Is the "Beckerian" quantity-quality tradeoff regarding the offspring always true?*

Éva Berde - Izabella Kuncz

1 Introduction

Modelling the consequences of the individual choice in childbearing decision and taking into consideration the diminishing fertility rate we concentrate on the two main theoretical directions used in fertility analysis: the "Beckerian" theory and the "synthesis" framework. Although evidence-based studies could be found verifying either theory, we are in favour of the latter one. Comparing the fertility trends of different developed countries – with fertility rates close to replacement level (such as Sweden) and with very low fertility rates (such as Austria) – supports the "synthesis" approach.

The original works of Becker (1960), Becker and Levis (1973) and Willis (1973) unambiguously support the idea of quantity – quality tradeoff. However almost at the same time when these papers were published, Adelman (1963), Freedman (1963), Silver (1965), Freedman and Coombs (1966a 1966b) and Easterlin (1968) found mixed empirical evidence. They also highlighted among other factors the role of women's opportunity cost of childbearing, the differences in preferences and the cultural heritage in the countries. The recent literature takes into consideration many other possible causes of women's fertility decisions, like female employment, economics downturn, individual comfort, intertemporal influence of peers, the joy of having children, etc. (For example

*Our research was financed by Pallas Athéné Domus Animae Foundation. We are grateful for András Semjén's help in the the language compilation of the present text.

see the studies of Dang and Rogers (2016), Colleran et al. (2015), Sobotka et al. (2011), Luci and Thevenon (2010), Ellis (2008), Black et al. (2005), Kaplan (1994).)

One way to decide the relevance of quantity – quality choice is to find out the relationship between the families' human investments into their children and the fertility rate of the same families. Data usually do not allow this kind of analysis, because the determination of the level of human investment is a very difficult task. The pioneering work of Lee and Mason (2010), using Lee and Mason (2011) data, found a negative and significant correlation between the number of children and the human capital investment into children. In our paper we argue that their analysis could not lead to a precise interpretation because they deal only with average data.

We show that by calculating the correlation between the human investment and fertility rate using a continuous relationship between the two variables, a modified result is obtained. In the first part of our paper we build an overlapping generation (OLG) model, similar to that in Lee and Mason (2010), but with altered human capital elasticity. Additionally we include two other modifications: i) four overlapping generations instead of three are introduced, and ii) transfer to oldest generation is provided by the second and the third generations. We seek answers to the following question: How much consumption is provided for the youngest and the oldest generations as a function of the fertility and survival rates?

2 The model

2.1 Demography

In this economy four generations live together: children (N_0), younger workers (N_1), older workers (N_2) and elderly (N_3). The younger workers are in their childbearing period and each of them has F_t child on average. One period later children become active on the labour market as younger workers, and younger workers will become older workers. From older workers at period $(t-1)$ $100s_t$ percentage becomes elderly at period t. At the end of their last period they die.

$$N_t^0 = F_t \cdot N_t^1$$

$$N_t^1 = N_{t-1}^0$$

$$N_t^2 = N_{t-1}^1$$

$$N_t^3 = s_t \cdot N_{t-1}^2$$

The total population is $N_t = N_t^0 + N_t^1 + N_t^2 + N_t^3$ in period t.

2.2 Wages, human capital and consumption

The annual wage of young workers depends on human capital investment by the two worker age groups during the preceding period:

$$W_t^1 = g(H_t) \tag{2.1}$$

where $g'(H_t) > 0$ and $g''(H_t) < 0$. The annual wage of older workers depends on the young workers' wage.

$$W_t^2 = f(W_t^1) \tag{2.2}$$

The amount of human capital invested in one child depends on the workers wages and the human capital investment rate (h_{t-1}). The investment rate can be higher, if the workers have to support less children:

$$H_t = h(F_{t-1}) \cdot (W_{t-1}^1 + W_{t-1}^2) \tag{2.3}$$

where $h'(F_{t-1}) < 0$ and $h''(F_{t-1}) > 0$.

The workers have to finance the consumption of all age groups and the human capital of their children, so the budget constraint is

$$W_t^1 \cdot N_t^1 + W_t^2 \cdot N_t^2 \geq C_t^0 \cdot N_t^0 + C_t^1 \cdot N_t^1 + C_t^2 \cdot N_t^2 + C_t^3 \cdot N_t^3 + H_{t+1} N_t^0$$

The human capital investment is

$$H_t = \alpha \cdot F_{t-1}^{\beta_{t-1}} \cdot (W_{t-1}^1 + W_{t-1}^2) \tag{2.4}$$

where $\beta_{t-1} < 0$.

$$W_t^1 = \gamma \cdot H_t^\delta = \gamma \cdot \Big(\alpha \cdot F_{t-1}^{\beta_{t-1}} \cdot (W_{t-1}^1 + W_{t-1}^2)\Big)^\delta \tag{2.5}$$

where $0 < \delta < 1$ and $\gamma > 0$.

$$W_t^2 = \varphi \cdot W_t^1 \tag{2.6}$$

where $\varphi > 0$.

Transfers to the elderly (TR_t) are paid from the labour income of the workers. If the elderly age group is not employed, this transfer is their only income to spend on their consumption. The transfer is the 25 % of the working generation's wage after deducing the human capital investment (disposable income in this model). The percentage is modified upon the size of fertility and survival rate, and the original 0.25 ratios for the two working groups apply only in the theoretical case when generations are distributed equally.

$$\frac{TR_t}{N_t^3} = \psi_t \cdot \Big(W_t^1 \frac{N_t^1}{N_t^3} + W_t^2 \frac{N_t^2}{N_t^3} - H_{t+1} \cdot \frac{N_t^0}{N_t^3}\Big)$$

where

$$\psi_t = \min\Big(0.25; \quad 0.25 \cdot \frac{0.25}{\frac{N_t^0}{N_t}} \cdot s_t^2 \cdot \frac{1}{F_t^2}\Big)$$

The children's consumption is financed by the two worker-age groups. The members of the younger workers give them the $100\mu_t$ percentage of the disposable income and the those belong to older group provide $100\nu_t$ percentage of their income.

The consumption of a child is

$$C_t^0 = \mu_t\Big(W_t^1 \cdot \frac{N_t^1}{N_t^0} - \alpha \cdot F_t^{\beta_t} \cdot W_t^1\Big) + \nu_t\Big(W_t^2 \cdot \frac{N_t^2}{N_t^0} - \alpha \cdot F_t^{\beta_t} \cdot W_t^2\Big) \tag{2.7}$$

Workers can spend the remainder on their own consumption. The per capita values are

$$C_t^1 = (1 - \mu_t - \psi_t)\Big(W_t^1 - \alpha \cdot F_t^{\beta_t} \cdot W_t^1 \cdot \frac{N_t^0}{N_t^1}\Big) \tag{2.8}$$

$$C_t^2 = (1 - \nu_t - \psi_t)\Big(W_t^2 - \alpha \cdot F_t^{\beta_t} \cdot W_t^2 \cdot \frac{N_t^0}{N_t^2}\Big) \tag{2.9}$$

2.3 Dynamics

Using the above equations of the model, the dynamics of the wages - the ratio of the future and current wages - is as follows:

$$\begin{aligned} \frac{W_{t+1}^1}{W_t^1} &= \frac{\gamma \cdot \Big(\alpha \cdot F_t^{\beta_t} \cdot (W_t^1 + W_t^2)\Big)^{\delta}}{W_t^1} = \gamma \cdot \Big(\alpha \cdot F_t^{\beta_t}\Big)^{\delta} \cdot \Big(\frac{W_t^1 + \varphi \cdot W_t^1}{W_t^1}\Big)^{\delta} = \\ &= \gamma \cdot \Big(\alpha \cdot F_t^{\beta_t}\Big)^{\delta} \cdot (1+\varphi)^{\delta} \cdot {W_t^1}^{\delta-1} \end{aligned} \tag{2.10}$$

$$\frac{W_{t+1}^2}{W_t^2} = \frac{\varphi \cdot W_{t+1}^1}{\varphi \cdot W_t^1} \tag{2.11}$$

If the wages do not change from one period to the other, the economy is in steady state. In steady state the level of younger workers' wages is

$$W_t^{1^*} = \Big(\frac{1}{\gamma \cdot \alpha^{\delta}}\Big)^{\frac{1}{\delta-1}} \cdot F_t^{\frac{\beta_t \cdot \delta}{1-\delta}} \cdot \Big(\frac{1}{(1+\varphi)^{\delta}}\Big)^{\frac{1}{\delta-1}}. \tag{2.12}$$

3 Estimate of elasticity

Lee and Mason (2010), using NTA data, found a negative and significant correlation between the number of children and the human capital investment into children, but they dealt only with average data.

In our estimation of the elasticity (β) we used the UNESCO database for the human capital (average year of schooling) and the World Bank database (fertility rates). In total we used data from 98 countries. The countries were split into two groups: with lower fertility (total fertility rate under 2.1, the replacement level) and with higher fertility (total fertility rate above 2.1). Then separate ordinary least square (OLS) regression was calculated for the two groups of countries. Significantly higher regression coefficient (in absolute value) was found in the higher fertility case.$\beta = -0.8348$ versus $\beta = -0.273$.[1] We supposed a linear relationship between β and the fertility rate (F). Then β is obtained as:

$$\beta_t = 0.2221 - 0.6026 \cdot F_t.$$

4 Simulation

With the help of the previous equations we simulated the different paths of the economy . Starting from the steady state in the first period, we calculated 15 periods of the model (every period equals to 20 years). Table 1 shows our parameter values.

[1]The regression in the higher fertility case is $\ln(H) = 2.891532 - 0.8348 \cdot \ln(\mathrm{TFR})$ and in the low fertility case is $\ln(H) = 2.4987 - 0.273 \cdot \ln(\mathrm{TFR})$, where H is the average year of schooling, and TFR is the total fertiltiy rate. Both are significant at 0.1 level.

Table 1: Parameter values

Value	**Source**
$\alpha = 0.1$	Lee and Mason (2010)
$\gamma = 1$	Lee and Mason (2010)
$\delta = 0.33$	Mankiw, Romer, Weil (1992), Lee and Mason (2010)
$\varphi = 1.1$	NTA database[2]
$\mu = 0.2$	See text
$\nu = 0.2$	See text

In addition to the parameter values of the Table 1 we assumed that workers spend the 20% of their disposable income on the children's consumption (The additional approximately 5 % of income spent on the human capital investment of children adds up to the quarter of their income)

Examining a lot of cases (low fertility, high fertility, diminishing fertility, rising fertility, low survival, high survival, and so on), we show some of the most interesting paths. Three typical simulation results could be seen in the Appendix.

[2]Using NTA income data of 40 countries we found that an older worker's (41-60- year-old) labour income is in average approximately 10 percent higher than the income of a younger worker (21-40-year-old).

5 Results

Based on the simulation results of our model we can conclude that lower fertility causes higher human capital investment, which results in higher wages. However when fertility is too low (usually less than 1.4 in our model) the higher wage is not enough for the proper transfer to the elderly. In low fertility cases after a delay period the retired can not consume enough receiving only the transfer. They are better off if they work at least part time and receive a certain amount of wage. The higher the survival rate the sooner the critical level of the transfer is reached.

The increase of the fertility rate from the lowest low level (between 1.2 and 1.4) could increase the transfer to the elderly, but the elderly still need to work if the increase of the fertility rate is modest, and fertility does not reach the replacement level. If the fertility rate decreases constantly from an extremely high level (above 8) then even the high survival rate does not result in unacceptable transfers while the fertility rate is still above the replacement level. When fertility falls below the replacement level, critical amount of the transfer is experienced soon.

In the opposite direction, slow increase of the fertility rate from below the replacement level and high survival rate meanwhile does not increase the transfers properly in the first few periods. Elderly people must still work.

In the context of child rearing, when fertility is high (usually more than 5) there is not enough money for non-workers. Neither transfer for elderly, nor the consumption of the children reach the adequate level, and furthermore the human capital investment to children is extremely low.

The different paths of our model show that there are difficulties if the fertility rate is extremely high or extremely low. In both cases elderly could improve their own situation if they work and receive labour income. Children can consume at a proper level only if the fertility rate is not remarkably high.

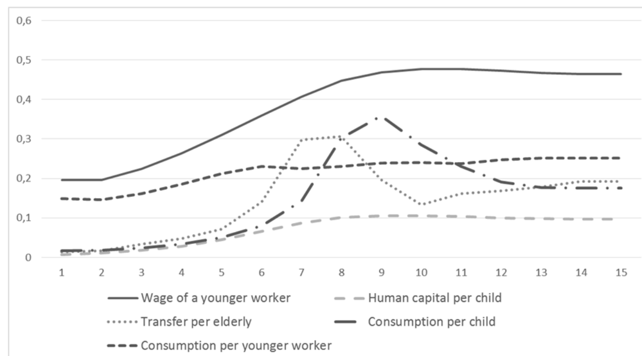
Appendix

Case 1

Period	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15
F[3]	3.00	2.66	2.31	1.97	1.63	1.29	0.94	0.60	0.70	0.80	0.90	1.00	1.00	1.00	1.00
s	0.6	0.6	0.8	0.8	0.8	0.9	0.9	0.9	0.9	0.9	0.9	0.9	0.9	0.9	0.9

Table 2: First decreasing then slightly increasing fertility rates

(a) Generations' shares



(b) Consumptions and transfers

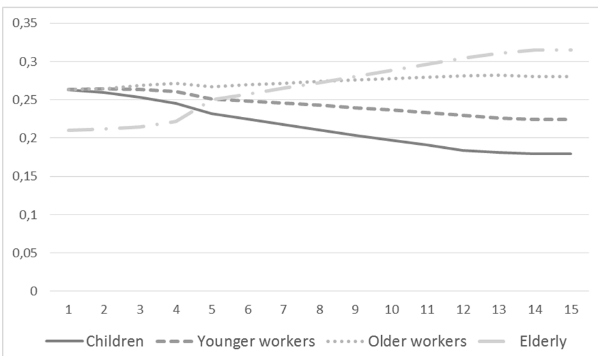
Figure 1: First decreasing then slightly increasing fertility rates

[3]F is the fertility of an average young worker, which equals to the half of the female fertility. In our model there is no death until the end of 3rd period, and replacement level belongs to the F=1.

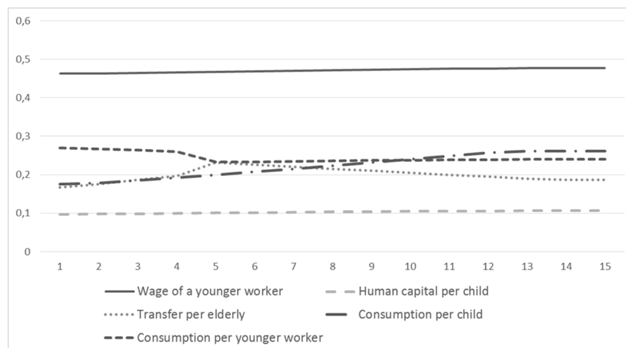
Case 2

Period	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15
F	1.00	0.98	0.96	0.94	0.92	0.90	0.89	0.87	0.85	0.83	0.82	0.80	0.80	0.80	0.80
s	0.8	0.8	0.8	0.8	0.9	0.9	0.9	0.9	0.9	0.9	0.9	0.9	0.9	0.9	0.9

Table 3: Continuously decreasing fertility rates



(a) Generations' shares



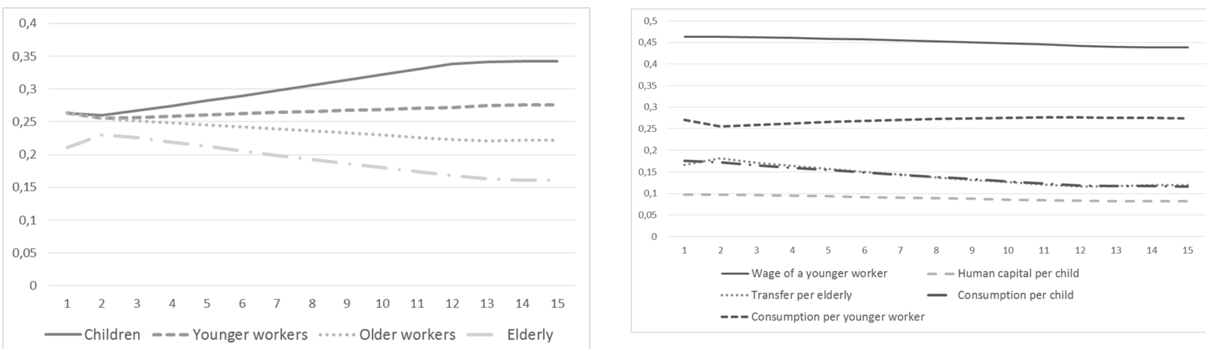
(b) Consumptions and transfers

Figure 2: Continuously decreasing fertility rates

Case 3

Period	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15
F	1.00	1.02	1.04	1.06	1.08	1.10	1.13	1.15	1.17	1.20	1.22	1.24	1.24	1.24	1.24
s	0.8	0.9	0.9	0.9	0.9	0.9	0.9	0.9	0.9	0.9	0.9	0.9	0.9	0.9	0.9

Table 4: Continuously increasing fertility rates



(a) Generations' shares

(b) Consumptions and transfers

Figure 3: Continuously increasing fertility rates

Bibliography

Adelman, I. (1963). An econometric analysis of population growth. The American Economic Review, 314-339.

Becker, G. S. (1960). An economic analysis of fertility. In Demographic and economic change in developed countries (pp. 209-240). Columbia University Press.

Becker, G. S., & Lewis, H. G. (1973). On the Interaction between the Quantity and Quality of Children." journal of Political Economy. 81: 2, pp. S279-88.

Black, S. E., Devereux, P. J., & Salvanes, K. G. (2005). The more the merrier? The effect of family size and birth order on children's education. The Quarterly Journal of Economics, 669-700.

Colleran, H., Jasienska, G., Nenko, I., Galbarczyk, A., & Mace, R. (2015). Fertility decline and the changing dynamics of wealth, status and inequality. Proceedings of the Royal Society of London B: Biological Sciences, 282(1806), 20150287.

Dang, H. A., & Rogers, F. H. (2013). The decision to invest in child quality over quantity: household size and household investment in education in Vietnam. World Bank Policy Research Working Paper, (6487).

Easterlin, R. A. (1973). Does money buy happiness?. The public interest, (30), 3.

Ellis, J. (2008). Culture, fertility, and son preference (Doctoral dissertation, London School of Economics).

Freedman, D. (1963) The relation of economic status to fertility. The American Economic Review 53 (June):414-426.

Freedman, R., & Coombs, L. (1966a). Childspacing and family economic position. American Sociological Review, 631-648.

Freedman, R., & Coombs, L. (1966b). Economic considerations in family growth decisions. Population Studies, 20(2), 197-222.

Kaplan, H. (1994). Evolutionary and wealth flows theories of fertility: Empirical tests and new models. Population and development Review, 753-791.

Lee, R., & Mason, A. (2010). Fertility, human capital, and economic growth over the demographic transition. European Journal of Population/Revue européenne de Démographie, 26(2), 159-182.

Lee, R., & Mason, A. lead authors and editors (2011). Population Aging and the Generational Economy: A Global Perspective (Cheltenham, UK and Northampton, MA: Edward Elgar).

Luci, A., & Thévenon, O. (2010). Does economic development drive the fertility rebound in OECD countries?. National Transfer Accounts (NTA) project http://www.ntaccounts.org/web/nta/show

Mankiw, N. G., D. Romer, & D. N. Weil. (1992). A contribution to the empirics of economic growth. Quarterly Journal of Economics 107(May):407-437.

Silver, M. (1965). Births, marriages, and business cycles in the United States. The Journal of Political Economy, 237-255.

Sobotka, T., Skirbekk, V., & Philipov, D. (2011). Economic recession and fertility in the developed world. Population and development review, 37(2), 267-306.

Willis, R. J.. (1973). A New Approach to the Economic Theory of Fertility Behavior. Journal of Political Economy, 81(2), S14–S64. Retrieved from http://www.jstor.org/stable/1840411