac{\partial M}{\partial Q} \frac{\partial Q}{\partial E} \frac{\partial E}{\partial X} \frac{\partial X}{\partial t_x} + \frac{\partial A}{\partial t_x} + \left. \frac{\partial A}{\partial Q} \frac{\partial Q}{\partial E} \frac{\partial E}{\partial X} \frac{\partial X}{\partial t_x} \right] + \left[ \frac{\partial l}{\partial t_L} + \frac{\partial l}{\partial Q} \frac{\partial Q}{\partial E} \frac{\partial E}{\partial X} \frac{\partial X}{\partial t_L} + \frac{\partial S}{\partial M} \frac{\partial M}{\partial t_L} \right. \\
 &+ \frac{\partial S}{\partial M} \frac{\partial M}{\partial Q} \frac{\partial Q}{\partial E} \frac{\partial E}{\partial X} \frac{\partial X}{\partial t_L} + \frac{\partial S}{\partial A} \frac{\partial A}{\partial t_L} + \frac{\partial S}{\partial A} \frac{\partial A}{\partial Q} \frac{\partial Q}{\partial E} \frac{\partial E}{\partial X} \frac{\partial X}{\partial t_L} + \frac{\partial S}{\partial Q} \frac{\partial Q}{\partial E} \frac{\partial E}{\partial X} \frac{\partial X}{\partial t_L} + \frac{\partial X}{\partial t_L} + \frac{\partial Y}{\partial t_L} \\
 &+ \left. \frac{\partial Y}{\partial Q} \frac{\partial Q}{\partial E} \frac{\partial E}{\partial X} \frac{\partial X}{\partial t_L} + \frac{\partial M}{\partial t_L} + \frac{\partial M}{\partial Q} \frac{\partial Q}{\partial E} \frac{\partial E}{\partial X} \frac{\partial X}{\partial t_L} + \frac{\partial A}{\partial t_L} + \frac{\partial A}{\partial Q} \frac{\partial Q}{\partial E} \frac{\partial E}{\partial X} \frac{\partial X}{\partial t_L} \right] \frac{dt_L}{dt_x} = 0
 \end{aligned} \tag{A1}$$

2. The government budget constraint is given by:



$$G = t_x X + t_L \cdot (w \cdot \sum_i L_i) - sA = t_x X + t_L w \cdot (T - l - S) - sA =$$

$$t_x X + t_L w \cdot (T - l - S(M, A, Q)) - sA$$

Totally differentiating  $G$  and imposing the revenue neutrality (i.e.,  $dG = 0$ ) to get

$$dG = X dt_x + t_x \frac{\partial X}{\partial t_x} dt_x + t_x \frac{\partial X}{\partial t_L} dt_L + Tw \cdot dt_L + t_L w \cdot dT - l \cdot w dt_L - t_L w \frac{\partial l}{\partial t_x} dt_x$$

$$- t_L w \frac{\partial l}{\partial t_L} dt_L - t_L w \frac{\partial l}{\partial Q} \frac{\partial Q}{\partial E} \frac{\partial E}{\partial X} \frac{\partial X}{\partial t_x} dt_x - t_L \frac{\partial l}{\partial Q} \frac{\partial Q}{\partial E} \frac{\partial E}{\partial X} \frac{\partial X}{\partial t_L} dt_L - S w dt_L$$

$$- t_L w \frac{\partial S}{\partial M} \frac{\partial M}{\partial t_x} dt_x - t_L w \frac{\partial S}{\partial M} \frac{\partial M}{\partial t_L} dt_L - t_L w \frac{\partial S}{\partial M} \frac{\partial M}{\partial Q} \frac{\partial Q}{\partial E} \frac{\partial E}{\partial X} \frac{\partial X}{\partial t_x} dt_x$$

$$- t_L w \frac{\partial S}{\partial M} \frac{\partial M}{\partial Q} \frac{\partial Q}{\partial E} \frac{\partial E}{\partial X} \frac{\partial X}{\partial t_L} dt_L - t_L w \frac{\partial S}{\partial A} \frac{\partial A}{\partial t_x} dt_x - t_L w \frac{\partial S}{\partial A} \frac{\partial A}{\partial t_L} dt_L$$

$$- t_L w \frac{\partial S}{\partial A} \frac{\partial A}{\partial Q} \frac{\partial Q}{\partial E} \frac{\partial E}{\partial X} \frac{\partial X}{\partial t_x} dt_x - t_L w \frac{\partial S}{\partial A} \frac{\partial A}{\partial Q} \frac{\partial Q}{\partial E} \frac{\partial E}{\partial X} \frac{\partial X}{\partial t_L} dt_L$$

$$- t_L w \frac{\partial S}{\partial Q} \frac{\partial Q}{\partial E} \frac{\partial E}{\partial X} \frac{\partial X}{\partial t_x} dt_x - t_L w \frac{\partial S}{\partial Q} \frac{\partial Q}{\partial E} \frac{\partial E}{\partial X} \frac{\partial X}{\partial t_L} dt_L - Ads - s \frac{\partial A}{\partial t_x} dt_x$$

$$- s \frac{\partial A}{\partial t_L} dt_L - s \frac{\partial A}{\partial Q} \frac{\partial Q}{\partial E} \frac{\partial E}{\partial X} \frac{\partial X}{\partial t_x} dt_x - s \frac{\partial A}{\partial Q} \frac{\partial Q}{\partial E} \frac{\partial E}{\partial X} \frac{\partial X}{\partial t_L} dt_L = 0$$

Rearranging the above equation and divided both sides by  $dt_x$  leads to the

following:

$$[X + t_x \frac{\partial X}{\partial t_x} - t_L w \frac{\partial l}{\partial t_x} - t_L w \frac{\partial l}{\partial Q} \frac{\partial Q}{\partial E} \frac{\partial E}{\partial X} \frac{\partial X}{\partial t_x} - t_L w \frac{\partial S}{\partial M} \frac{\partial M}{\partial t_x} - w t_L \frac{\partial S}{\partial M} \frac{\partial M}{\partial Q} \frac{\partial Q}{\partial E} \frac{\partial E}{\partial X} \frac{\partial X}{\partial t_x}$$

$$- t_L w \frac{\partial S}{\partial A} \frac{\partial A}{\partial t_x} - t_L w \frac{\partial S}{\partial A} \frac{\partial A}{\partial Q} \frac{\partial Q}{\partial E} \frac{\partial E}{\partial X} \frac{\partial X}{\partial t_x} - t_L w \frac{\partial S}{\partial Q} \frac{\partial Q}{\partial E} \frac{\partial E}{\partial X} \frac{\partial X}{\partial t_x} - s \frac{\partial A}{\partial t_x} - s \frac{\partial A}{\partial Q} \frac{\partial Q}{\partial E} \frac{\partial E}{\partial X} \frac{\partial X}{\partial t_x}]$$

$$+ \frac{dt_L}{dt_x} [t_x \frac{\partial X}{\partial t_L} + wT - wl - wS - t_L w \frac{\partial l}{\partial t_L} - t_L w \frac{\partial l}{\partial Q} \frac{\partial Q}{\partial E} \frac{\partial E}{\partial X} \frac{\partial X}{\partial t_L} - t_L w \frac{\partial S}{\partial M} \frac{\partial M}{\partial t_L}$$

$$- t_L w \frac{\partial S}{\partial M} \frac{\partial M}{\partial Q} \frac{\partial Q}{\partial E} \frac{\partial E}{\partial X} \frac{\partial X}{\partial t_L} - t_L w \frac{\partial S}{\partial A} \frac{\partial A}{\partial t_L} - t_L w \frac{\partial S}{\partial A} \frac{\partial A}{\partial Q} \frac{\partial Q}{\partial E} \frac{\partial E}{\partial X} \frac{\partial X}{\partial t_x} - t_L w \frac{\partial S}{\partial Q} \frac{\partial Q}{\partial E} \frac{\partial E}{\partial X} \frac{\partial X}{\partial t_L}$$

$$- s \frac{\partial A}{\partial t_L} - s \frac{\partial A}{\partial Q} \frac{\partial Q}{\partial E} \frac{\partial E}{\partial X} \frac{\partial X}{\partial t_L}] = 0$$

Let  $w = 1$ . It is straightforward to get

$$\frac{dt_L}{dt_x} = -\frac{\phi}{\Omega}, \quad (A2)$$

where







$$\begin{aligned}
\frac{1}{\lambda} \frac{dU}{dt_x} &= \left[ \left( -\frac{\partial X}{\partial t_x} \right) \left( MSD - t_x + [1 + (1 - t_L) \frac{\partial l}{\partial S}] \frac{\partial S}{\partial Q} \frac{\partial Q}{\partial E} \frac{\partial E}{\partial X} \right) - t_L \left( \frac{\partial l}{\partial t_x} + \frac{\partial l}{\partial Q} \frac{\partial Q}{\partial E} \frac{\partial E}{\partial X} \frac{\partial X}{\partial t_x} \right) \right. \\
&\quad - t_L \left( 1 + \frac{\partial S}{\partial M} \right) \left( \frac{\partial M}{\partial t_x} + \frac{\partial M}{\partial Q} \frac{\partial Q}{\partial E} \frac{\partial E}{\partial X} \frac{\partial X}{\partial t_x} \right) - \left( t_L \frac{\partial S}{\partial A} + s \right) \left( \frac{\partial A}{\partial t_x} + \frac{\partial A}{\partial Q} \frac{\partial Q}{\partial E} \frac{\partial E}{\partial X} \frac{\partial X}{\partial t_x} \right) \left. \right] + MWC \left[ X + t_x \frac{\partial X}{\partial t_x} + \right. \\
&\quad - t_L \frac{\partial l}{\partial t_x} - t_L \frac{\partial l}{\partial Q} \frac{\partial Q}{\partial E} \frac{\partial E}{\partial X} \frac{\partial X}{\partial t_x} - t_L \frac{\partial S}{\partial M} \frac{\partial M}{\partial t_x} - t_L \frac{\partial S}{\partial M} \frac{\partial M}{\partial Q} \frac{\partial Q}{\partial E} \frac{\partial E}{\partial X} \frac{\partial X}{\partial t_x} - t_L \frac{\partial S}{\partial A} \frac{\partial A}{\partial t_x} - t_L \frac{\partial S}{\partial A} \frac{\partial A}{\partial Q} \frac{\partial Q}{\partial E} \frac{\partial E}{\partial X} \frac{\partial X}{\partial t_x} \\
&\quad \left. - t_L \frac{\partial S}{\partial Q} \frac{\partial Q}{\partial E} \frac{\partial E}{\partial X} \frac{\partial X}{\partial t_x} - s \left( \frac{\partial A}{\partial t_x} + \frac{\partial A}{\partial Q} \frac{\partial Q}{\partial E} \frac{\partial E}{\partial X} \frac{\partial X}{\partial t_x} \right) \right] \\
&= \left( -\frac{\partial X}{\partial t_x} \right) \left( MSD - t_x + [1 + (1 - t_L) \frac{\partial l}{\partial S}] \frac{\partial S}{\partial Q} \frac{\partial Q}{\partial E} \frac{\partial E}{\partial X} \right) \\
&\quad + MWC \left( X + t_x \frac{\partial X}{\partial t_x} \right) \\
&\quad - (MWC + 1) t_L \frac{\partial l}{\partial t_x} \\
&\quad - \left[ (MWC + 1) \cdot t_L \frac{\partial l}{\partial Q} + MWC \cdot t_L \frac{\partial S}{\partial Q} \right] \frac{\partial Q}{\partial E} \frac{\partial E}{\partial X} \frac{\partial X}{\partial t_x} \\
&\quad - t_L \left[ 1 + \frac{\partial S}{\partial M} (1 + MWC) \right] \left( \frac{\partial M}{\partial t_x} + \frac{\partial M}{\partial Q} \frac{\partial Q}{\partial E} \frac{\partial E}{\partial X} \frac{\partial X}{\partial t_x} \right) \\
&\quad - (MWC + 1) \left( t_L \frac{\partial S}{\partial A} + s \right) \left( \frac{\partial A}{\partial t_x} + \frac{\partial A}{\partial Q} \frac{\partial Q}{\partial E} \frac{\partial E}{\partial X} \frac{\partial X}{\partial t_x} \right)
\end{aligned}$$

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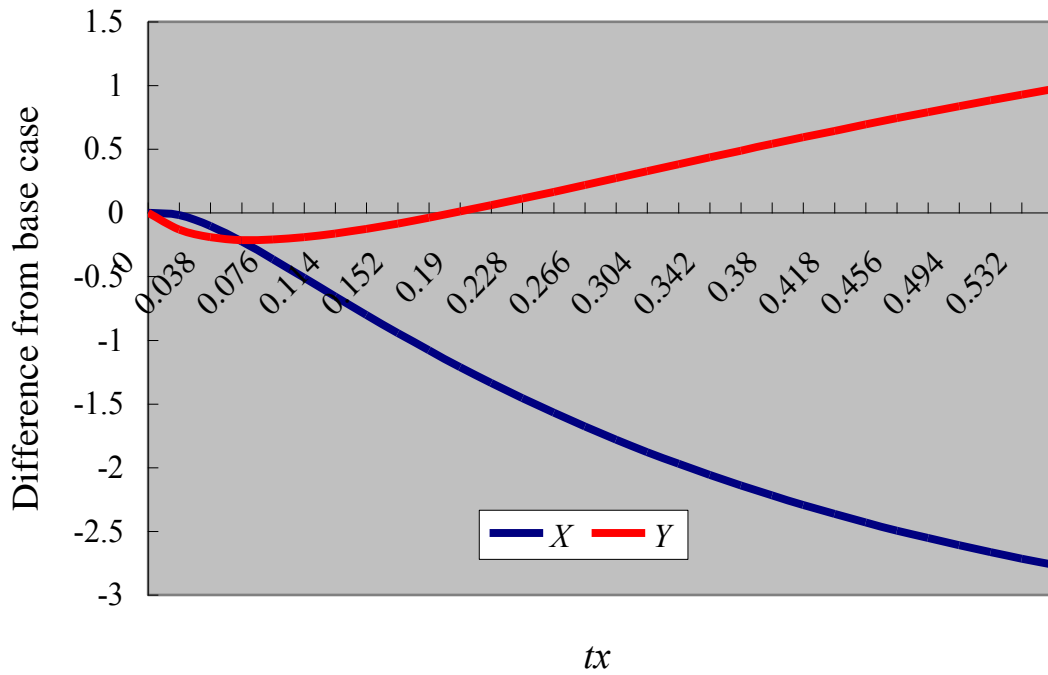


Figure 1. Effects of environmental tax on commodities: difference from base case

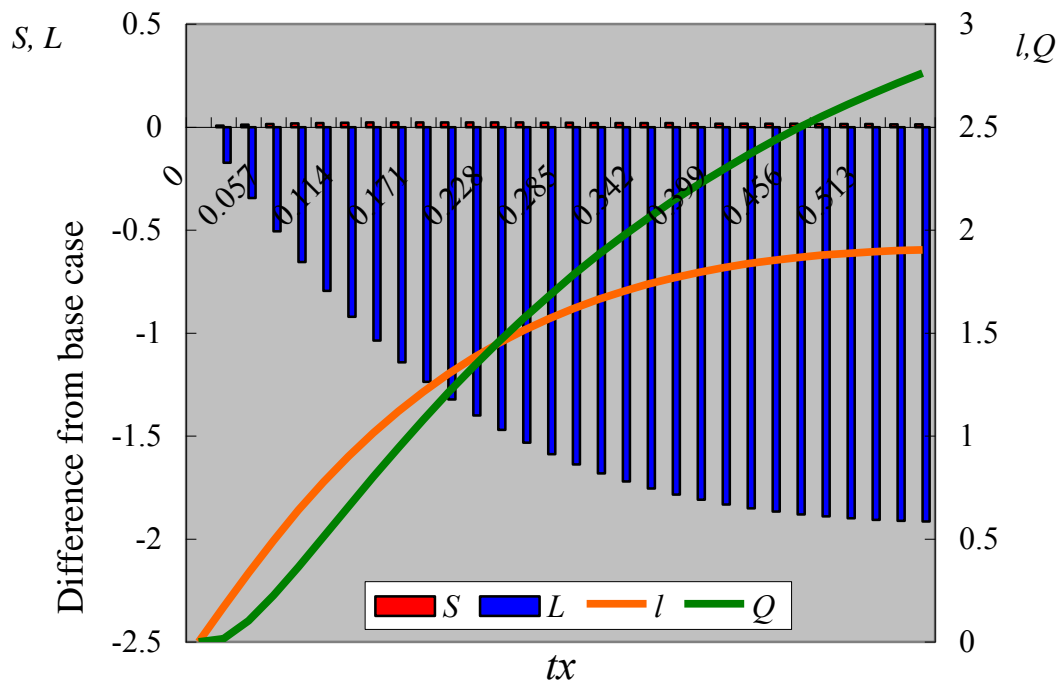


Figure 2. Effects of environmental tax on time allocation and environmental quality: difference from base case



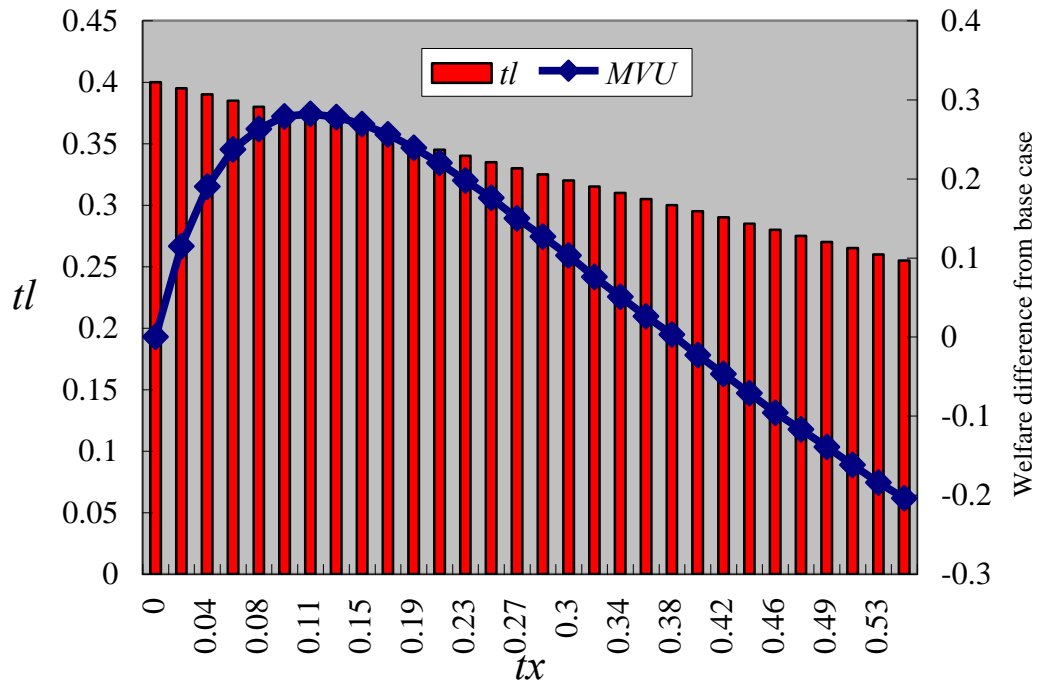


Figure 3. Effects of environmental tax on welfare: difference from base case