Estimating a Banking-Macro Model for Europe Using a Multi-Regime VAR*

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Abstract

This paper introduces a Banking-Macro Model and estimates the linkages through a Multi-Regime VAR (MRVAR). We introduce a dynamic model which is akin to the Brunnermeier and Sannikov (BS) model (2010). The banking sector is exposed to instability due to adverse movements of asset prices and their impact on risk premia and credit spreads. In contrast to the standard model of the financial accelerator, exhibiting mean reversion, our model, similarly to BS (2010), exhibits local instability. Whereas the standard model leads, in terms of econometrics, to a one-regime VAR we argue for the use of a MRVAR. We estimate our model for EU countries with a MRVAR using a constructed financial stress index and industrial production for those countries. We undertake impulse-response studies with a MRVAR and explore regime dependency of shocks. We show that the shocks have asymmetric effects, depending on the growth regime of the economy, and on the size of the shocks. Small financial stress shocks may not matter, but large shocks are likely to have magnifying effects.

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1 Introduction

At the center of the large financial meltdown of the years 2007-2008 in the US was the banking system. As Reinhard and Rogoff (2009) and Gorton (2009, 2010) have demonstrated, the banking sector is often at the center of a financial crisis. Most crises have ended up as a meltdown of the banking sector, and the banking sector has usually exacerbated and amplified the crisis whatever origin it had. As Gorton (2010) shows, in the past, loan losses and bank runs where the conventional mechanisms by which the crises where triggered, but more recently, banking crises seem to be strongly related to adverse shocks in asset value losses and financial stress.

Important versions of such studies of the destabilizing effects of the banking sector were put foreward after the "great recession" of the years 2007-2009 in the US.[1] Not many of such studies can be found for Europe.[2] It is of great interest now whether such adverse feedback movements could currently also occur to the European banking system. We want to study how this destabilizing mechanisms might work in a model of a banking-macro link and apply this to an EU Data set that is now made available from IMF (2011).[3]

For the US there were recent studies that work with the financial accelerator to capture the financial-real linkage, but the destabilizing dynamics is not sufficiently captured in those models. Also, so far the financial accelerator theory has mainly been applied to firms and households. Bernanke, Gertler and Gilchrist (1999) have shown that the financial market can have amplifying effects. Yet, in the DSGE tradition there is only a locally magnifying effect, through collaterals. Collateral value rises at high level of economic activity, making credit available and cheap, and the reverse happens at low level of economic activity.

Technically, the models are solved through local linearizations about a unique and stable steady state, and the amplifying effects occur only with respect to deviations from the steady state. Also, mostly no debt dynamics is tracked.[4] The departure

[1] See Brunnermeier and Sannikov (2010, 2011), He et al (2008), and Adrian et al (2010).

[2] An interesting recent paper is Monnin and Jakipii (2010), who work with a distance of default model to study the instability of banking for selected EU countries.

[3] See the IMF's (2011) Financial Stability Index (FSI).

[4] Empirically the debt to asset value ratio is predicted to fall in the boom and rise in reces-

from the steady state is eventually mean reverting. Although the economy is accelerating, it will revert back to the steady state. Empirically, this is often shown in a one-regime VAR, see Gilchrist et al. (2009, 2010), Christensen and Dib (2008), and Del Negro et al. (2010).

As the meltdown of the years 2007-8 has demonstrated, shocks to banks seem to be destabilizing rather then mean reverting.[5] Important papers in this context are Brunnermeier (2009) and Brunnermeier and Pederson (2009), that show that banks often have to liquidate their capital, when asset prices get depressed and margin requirements in the money market rise, which forces the financial intermediaries to take a hair cut and to delever further, with another subsequent fall of asset prices reinforcing the downward trajectory. This has started new research on financial instability putting asset prices and their volatility at the center.[6]

Models attempting to capture such mechanisms often stress that the falling asset prices, generate by fire sales of assets by some intermediaries, have external effects on the financial industry. The possibility of a downward spiral then comes from interconnectedness, interlinkages and contagion. Such studies have started with Greenwald and Stiglitz (1996) and continued with Adrian et al. (2010), Gorton (2010), Geanakoplos (2010), Geanakoplos and Farmer (2009), and Brunnermeier and Sannikov (2010, 2011). Those papers argue that this dynamics will create an endogenous generated jump in risk which is usually triggered by large changes in asset price movements.

This process primarily works through the balance sheets of banks. Banks, in the

sions.This is for example empirically stated in Gilchrist et al. (2009). Yet, as Geanakoplos (2010) mentions, the empirical measure is distorted through the way the debt asset ratio is measured, namely as total assets over equity. Equity value rises in the boom and falls in a recession. A model of the interaction of asset prices and leveraging is presented by Semmler and Bernard (2011).

[5]Many students of the great depression developed the perception that locally destabilizing effects, arising from the banking sector, are missing in modern macroeconomic modeling. There are the earlier non-conventional studies by Kindleberger and Aliber (2005) and Minsky (1976, 1982) that view the role of credit as significantly amplifying forces. In Kindleberger it is the instability of credit, and in Minsky it is the way financing becomes de-linked from collaterals that contributes to a downward spiral once large real or financial shocks occur. This is surely an important tradition that captured many of the aspects of the banking-macro link.

[6]We may include here what has been called by Gorton (2010) the shadow banking system, such as investment firms, brokers and money market dealers. Those have been growing rapidly in the US in the last 15 to 20 years.

first instance, may have loan losses.[7] This may be arising from default of the firm or household sector, the foreign sector or resulting from sovereign debt. On the other hand, large shocks to asset prices and financial stress will affect banks – the asset and liability side of their balance sheets – impacting the trust and the availability of credit in the interbank money market. The fire sale of assets by some intermediaries, make the capital basis of others even weaker. As the financial stress rises, so will the risk premia, repo rates, the Ted spreads and credit spreads. This spillover effects to other intermediaries (as well as firms and households) creates what BS (2010) call endogenous risk. Following BS (2010), in a dynamic model of the banking sector, we show that such an unstable dynamics on the downside is likely to occur.

As to the empirics of such unstable dynamics one would expect that it is regime dependent: There will be high financial stress and a rise of credit spreads in a period of low economic activity, but low financial stress and narrow credit spreads in a period of high economic activity. To explore this instability empirically, we use a Financial Stress Index (FSI) recently provided by the IMF (2011).[8] The FSI by the IMF is available for a large number of EU countries and the US. We will use this data set for the empirical part of our study.

Recent models on the banking-macro link, for example Brunnermeier and Sannikov (2010, 2011), refer to asset prices and their volatility in constructing the downward destabilizing effect, triggered by financial intermediaries, we want to refer in our model below – and also in our empirical work – to the movement of risk premia and credit spreads to explore the downward instability. We can justify our focus on those measures of financial stress, since in theoretical as well as empirical studies it has been shown that factors such as large asset price falls, rising volatility, higher risk premia, and a run into liquidity are highly correlated with movements in discount rates. The different factors driving discount rates – and through that asset prices – are extensively discussed in Cochrane (2011).

Yet, as we also will show the triggering of the downward instability also depends on the constraints that are set for the banking sector, for example, on their asset

[7]The exposure of banks to the borrowing of the public sector, private firms and households – as well other banks — is likely to matter for the insolvency or liquidity risk of banks, As Gorton (2010) shows, this is often accompanied by bank runs.

[8]See IMF (2011) for EU countries. For other important financial stress measures, specifically for the US Economy, see the St. Louis Fed (2011), and Kansas City Fed (2011).

accumulation and payouts, affecting risk taking, equity formation and leveraging of the banks. Higher payoffs, for instance, may encourage more risk taking and risk transfer, generating eventually higher aggregate risk and risk premia to be paid by all. So we will explore the behavior of our dynamic model for different constraints on the banks decision variables.

The remainder of the paper is organized as follows. Section 2 builds up a model that reflects the above features. Section 3 solves numerically some model variants using dynamic programming. Section 4 discusses the quality of empirical variables to capture the interaction of banking financial stress and real output. Section 5 presents the MRVAR estimation procedure and reports the impulse-response studies for VAR as well MRVAR for EU countries and the USA. Section 6 concludes the paper. The appendix describes our procedures and presents in detail the multi-country results.

2 The Banking Model and its Dynamics

Next, let us present the above developed ideas in a more formal model, which is closely related to the BS model (2010). We build this model on the balance sheets of the financial intermediaries.

2.1 Basic Model

We are introducing a basic model, akin to the model by BS (2010), by referring to the balance sheets of the banks. On the left hand side of table 1there are assets, valued at current asset prices. On the right hand side there is debt d_t and net worth $n_t = p_t k_t - d_t$.

The equity might be divided up into inside equity $\alpha(p_t k_t - d_t)$ and outside equity $(1 - \alpha)(p_t k_t - d_t)$. The latter may be state dependent.

Next we introduce the dynamics of the variables. The asset price, the capital stock and the debt may evolve as defined in equs (1)-(3).

Assets	Liabilities
$p_t k_t$	d_t
	$n_t = p_t k_t$-d_t
total assets	$\alpha(p_t k_t - d_t) + (1-\alpha)(p_t k_t - d_t)$

Table 1: The Balance Sheet of Banks

$$dp_t = \mu_t p_t dt + \sigma_t p_t dZ_t \tag{1}$$

$$dk_t = (\varphi(i_t/k_t) - \delta)k_t dt + \sigma_t k_t dZ_t \tag{2}$$

$$dd_t = (r_t d_t - (ak_t - i_t))dt \tag{3}$$

In BS (2010) the growth rate of asset prices in equ. (1) follows a geometric Brownian motion, but it is affected by time varying volatility, σ_t, for example as a result of rapid sales of assets. In their model they define endogenous risk as the major driving force of the instability which is represented by time a varying volatility of asset prices, σ_t. Actual price movements, generated by others, may create shocks to financial intermediaries. The asset price shocks will reduce the collateral value of the financial intermediaries, and fast depreciation of asset prices— possibly triggered by a fire sale of assets– may have extensive externality effects on other intermediaries, leading to a general loss of net worth. As mentioned, instead of using asset price movements and volatility we employ risk premia and credit spreads to capture those effects.[9]

Assets of the financial intermediaries in equ. (2) will be increased by investment, i_t/k_t, the function $\varphi(i_t/k_t)$ includes some adjustment cost which is concave in the argument, and δ is a deterioration rate of assets.[10] The actual gross capital of the

[9] As mentioned above, in empirical studies it is shown that large asset price falls, rising volatility, higher risk premia as well as higher discount rates are highly correlated. The relationship of those are discussed in Cochrane (2011).

[10] In their recent version, Brunnermeier and Sannikov (2011) write a model with capital assets, that could be banking capital or real capital. The model is equivalent of there is perfect substitution among them.

bank increases at the rate i_t/k_t. The debt evolves at a rate that is essentially determined by the excess spending of investment over capital income, which is defined here as ak_t. Investment in equ. (2) will increase the stock of assets for financial intermediaries, but the high rate of purchase of assets will also increase their debt, once the investment spending exceeds their income. The interest rate to be paid on debt, r_t, includes a risk premium reflecting asset price shocks and financial stress of banks. It will be made endogenous being state or time depending risk premium. Note that only equs. (1) and (2) are stochastic.

So far we have neglected payouts, bonus payments for executives, which can be viewed to serve the consumption stream of the executives.[11] We can define the executives bonuses as an optimal consumption stream, to be derived optimally through some intertemporal decision making process. We can also have the investment being computed as optimal, with $g_t = i_t/k_t$. Then we have a dynamic decision problem such as:

$$V(k,d) = \max_{c_t, g_t} E \int_0^\infty e^{-\rho t} U(c_t) dt \tag{4}$$

s.t.

$$dp_t = \mu_t p_t dt + \sigma_t p_t dZ_t \tag{5}$$

$$dk_t = (\varphi(i_t/k_t) - \delta) k_t dt + \sigma_t k_t dZ_t \tag{6}$$

$$dd_t = (r_t d_t - (ak_t - i_t - c_t)) dt \tag{7}$$

The latter model includes now payouts, c_t, which is used for a consumption stream,[12] whereby future payouts are discounted at a rate ρ. Note that we have here $g_t = i_t/k_t$.

[11] In Semmler and Bernard (2011) bonus payments of the six largest US investment banks are computed. Bonus payment, as a percent of revenues, went up from roughly 10 percent in 2000 to 35 percent in 2007.

[12] In recent attempts of financial market reforms in Europe the cash payment of bonus payments is planned to be restricted to 20 percent of total bonus payments, the remaining part is only allowed to be paid out in subsequent years via common stocks. In our model we leave aside those complications.

Note also that in equ. (7) if the excess of spending for new assets and payouts exceeds the income generated, then the debt of the financial intermediary will rise.[13]

We want to remark that the above is a standard model of wealth management, now commonly used to study wealth management of financial institutions, see He and Krishnamurthy (2008). If we replace the constant income for a unit of wealth, a in ak_t, by a weighted average of risky and risk free returns of a wealth fund k_t, then the remaining parts of the equations above are reasonably familiar from the wealth management literature, see also Semmler et al. (2009). Yet the explicit equation for the evolution of debt of the financial intermediary, as represented in equ. (7), is usually missing. This reflects the innovative part of the model by BS (2010) and other recent literature.[14]

Now let us derive a dynamic equation for the debt-asset ratio.[15] Let us take as the debt-asset ratio: d_t/k_t: We can rewrite this, for convenience, as $\omega = -(d_t/k_t)$.[16] Taking log and time derivative of this, we can write the asset accumulation and debt dynamics with the previous objective function of the financial intermediaries as:[17]

$$V(\omega_t) = \max_{\tilde{c}_t, g_t} E \int_0^\infty e^{-\rho t} U(\tilde{c}_t) dt \tag{8}$$

$$d\omega_t = ((g_t - r_t + \sigma^2)\omega_t + a - \tau(g_t))dt - \tilde{c}_t + \sigma_t \omega_t dZ_t \tag{9}$$

Hereby $\tilde{c}_t$ is the new control variable.[18] Term $\tilde{c}_t$ is the consumption wealth ratio, $\frac{c}{k}$. The expression $\tau(g_t)$ represents a convex adjustment cost which is affecting the size of borrowing to achieve a growth rate g_t. This is modeled by following the capital

[13] As mentioned before, for the problem of a social planner, which is equivalent to a monopoly problem of the financial intermediary, the prices are endogenous and do not play an additional role at first, see BS (2010)

[14] See for example Hall (2010) who also includes an equation for the evolution of debt. Note that recently, as also BS (2010) are arguing, financial intermediaries have been encouraged by more risk taking through transfer of risk to outside investors, for example through CDO‘s and consequently the financial intermediaries will build up their debt – thus their default risk – more.

[15] Note that we use stocks of assets and debt, in contrast to Geanokoplos (2010) who uses flows as leverage measure, hereby then leveraging is highly positively correlated with booms.

[16] See Brunnermeier and Sannikov (2010)

[17] For a similar approach, see BS (2010) and Hall (2010).

[18] A derivation of a dynamic equation in the stochastic case, using Itoh‘s lemma, is given in BS (2010). The term σ^2 comes in through Itoh‘s lemma.

adjustment cost literature. Yet, of course only the growth of wealth g_t appears in the equation for the evolution of assets k_t. The other expressions in equ. (9) are straight forward derivations from the negative of the growth rate of the debt-asset ratio as stated above.

2.2 Some Specifications

Next we undertake some specifications of the basic model that may help to highlight the inherent amplification mechanisms. A first type of specification pertains to the impact of financial stress on the banks decisions, in particular to role risk premia or credit spreads, banks are facing. BS (2010) provide basic proofs of the probability of instability with endogenous asset prices. But what one could be interested in, as indicated above, are significant variation in credit spreads– due to the rise and fall of financial stress. Considering net worth, such as $p_t k_t - d_t$, on the balance sheets of banks, an adverse asset price shock reduces the collateral value of the financial intermediaries, and they will face a greater haircut, greater Ted spread, higher repo rates and this greater default premia in the credit market.[19] Though, at first sight, one might look at asset price volatility but what actual might magnify[20] the downward spiral, is the financial stress and credit spreads that the capital market requires.[21]

A first simple way to capture such varying credit spreads is to introduce state de-

[19] Movements of asset prices resulting for example from fire sales of assets, imposing some externality effects on third parties through some rise of endogenous risk, could also result from sentiments, as for example studied in Lux (2009). A market sentiment is also at play in the theory of Geanokoplos (2010), where leveraging drives asset prices, but risk premia will appear on the credit market. The role of heterogeneous expectations and trading strategies for market price movements could be explored as well, as in Chiarella et al. (2009: chs 6-9). Yet, there are more general effects that can make the market price of the asset deviate from its fundamental price, as present value of future cash flow, for example liquidity problems, fire sales of assets and market dysfunctions, all giving rise to higher risk premia, see Geneva Report (2009).

[20] The rising volatility has also been included by numerous the financial stress indices developed by the KCFSI and the IMF FSI, to be discussed in sect. 4, where it is as own to affect credit spreads. Volatility is also relevant in a distance to default model where it is shown that the distance to default shrinks with rising volatility, and thus the risk premia rises, see Semmler (2011, ch. 19), and Bonnin et al (2010).

[21] Furthermore, it is very likely that positive and negative asset price shocks may have asymmetric effects. This is also discussed in Basel III. BS (2010) have studied also the effect of a rising volatility σ_t on the spread.

pendent credit spreads, determined by leverage ratios. We make credit spreads a function of leveraging, but this will be a bounded function, with a floor and ceiling. The floor will be a risk free interest rate, and the ceiling will be given by an upper bound.[22] We take take for the credit spread

$$r_t = r_{\varpi}(arctan(\omega_t)). \tag{10}$$

The *arctan* function above avoids the extreme instabilities arising in the asset price leveraging dynamics as might be implied in Geanokoplos and BS (2010), yet it makes the credit spreads depending on leveraging ω_t. Here the spread is made endogenous but it has bounds. Banks face credit spreads, but the impact on the debt dynamics is initially low, and then rising with leverage. We thus have that the decisions on consumption and asset growth are depending on state dependent credit spreads that the financial intermediaries are exposed to.[23] This will be built into our DP solution algorithm presented in appendix 1.

Yet, as above mentioned, there are other factors affecting financial stress and risk premia, due to externalities and contagion effects from asset price movements.[24] So, the alternative is to take a time varying risk premium that captures those factors. In order to capture those effects we extract low frequency components from our financial stress index and use this as proxy for time varying risk premia.[25] This is obtained by using Fourier analysis to estimation low frequency movements in the

[22]See Beja and Goldman (1980), and for a recent use see Chiarella et al. (2002), and Chiarella (2009, ch. 6 and 8, where extensions to heterogeneous agents and the stochastic case are provided.

[23]Since the publication of the financial accelerator principle by Bernanke et al (1999) the economists have been greatly concerned with the fact that borrowing cost moves counter-cyclically, and the ease of lending standards cyclically. Accordingly, we have proposed a state dependent risk premium $r(\omega_t)$. The risk premium, and thus the credit spread, is hereby made state dependent thus $r(\omega_t)$ rises with the leveraging. If the involved parameters are appropriately chosen, the risk premium goes to zero and a constant (risk free) interest rate will re-emerge. The constant interest rate, as assumed in some version of BS (2010), is a limit case of the above scenario. For a model with a state dependent risk premium, see Gruene et al. (2004).

[24]Adrian et al. (2010) have defined such a risk premium as a macro economic risk premium. They summarize the macro risk in one indicator using principle component analysis.

[25]As mentioned, important components of the financial stress index are credit spreads, for example the BAA/AAA spread or the BAA/T-Bill spread. Many studies have worked with those measures, see for example, Gilchrist et al. (2009), see also the important role of credit spread in the IMF (2012) FSI.

IMFFSI.[26] In our DP of appendix 1 we would then have one additional state variable such as

$$dx = 1dt \tag{11}$$

This represents a time index to capture time varying risk premia. The low frequency components in the credit spread is indexed on the variable x, representing time in the DP algorithm. It can be computed and included in the numerical procedure. We would thus have in equ.(9):

$$r_t = r_x(x_t). \tag{12}$$

Formally our stochastic dynamic decision problem will then have two decision variables and three state variables, the leverage ratio ω_t, the time index x_t and the stochastic term dZ_t in the above variant, with r_t time varying. Details of the estimation are discussed in appendix 2.

Another type of specification pertains to constraints on decision variables, for example on payoffs, resulting in consumption stream, $\tilde{c}_t$. We could assume that payoffs are be constrained by some financial market regulation; for example, if the net worth, as a ratio of net worth to total assets, falls below a certain safe threshold, then the bonus payments could be reduced. Equivalently we could postulate that if the debt to asset ratio moves below some threshold, lets say $\omega = -(d_t/k_t) \leqq \varpi$, then the bonus payments are decreased or set to zero. It could hold that bonus payments are used to give the managers an incentive to reduce leverage, so when the leverage is lower, a higher bonus payments could be allowed.[27] This might be considered as a penalty on risk taking and high leveraging – the former resulting from leveraged asset purchases.[28] The dynamics of the debt-wealth ratio, once those constraints on

[26]This has been done in Semmler and Hsiao (2009) to estimate time varying asset returns and can be employed here, see appendix 2

[27]This is for example planned by Basil III, where it refers to "linkages of the total variable compensation pool to the need ...to maintain a sound capital base"

[28]A a further issue might be that the financial intermediaries have in fact transferred risk to outside investors through securitization, i.e. through pooling and tranching of mortgage debt or other kind of liabilities, through MBSs or CDOs. Successfully undertaking the transfer of risk encourages them to take on more risk, but passes the verification cost on to someone else. The

decision variables are introduced, are likely to change.

Of course the inflow of new funds by the private sector, from abroad or the provision of broad based liquidity from the Central Bank could modify those amplifying effects.[29] Overall, the above specifications of the model make visible what possible amplifying effects can occur.

3 Solution Method and Numerical Results

As BS (2010) correctly state, the dynamics for a model such as represented by equs. (8)-(9) should not be studied by common linearization techniques. The first or even second order Taylor approximations to solve for the local dynamics of a model such as (4)-(7) or (8)-(9) will not properly capture the global instabilities of the model in particular in some regions of the state space. We have used the dynamic programming method by Gruene and Semmler (2004) to study the dynamics of the stochastic version of the model (8)-(9). Here, the debt to asset ratio is the state variable, and the control variables are the growth rate of assets and payouts for consumption, for which we will introduce constraints, and we will use state dependent and time dependent credit spreads as specified in sect. 2.2.

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verification cost usually defines the amount that financial intermediaries have to pay, but if it is passed on, they can generally borrow at a lower risk premium, see BS (2010: sect. 4) for details of such considerations. Higher bonus payouts may encourage more risk taking and risk transfer, generating eventually higher aggregate risk and greater risk premia.

[29] One could consider a further modification that takes into account the availability of funds for the financial intermediaries. There might be a fraction of households that accumulate risky assets, which will provide funds for the financial intermediaries. A fraction of funds could also come from capital inflows, see Caballero and Krishnamurthy (2009). In this context, the inflow of funds from the Central Bank could be considered, which for example took place in the US in the years 2008 and 2010 when the Fed employed an unconventional monetary policy, called quantitative easing, buying bad — and rapidly declining — assets from the financial intermediaries. The ECB provision of a three year low interest rate liquidity for the EU banks in December 2011 is a similar case. This has a mitigating effect on the unstable forces generated by the banking system. An estimation of this effect will be presented in section 5. On the other hand, the precautionary motives of households (and firms), the “run into high quality assets”, would lead to a reduction of financial funds for the financial intermediaries.

3.1 Solution Method and Model Variants

When we use a dynamic programming method to explore the local and global dynamics we use a coarse grid for a larger region of the state space, and then employing grid refinement for smaller region. The DP in the appendix 1 can provide us with information on the truly global dynamics in a larger region of the state space without losing much accuracy (see Becker *et al.*, 2007). In contrast, local linearization, as has been argued there, and also in BS (2010), does not give sufficient information on the global dynamics. When we study the basic model variants we explore the stability properties of each variant. We consider two ways of formulating credit spreads— state depending and time depending credit spreads.

State dependent credit spreads is defined as a function of leveraging, ω_t. To make the risk premia and credit spreads state dependent we take

$$r_t = \kappa \; arctan(\omega_t). \tag{13}$$

The *arctan* function above, with $\kappa > 0$, avoids the extreme instabilities arising in the asset price leveraging dynamics as might be implied in Geanokoplos (2010) and BS (2010), yet it lets the credit spread rising with leveraging ω_t.We restrict our considerations here to the simplest case, as in equ (13), where the bank pays a risk premium formulated in a simple way[30] which is built into our DP algorithm.

The alternative is to introduce a time varying credit spread which is discussed in detail in appendix 2. We take here the case of Germany and proxy the time varying risk credit spread by the IMF (2012) FSI.[31] which by and large reflects the varying risk premia. We estimate low frequency components of the credit spreads by:

$$r_t = \alpha_1 - \alpha_2(t - t_0) + \sum_{i=1}^{n} \Big(a_i \sin\big(\frac{2\pi}{\tau_i}(t - t_0)\big) + b_i \cos\big(\frac{2\pi}{\tau_i}(t - t_0)\big)\Big) \, . \tag{14}$$

[30] As mentioned above, we use the same *arctan* function-type for the risk premium. This has a lower limit, the risk free rate, and an upper limit. The upper limit of a premium charged is justified, since, as Stiglitz has always argued, with higher default premia, the lender might have loan losses at greater credit spreads.

[31] Note that the major components of the financial stress indices of the IMFFSI as well as the KCFSI, and the STLFSI, are variables that are capturing risk premia and thus credit spreads.

Note that the first two terms in the above equation represent a constant and time trend of credit cost, the next terms are the low frequency components. Appendix 2 reports how many periodic components are needed to properly proxy the actual time series of the credit spread and the coefficients of equ. (18). This is then used our DP algorithm.

As to the payouts[32], we introduce alternatively broader and narrower constraints, which will, however, always be non-negative.[33] When we define the payouts for, $\tilde{c_t}$, we let the choice to be taken from and interval: $\tilde{c_{min}} < \tilde{c_t} < \tilde{c_{max}}$. So, the payout is always positive but is constrained.[34] For the case of less constrained payouts we assume $0.01 < \tilde{c_t} < 0.3$ and for constrained payoffs, we assume that $0.01 < \tilde{c_t} < 0.05$. As concerning the asset growth we constrain the growth of assets to $-0.1 < g_t < 0.1$.

3.2 State-dependent Credit Spreads

We here report the results of the state-dependent credit spread and larger interval for payoffs. As to the parametrization of our model we take: $a = 0.5$, $\alpha = 0.3$, $\sigma = 0.008$, and $\gamma = 0.03$ and $\rho = 0.03$.

The figure 1 shows on the horizontal axis the state variable ω and on the vertical axis the stochastic shocks to the state variable ω. Since we have stochastic shocks, with pre-defined standard deviation $\sigma = 0.008$, the path of ω varies in the state space, and thus there is no unidirectional vector field, i.e. the path of ω_t is not a straight line. In our numerical procedure the shocks are drawn from a distribution having a pre-defined standard deviations $\sigma = 0.008$. As visible from the numerical solution

[32] As to the constraints on the the growth rate of assets (or certain types of assets) acquired by the banks, those are probably hard to constrain, unless there are borrowing constraints introduced as Geanokoplos (2010) seems to suggest. So we have used here rather broad constraints for the growth rate of assets.

[33] Regarding the payouts, we want to remark that BS (2010) conjecture that when the bonus payouts are chosen less constrained "the system is relatively stable near its "steady state" ... but becomes unstable below the steady state... "(BS, 2010:17). The reason for the result from unconstrained endogenous payout is: "With endogenous payout, the steady state naturally falls in the relative unconstrained region where amplification is low, and amplification below the steady state is high" (BS, 2010:18). BS make this statement with respect to the ratio of net worth to assets. Since we take the negative of the debt to asset ratio, the statements can be immediately translated into the properties of our model using the debt to asset ratio.

[34] Note we also could allow for dividend payments, in fact as our model is constructed the bonus payments can encompass dividend payments.

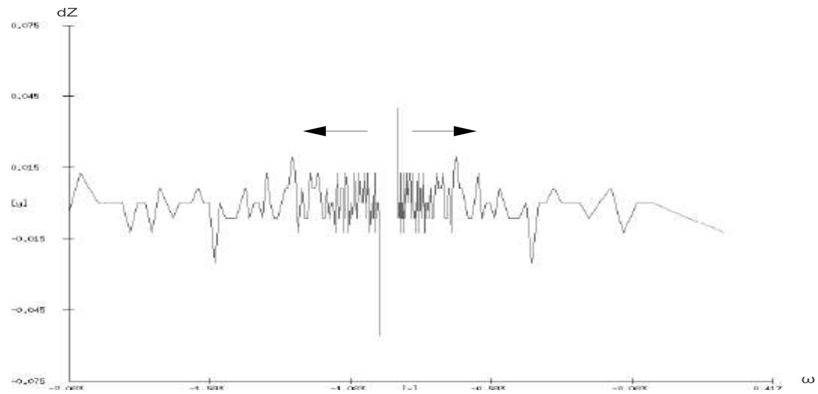

Figure 1: State-dependent credit spread, large payouts, and small domain of attraction

path in figure 1a, there is an unstable steady state roughly about a low level of debt to asset ratio $\omega^* = -0.85$.[35] In other words small initial leveraging will eventually end up at low level of debt to asset ratio, because the credit spread is low, but larger leveraging – a shock that moves the leverage above $\omega^* = -0.85$ makes both the leveraging as well as the credit spread rising, a vicious cycle: higher leverage creates higher credit spreads and higher credit spreads results in higher leveraging.[36] On the other hand, there is a small domain of attraction: if the financial intermediary starts with low leveraging and low credit spread, both may be reduced further.

It is the debt to asset ratio, and its accompanying credit spread, as well as the payoffs, that are amplifying. Thus, under those conditions a leveraging ratio beyond a threshold, is likely to be dynamically unstable and will, as BS (2010) predict, be

[35] Above the large credit spreads and larger pay-offs make the debt to asset ratio rise, moving the ω_t toward another steady state, which is roughly at $\omega^{**} = -3.8$

[36] Note that we do not pursue the issue here at what leverage ratio bankruptcy would occur. This depends on the distance to default, which is defined by the KMV model by the distance of the asset value of the bank to the debt, divided by the standard deviation (volatility) of the asset value. We are not pursuing this question here, since we do not explicitly computing the asset value of the financial intermediary. This is issue is pursued in Gruene and Semmler (2005).

amplifying.[37]

3.3 Time-depending Credit Spreads

Next we consider time dependent credit spreads but also tighter constraints on payouts. BS (2010:32) state that allowing the debt to asset ratio rise too much, driven by the incentives of the intermediaries to take on too much risk for the sake of short term profits, paying out high bonuses, and neglecting externalities may lead to damages and downturns. In their view the triggering of the downturn in the financial, product and labor markets, and the higher asset price volatility, results from not taking into account the full extent of the externalities.

They thus state that limiting payouts should be welfare improving. More explicitly they say: "We would like to argue that a regulator can improve social welfare by a policy that limits bonus payments within the financial sector. Specifically, suppose that experts are not allowed to pay themselves as long financial intermediaries are not sufficiently capitalized" (BS, 2010: 32). This type of regulatory effort would keep sufficient capital within the financial system and make it more stable.[38]

This conjecture can also be shown to hold using our DP solution algorithm. In order to explore this variant with time varying credit spreads and tighter payouts, we, as before, allow for negative and positive growth rates of the assets purchased by the financial intermediaries to be in the range$-0.1 < g_t < 0.1$, but we constrain the consumption to capital ratio by $0.01 < \tilde{c}_t < 0.05$. Again, the latter is always positive but it is constrained not to be too large. Under the condition that the growth rate of assets and the consumption rate can be chosen optimally, yet payouts will be constrained to be low.

Figure 2 shows the dynamics where the domain of attraction is increased, the steady state is now roughly at $\omega^* = -3.9$, which is also repeller: with lower leverage and low payoffs, the debt to asset ratio will go to zero. The domain of attraction of the zero debt to asset ratio is considerable enlarged. The dangers of large externalities,

[37] Since the the shape of the value function for this case is similar to the next case, it will be discussed in sect. 3.3.

[38] A similar view is present in the Geneva Report (2009, sect. 6.2) and Basel III.

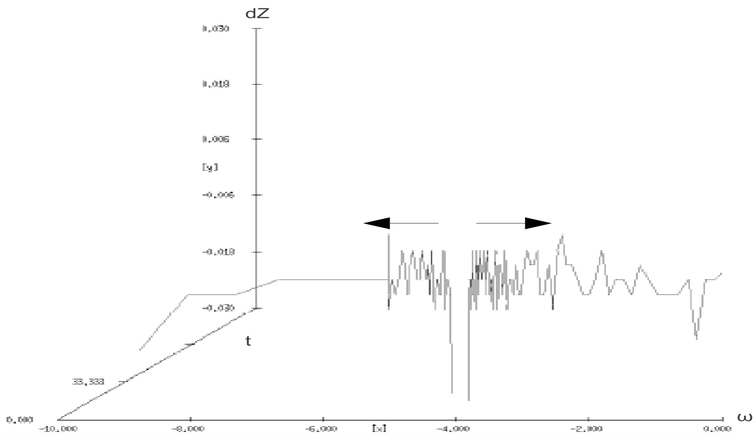

Figure 2: Time-varying credit spread, small payouts, and large domain of attraction

financial stress and meltdowns are reduced.[39] So, now a high debt to asset ratio can be stabilized if risk premia and payouts are small.

As to our figure 2, the shock dZ_t moves the trajectories, along the vertical axis, whereas the debt dynamics with credit spread moves the trajectories along the axis ω. Note that we have here now the shock drawn from the range $-0.1 < dZ_t < 0.1$, and we have used $-10 < \omega_t < 0$. For our third dimension, our time index x_t, the range is defined as $0 < t < 100$. But because we have a movement in a three dimensional space, the time axis, the third variable, the one that goes to the back, is fixed. We are showing here the projection of the trajectories to a 2 dimension space.

Figure 3 shows the corresponding value function, revealing the result that total welfare (for the financial intermediaries) is rising with lower debt to asset ratio.[40]

[39] There appears that an initial debt to asset ratio below that new threshold $\omega^* = -3.9$, with low payouts, and time varying risk premia, will produce always stability.

[40] The rise of the value function to the right of the steady state is reasonable, since the welfare from lower credit spread should be higher. It was computed through our numerical solution procedure. The value function for the case of sect. 3.2. was similar to the case above. It also increased with the debt to asset ratio falling, i.e. the value function to the right of ω=-0.85 in figure 1 was rising. Note that the shape of the value functions in the cases of sect. 3.3. as well as 3.2. are roughly the

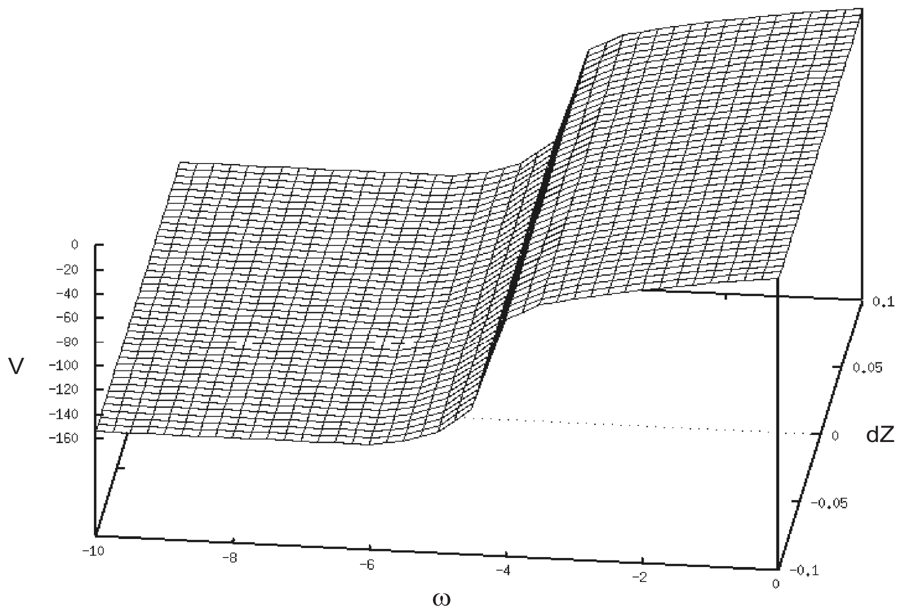

Figure 3: Value function for time-varying credit spread, small payouts, and large domain of attraction

Overall, the risk premium, and thus the credit spread, is, in our two cases, made state-and time-dependent. The state dependent credit spread is likely to trigger a vicious cycle. If the spread goes to zero interest rates tend to reflect the short term interest rate set by the central bank.[41] For the state-dependent credit spread and relatively larger payouts, we can observe that for a leverage beyond a threshold, the leverage will finally be built up quickly through a vicious cycle,[42] and for small payoffs and time-dependent credit spread there is a much larger domain of the zero

same as shown in BS (2010) in their figure 7, though we have negative values on the vertical axis, since we are taking log $\tilde{c}_t$, not $\tilde{c}_t$, in the preferences.

[41] A low borrowing interest rate for teh banks, according to the argument by BS (2010, sect. 4), may arise, if they can transfer risk through the securitization of loans and selling them as CDOs to a secondary risk market, see Semmler and Bernard (2012). This will not only reduce their risk exposure, but also give them less incentives for monitoring loans and increase leveraging and thus increase systemic risk: if idiosyncratic shocks are fully hedged out through securitization, the financial intermediaries then "face the cost of borrowing of only r ... Lower cost of borrowing leads to higher leverage and quicker payouts. As a result the system becomes less stable". (BS, 2010:39). They further argue that though in principle securitization may be good, since it allows for sharing of idiosyncratic risk, it also leads to the creation of severe leverage and the amplification of systemic risk.

[42] This situation creates a vicious circle of credit spreads and higher indebtedness, as often has been observed for companies, households as well as for sovereign debt, fir example for Greece and Italy in 2011.

attractor of the debt to asset ratio. Thus, larger shocks are likely not to matter as much as in the first case.

The downward instability and depends not only on the financial stress (and thus the size of the risk premia),[43] but also on the constraints of payouts: With high payoffs, the high leverage equilibrium can become an attractor, whereas with tighter constraints and low payouts the zero level leveraging can become the attractor for a large domain of attraction.[44]

4 Financial Stress Measures and Output

In the previous section we have postulated that the financial intermediaries are not only exposed to asset price movements and financial stress but may also amplify them, thus producing endogenous macro risk. Some variants of our model show that a shock to the banks‘ balance sheets can entail a considerable instability as considered in sect. 3.2, and a switch to financial stress, rising credit spread, and low growth regime or recession.

So far we have only considered that shocks to asset prices, to capital assets, $p_t k_t$, and net worth, $n_t = p_t k_t - d_t$, will increase debt, risk premia and credit spreads, and higher discount rates, and this might be amplified, particularly in the case of state-dependent credit spreads and large payouts, represented in sect. 3.2. For the case of sect. 3.23, we may observe some superior stability properties. Yet, the range of instability- or stability- is an empirical issue to be explored next.

The problem is what measures can one utilize to empirically evaluate the predictions of the model and undertake empirical estimates. What actual measurements should one take to evaluate how financial stress of banks is linked with the financial market, and what linkages should we look for the feedback loops to the macroeconomy?

[43]BS (2010) show that the high leverage can be become a tipping point for the instability and the downward movement.

[44]Yet, when and to what extent some exogenous risk is turned into endogenous risk, leading to a higher risk premium and discount rate, for all, may also, as BS (2011: 4) show, depend on the extent that the financial intermediaries are able to transfer risk. Large risk transfer might then show up, at least temporarily, as lower risk premia and credit spread for financial intermediaries.

Our model in sects. 2-3, may suggest take leverage ratios, stemming from the balance sheets of the financial intermediaries, as measuring those linkages: high leverage implying high financial stress and low leverage the reverse. However, there is an issue whether the ratio of net worth to capital assets, or the reverse measure, the leveraging ω, can be accurately measured and can be used as good measures of financial stress. this is greatly affected by the market valuation of assets as well as liabilities. In particular, asset valuation is heavily impacted by the confidence and estimate of income streams the asset generates, as well as presumed discount rates, and the liabilities such as bonds or short and long term loans are strongly affected by their corresponding risk premia.[45] Moreover, credit constraints, for example, as measured by the Fed index of changes in credit standards to determine the ease and tightness of obtaining credit as well as default premia and credit spreads and short term liquidity, are also important financial stress factors for financial intermediaries. All this will affect credit demand and supply of financial intermediaries. We thus need more extensive measures than only leverage to evaluate financial stress.

As aforementioned we propose to measure financial stress empirically by taking the IMF´s (2011) financial stress index, the FSI. This is available for a large number of EU countries.[46] The IMF‘s (2011) FSI[47] refers to three major sources and measures of instability, namely: 1) a bank related index – a banking beta as 12-month rolling

[45]This is implicit in Merton's risk structure of interest rates, see Merton (1974)

[46]As aforementioned the Federal Reserve Bank of Kansas City and the Fed St. Louis have thus developed a general financial stress index, called KCFSI and STLFSI respectively. The KCFSI and the STLFSI, take into account the various factors generating financial stress. The KC index is a monthly index, the STL index a weekly index, to capture more short run movements, see also Hatzius et al (2010). Those factors can be taken as substitutes for the net worth or leverage ratios as measuring financial stress of financial intermediaries. See also the Bank of Canada index for Canada, i.e. Illing and Lui (2006). Both the KCFSI and STLFSI include a number of variables and financial stress is related to an: 1) increase the uncertainty of the fundamental value of the assets, often resulting in higher volatility of the asset prices, 2) increase uncertainty about the behavior of the other investors, 3) increase the asymmetry in information, 4) increase the flight to quality, 5) decrease the willingness to hold risky assets, and 6) decrease the willingness to hold illiquid assets. The principle component analysis is then used to obtain the FSI. Linear OLS coefficients are normalized through their standard deviations and their relative weights computed to explain the FSI index. A similar procedure is used by Adrian and Shin (2010) to compute a macro economic risk premium. We want to note that most of the variables used are highly correlated with credits spread variables. They also have the highest weight in the index, for details see Hakkio and Keeton (2009, tables 2-3.).

[47]This is published for advanced as well for developing countries, see IMF (2008) and IMF FSI (2011)

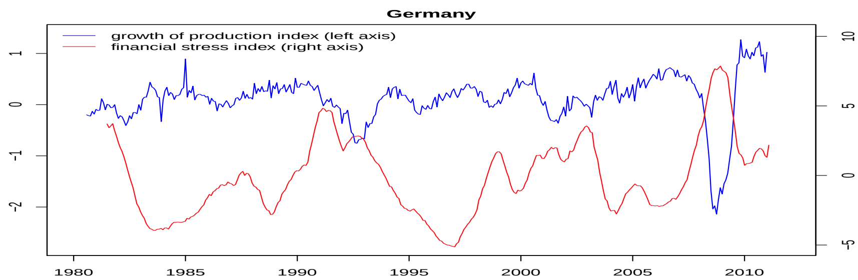

Figure 4: Financial stress and output for Germany: Financial stress index (IMFFSI, lower graph) plotted against growth rates of industrial production (3 month moving average, upper graph)

beta of bank stock index and a ted or interbank spread, 2) a security related index – a corporate bond yield spread, an inverted term spread, and a monthly stock returns (measured as declines), six-month rolling monthly squared stock returns and finally, 3) an exchange rate index – a six-month rolling monthly squared change in real exchange rates. All are are detrended and scaled with their standard deviations in order to normalize the measures.

As measure for the performance of the macroeconomy we take a monthly production index for the different countries, or what is more proper in the context of our model, the growth rate of the monthly production index of the various countries we are considering.

As concerning the IMF FSI, combining the three groups of variables with appropriate weight in a stress index and contrasting it with the monthly production index, one can observe clearly a counter-cyclical behavior. This is illustrated in figure 4 where the variables are shown for a three month moving average.

As the comparison of the smoothed growth rate of the production index and the stress index in figure 4 show there is less financial stress in good times, but more in bad times. Financial intermediaries are clearly doing better in economic booms then in recessions[48]. Given the apparent linkages between the FSI and economic activity, we would also expect a strong linkage between net worth, or leveraging, of financial intermediaries and economic activity, since the financial stress is affecting the balance sheets of financial intermediaries.[49]

A "one-regime VAR" has been used frequently to study the financial accelerator.[50] Yet those "one-regime VAR" studies presume only local behavior of the variables, symmetry effects of shocks and mean reversion after the shocks. What we will pursue here is an MRVAR. Our MRVAR[51] takes the IMF FSI as empirical measure of financial stress, and the growth rate of the monthly production index, the latter also used as a threshold variable, to define regimes for a selected number of EU countries.

5 An Empirical MRVAR Analysis

To empirically examine whether or not local instabilities are present and, if present to assess their consequences, we require a modeling framework that can accommodate varying dynamic patterns across alternative states of the economy. Our multi–regime vector autoregression (MRVAR) approach permits us to test for the presence of multiple regimes and to allow for regime–dependencies in the responses to shocks to the system. Shocks might occur during a regime with great instability, as, for example, in the case of high steady-state leverage ratio (or high stress) discussed in sect. 3, figure 1, the effects will be larger as compared to a regime with large domain of attraction for the zero leverage ratio (and low stress), see sect. 3, figure 2. As

[48]This coincides also with the empirical study by Gorton (2010) that there is more insolvency of financial institutions in bad times.

[49]The fact that the leverage ratio is rising in recessions and falling in booms, is documented in Gilchrist et al. (2009). We want to note that the financial stress index can also be linked to some broader index of economic activity, See Hakkio and Keeton (2009) see Hakkio and Keeton (2009).

[50]Estimating the financial accelerator for the macroeconomy with a "one regime VAR", see Christensen and Dib (2008). and for the application of the financial accelerator to study financial intermediaries in a "one regime VAR", see Hakkio and Keeton (2009) and Adrian et al. (2010).

[51]For using a MRVAR, see Mittnik and Semmler (2009) and Ernst, Mittnik and Semmler (2010).

a consequence, responses to positive and negative shocks may have different effects, as may variations in the size of the shocks imply over-proportional responses.

We estimate MRVAR models for altogether six countries: the five EU countries Germany, France, Italy, Spain, and the UK and the U.S. In this section we will discuss the results for Germany in more detail. The results fore the other countries are reported in Appendix 3.

5.1 MRVAR Approach

To assess the dependence of the responses to shocks to the stress index, we employ an MRVAR approach. A major limitation of conventional linear VAR models is that shock responses are independent of the state of the economy at the time a shock occurs. Also, VAR response profiles are invariant with respect to the sign and size of a shock. That is, responses to positive and negative shocks are sign–symmetric; and the response to shocks of different sizes are simply scaled versions of the response to a shock of size one. To capture state dependencies and asymmetries of shock responses, a nonlinear model needs to be specified. The "mildest" form of generalizing a linear, constant–parameter VAR is to adopt a piecewise linear VAR, such as Markov switching autoregressions (Hamilton, 1989) or threshold autoregressions (Tong, 1978, 1983). A characteristic of Markov switching autoregressions is that the states are unobservable and, hence, do not necessarily have an obvious interpretation. Also, a given observation cannot directly be associated with any particular regime. Only conditional probabilistic assignments are possible via statistical inference based on past information.

For our purposes, namely, state–dependent response analysis, states are associated with specific stages of the business cycle as measured, for example, in terms of output growth. MRVAR models in the form of threshold autoregression models of Tong (1978, 1983) or, in a vector setting, of multivariate threshold autoregressions (Tsay, 1998) are obvious candidates. In contrast to Markov switching autoregressions or standard multivariate threshold autoregressions, our approach assumes that we can, based on some observable variable, define upfront a meaningful set of regimes, which are not a result of some estimation procedure, but rather motivated by the objective of the empirical analysis. This is preferable in our setting, where we are interested in evaluating the potential effectiveness of policy measures for a particular state of the economy.

The MRVAR specification adopted here is given by

$$y_t = c_i + \sum_{j=1}^{p_i} A_{ij} y_{t-j} + \varepsilon_{it}, \text{ if } \tau_{i-1} < r_{t-d} \leq \tau_i, \ \varepsilon_{it} \sim NID(0, \Sigma_i), \ i = 1, \ldots, M, \quad (15)$$

where r_{t-d} is the value of the threshold variable observed at time $t-d$; and regimes are defined by the (prespecified) threshold levels $-\infty = \tau_0 < \tau_1 < \cdots < \tau_M = \infty$. In the following analysis we estimate a two–regime VAR, with the output–growth rate as the threshold variable, and the average growth rate delineating the threshold for the sample.

In addition to the more straightforward regime interpretation, MRVAR models are also more appealing than Markov switching autoregressions as far as estimation is concerned. Rather than EM–estimation, MRVARs with predefined threshold levels resemble conventional VARs and can be estimated regime by regime, using standard common least–squares—provided the regime–specific sample sizes permit this, or using Bayesian techniques.

Response analysis for linear VAR models is straightforward. Point estimates and asymptotic distributions of shock response can be derived analytically from the estimated VAR parameters (see Mittnik and Zadrozny, 1993). In nonlinear settings, this is, in general, not possible, and one has to resort to Monte Carlo simulations. Following Koop et al. (1996), the so–called generalized impulse responses, which depend on the overall state, z_t, type of shock, v_t, and the response horizon, h, are defined by

$$GIR_h(z_t, v_t) = \mathrm{E}\left(y_{t+h} \mid z_t, u_t + v_t\right) - \mathrm{E}\left(y_{t+h} \mid z_t, u_t\right), \quad (16)$$

where the overall state, z_t, reflects the relevant information set. For a Markov–switching VAR process, z_t comprises information about the past realizations of y_t and the states; for an MRVAR process with known threshold levels, only information about past realizations $y_{t-1}, \cdots, y_{t-p_{\max}}$, with $p_{max} = \max(p_1, \ldots, p_M)$, is required.

To understand the differences in the dynamic characteristics between the different regimes, regime–specific response analysis as in Ehrmann et al. (2003) is helpful. MRVAR models assume that the process remains within a specific regime during the next h periods. This is particularly reasonable when regimes tend to persist or when we are interested in short–term analysis, and helps to understand regime–specific dynamics.

5.2 Estimations

For our bivariate analysis of the six countries, we use monthly data on industrial production (IP) and the IMF's Financial Stress Index (FSI) covering more or less the period from mid 1981 to mid 2011.[52] In this section we focus on the results for Germany. The results for the other countries are detailed in appendix 3.

We estimate a standard VAR and an MRVAR model for the IP growth rate and absolute changes in the FSI, and define $y_t = (100\Delta \log IP_t, \Delta FSI_t,)'$. We use the AIC for model selection. For MRVAR model (15), the AIC is given by

$$AIC\,(M, p_1, \ldots, P_M) = \sum_{j=1}^{M} \left[T_j \ln|\hat{\Sigma}_j| + 2n \left(np_j + \frac{n+3}{2} \right) \right], \tag{17}$$

where M is the number of regimes; p_j is the autoregressive order of Regime j; T_j reflects the number of observations associated with Regime j; $\hat{\Sigma}_j$ is the estimated residual covariance matrix for Regime j; and n denotes the number of variables in vector y_t. Formulation (17) differs from that in Chan et al. (2004) in that we account for possible heterogeneity in the constant terms, c_j, and residual covariance, Σ_j, across regimes.[53]

For the case of Germany, the AIC suggests a fourth–order VAR, $p = 4$. Specifying a two–regime MRVAR with the threshold, τ, set to the sample mean of the monthly IP–growth rate, given by 0.1469%, we assign observations associated with below–mean (above–mean) growth rates to the High–Regime (Low–Regime). Then,

[52] We use seasonally–adjusted industrial–production data from OECD (2011) and the FSI data provided from the IMF (2011).

[53] When employing (17) to discriminate between an MRVAR and a standard VAR specification (i.e., a one–regime MRVAR), we need to include the n parameters in the intercept vector, c, and the $n(n+1)/2$ parameters in the residual covariance matrix for an equivalent parameter count.

Table 2: Specifications of VAR and MRVAR Models

	Germany	France	Italy	Spain	UK	USA
			VAR			
p	4	6	5	6	6	6
AIC	691.1	437.3	640.8	667.4	322.1	-8.179
T	362	357	361	360	361	362
			MRVAR			
Threshold	0.1469	0.0526	0.0346	0.0642	0.0572	0.1600
p_{lo}	3	4	3	3	1	4
p_{hi}	3	2	2	5	3	4
AIC	393.2	170.4	364.5	439.9	31.27	-266.0
T_{lo}	176	174	183	177	167	166
T_{hi}	186	183	178	183	194	196
Sample period	06/1981 07/2011	10/1981 06/2011	06/1981 06/2011	07/1981 06/2011	07/1981 07/2011	06/1981 07/2011

the AIC suggests an autoregressive order of three for both regimes. Although the MRVAR has quite a few more free parameters than the fitted VAR (35 vs. 21 parameters), the AIC favors the two–regime MRVAR with AIC $(M = 2, p_{lo} = 3, p_{hi} = 3) =$ 393.2 (and regime–specific sample sizes $T_{lo} = 176$ and $T_{hi} = 186$) over a standard VAR with AIC $(M = 1, p = 4) = 691.1$.

The model specifications for Germany as well as the other five countries are summarized in table 2. As for Germany, the AIC strongly favors the MRVAR model over the conventional, one–regime VAR. The mean of the six VAR–AICs is 458.4 and substantially exceeds that of the MRVAR–AICs, given by 188.9.

5.3 Response Analysis for Germany

To assess the effects of linear versus nonlinear model specification, we first look at the estimates of the cumulative unit–shock responses for the VAR model and the regime–specific responses for the MRVAR model. To derive structural responses, we assume that a shock to IP simultaneously affects the soundness index, whereas IP reacts with a one–period delay to an FSI shock. The cumulative responses due to a

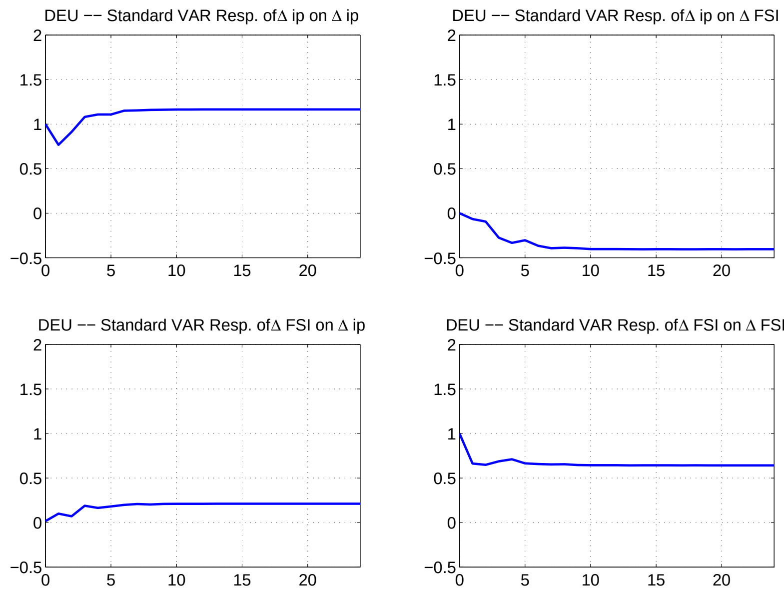

Figure 5: Standard VAR responses for Germany

unit shock implied by the estimated VAR model are shown in Figure 5.[54]

The results for the conventional VAR model (Figure 5) suggests that a positive one–standard–deviation stress shock has an increasingly negative cumulative IP–growth effect which settles at -0.37% after about seven months. The cumulative response of IP to a unit shock in IP itself settles at 1.16%; and the stress index responds positively to a positive IP shock (0.19%), but more so to a positive FSI shock itself (0.65%).

Next, we explore the *within–regime* response behavior. Clearly, the assumption to stay within a particular regime for an extended period is not very realistic as shocks and regime dynamics may induce regime migration. Also, by looking at the within–regime dynamics, we solely focus on the regime–specific autoregressive parameters and ignore the level effects induced by the differences in the regime intercepts. Any differences in the regimes' intercepts will induce additional variation in the overall dynamics when the process switches between regimes. However, a regime–specific

[54]In the following initial discussion discussion of the general results for the response analysis, we, first, focus solely on the responses' point–estimates. We will consider interval estimates when we discuss the results specific to the question under investigation.

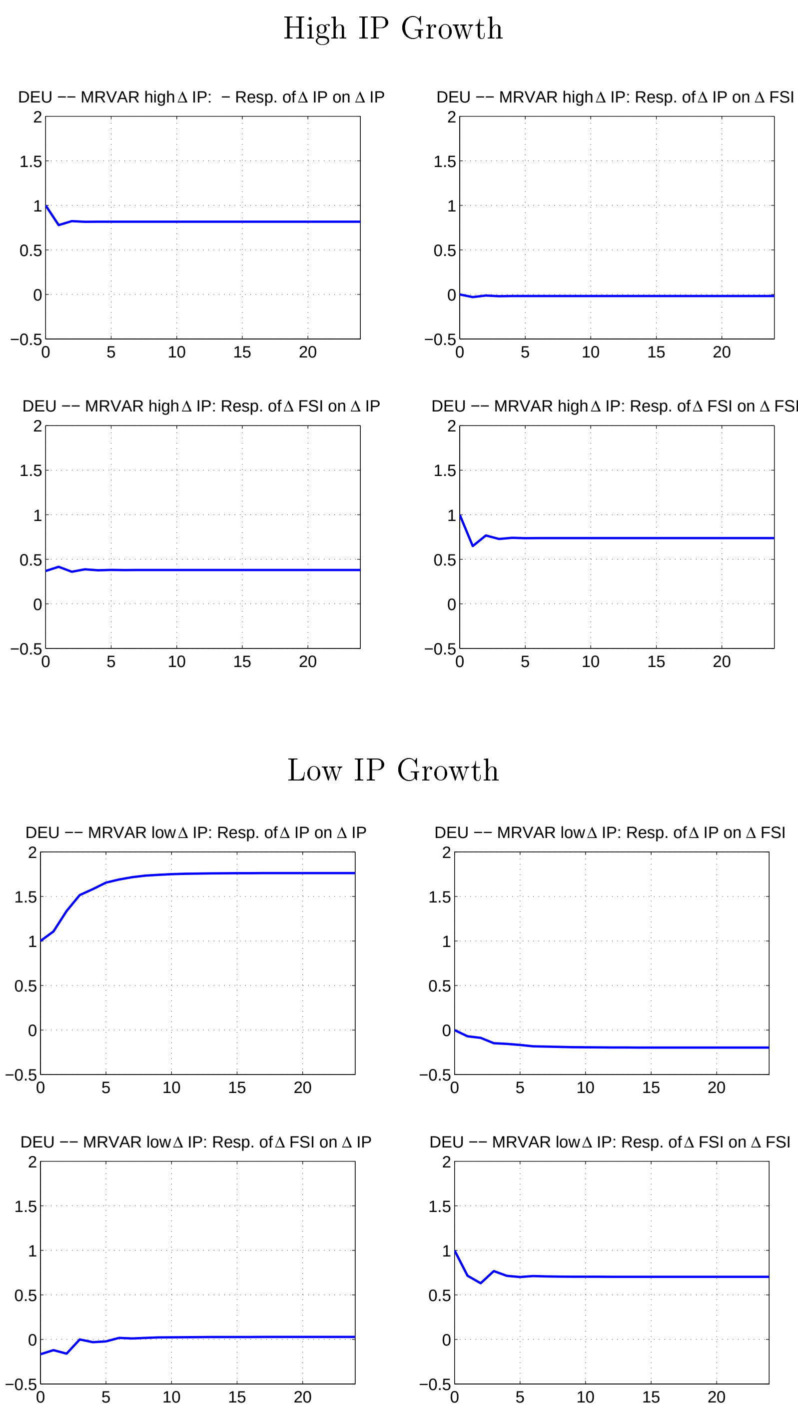

Figure 6: MRVAR Responses for Germany in a high–growth (top half) and low–growth regime (bottom half)

response analysis is useful as it helps to better understand the short–term dynamics associated with the estimated regimes.

The within–regime MRVAR responses are presented in MRVAR in Figure 6. Here, the cumulative responses to unit shocks implied by each of the two MRVAR regimes are somewhat different. Within the high IP–growth regime (upper half in Figure 6) a positive one–standard–deviation stress shock has practically no impact on IP (-0.02%). Compared to the VAR analysis, IP responds less positively to a unit shock in IP itself, settling at 0.71% rather than unity. The stress index responds somewhat higher to a positive IP shock (0.33%); and the cumulative MRVAR response to a positive FSI shock is with 0.74% versus 0.65% marginally higher than the VAR–implied. Within the low IP–growth regime (lower half in Figure 6) a stress shock has a weak negative impact on IP (-0.17%). With 1.82%, the IP response to positive shock in IP is much stronger than in the high–growth regime or the VAR–implied case. The stress index hardly reacts an IP shock (0.07%); and, with 0.70%, the response to an FSI corresponds to that obtained for the high–growth regime.

Given our objective, the evaluation of the impact of banking–sector stress on economic growth, measured in terms of IP growth, our subsequent analysis will focus on the response of IP to shocks to the financial stress index.

In order to investigate the economy's overall growth effects due to FSI shocks, we, first, simulate generalized cumulative response functions to unit–impulse shocks. We do this for specific states at which the shock is assumed to occur. The two specific states we select are the sample averages observed for the two regimes, as they can be viewed as representatives for low–and high–growth states of the economy. The mean in low–growth regime is $\bar{y}_{low} = (-1.0542, -0.0216)'$, and that for high–growth $\bar{y}_{high} = (1.2834, 0.0051)'$. For each case we simulate both a positive and a negative unit–shock to the stress–index. The mean cumulative responses to IP together with one–standard deviation confidence bands are shown in Figure 7.[55]

The estimated unit–shock responses strongly suggest that the impact of an FSI shock on output varies with the state of the economy. A *positive* unit–shock in the average high–growth state (top left plot in Figure 7) causes IP to drop by about -1.9% after

[55]The generalized cumulative responses were simulated based on 200 replications, which were repeated 2000 times to approximate the standard errors of the responses.

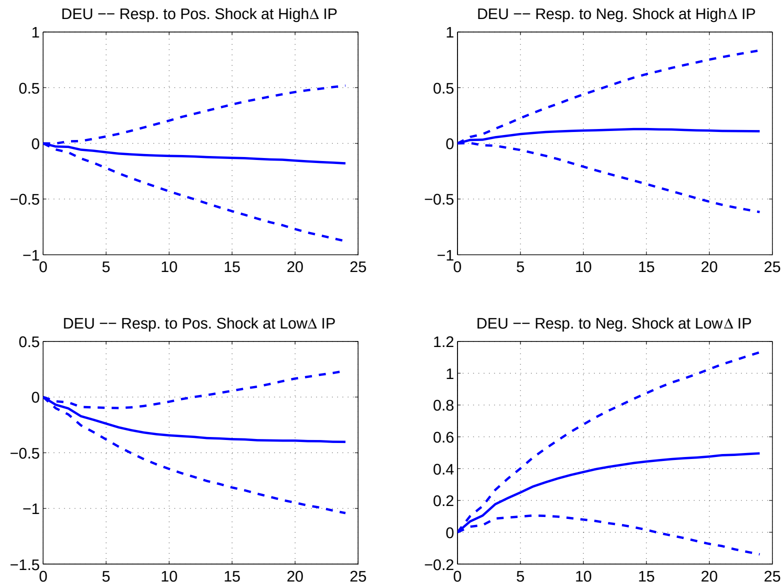

Figure 7: Cumulative MRVAR responses to positive (left panel) and negative FSI shocks (right panel) in high– (upper panel) and low–growth states (lower panel)

two years. However, only for the first–month response does the confidence band boarder the zero line, indicating some significance. The same shock applied in the average low–growth state (bottom left plot), results in an IP contraction, which is twice as large (-0.41%). This result possesses more significance, as the confidence band excludes the zero line for 12 months. Thus, in a recession period an increase in financial stress curbs German IP more severely than during a boom.

If the same FSI–shock is negative, we obtain pretty much the reverse results. In absolute terms, a negative FSI shock during low growth (bottom right plot) has a stronger impact than during growth. Thus, at least for one–standard–deviation shocks, the German IP responses are mildly sign–asymmetric. Though, only in the low–growth regime do the confidence bands suggest significance beyond the first month.

Next, we investigate to what extent the *size* of the shock to financial stress matters. In addition to simply a *unit* shock to the stress index, we simulate the cumulative IP responses to stress shocks with different sizes. Specifically, we impose positive and negative shocks from one through six standard deviations.

The consequences of positive shocks after 24 months differ quite dramatically with the magnitude of the shocks. Figure 8 compares the response profiles. Large negative FSI shocks in the low–growth regime (top right graph in Figure 8) boost IP growth by about three times as much as in an average high–growth period (top left graph). On the other hand, in a low–growth state, large positive shocks reduce IP only about 1.5 times as much as in a high–growth period. Not only do large positive shocks have quite a different relative impact compared to small positive shocks. Their relative effect also varies strongly with the regime at which the shock occurs.

As figure 8 shows, we find an analogous but somewhat less extreme divergence for negative shock scenarios. For small negative shocks (-0.25 and -0.5) IP responds more or less identically. Larger stress reductions, however, have a much stronger positive effect on IP growth when the economy is in a recessionary rather than a boom period, a phenomenon observed earlier for the unit shock. In case of larger shocks (-1.25 and -1.5), the impact in low–growth is about 50% larger than in the high–growth.

A comparison of the left plots in figure 8 reveals that—as in the unit–shock experiment—there is a stronger growth effect on IP due to stress reduction when the shock size varies. This holds especially, for large shocks. A 1.5 standard–deviation reduction in the stress index raises IP by about three times as much as a stress–increase of the same size would lower IP. This seems to us a very relevant observation concerning the asymmetric impact of monetary policy on the economy in different growth regimes. Monetary policy shocks –in particular what has recently been called unconventional monetary policy[56] – is likely to have large effects if the shocks are large and are undertaken in the low growth regime.

Our empirical MRVAR results strongly suggest that the timing of policy actions affecting financial stress is very influential on the success of such measures. The findings are compatible with recent studies which argue that unconventional monetary policy is needed in a depressed economy that is accompanied by a sharp rise in credit spreads, which, more so than asset–price volatility, constitute the dominant

[56]For example of quantitative easing, as pursued by the Fed since 2008.

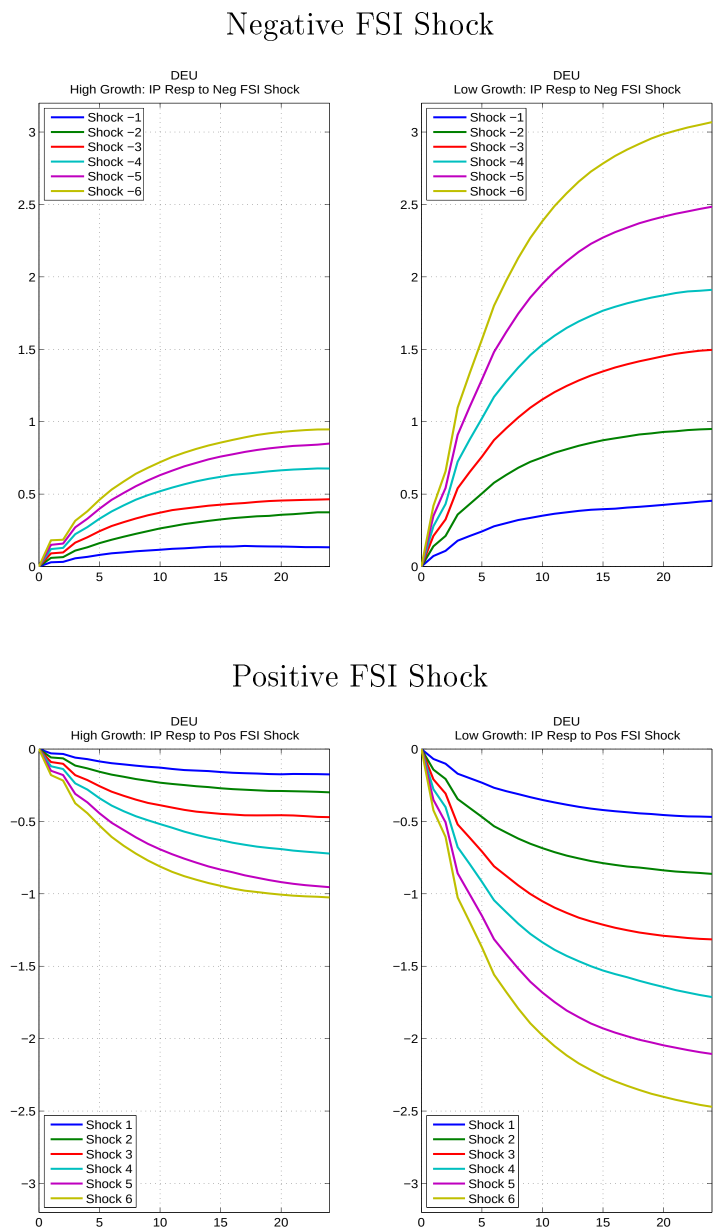

Figure 8: Cumulative MRVAR Responses to Negative (top) and positive (bottom) Stress–index Shocks in Average High– (left) and Low–growth States (right)

component of the stress index.[57] The results suggest that not only a decrease in the interest rate but also a reduction in financial stress and in credit spreads are required to induce significant expansionary effects.

5.4 Response Analysis for the Other EU Countries and the USA

Overall information on the VAR and MRVAR estimations for Germany, France, Italy, Spain, UK and the USA are provided in table 2 above. There is reported for each country the order of the VAR and MRVAR estimations, the AIC values, the threshold values and the number of observations for the high and low growth regimes. Next we give an interpretation of the plots for the smoothed IMF FSI and and IP (Industrial Production) and the impulse-responses of the MRVAR for each EU country and the USA. May summarize our results from the plots in appendix 3 as follows. There are similarities in the results but there is also country heterogeneity.

Common Features

1. As to the growth regimes: the FSI shocks have asymmetric effects and are state dependent, which means that positive and negative shocks of the same size reveal different effects, except for the UK. A reduction of financial stress (negative shock) in the low growth regime has a stronger effect on output than in the high growth regime

2. As to the size effects of shocks: Small shocks often show small effects, and large shocks reveal over-proportionally large effects of shocks. This points to some corridor stability: for small shocks the variables may quickly revert to their mean where as for large shocks not. So, in the low growth regime large shocks (large stress reduction) is required to affect output significantly. This mostly holds, except for the UK.

[57] See, for example, Curdia and Woodford (2009).

Country Heterogeneity

We can observe quite a significant heterogeneity between countries as to the way how negative or positive financial stress shocks affect output in the low or high growth regimes and to what extent the size of financial shocks matter.

1. For Germany, looking at the FSI and IP index, we can observe that the output reaction to the 2007-8 financial meltdown shows a very strong effect of FSI on output (as strong as for the US). As to the MRVAR impulse-responses, we can observe that the shocks have asymmetric effects: stress shocks in the high growth regime are much smaller. Moreover, greater shocks in the high growth regime do not have an over-proportional effects as compared to small shocks. There are, however, over-proportional effects of positive and negative shocks in the low growth regime, and the reduction of stress in the low growth regime is very effective with large shocks. This can be interpreted that the German economy has a strong manufacturing sector and a positive financial stress (and credit) shock in a high growth regime will have no strong effects, but in a low growth regime Germany maybe quite vulnerable to positive FSI shocks. On the other hand, in the low growth regime large stress reduction have a large effect on output improvement. Overall, the MRVAR impulse-response behavior is similar to the US.

2. France shows little response of the FSI during the financial market meltdown 2007-8, and also the output falls less than in Germany. This shows also up in the impulse-response diagrams. There is some stronger effects of negative shocks in high growth regimes, but there are only little effects of positive shocks to low growth regimes. There are obviously some stronger downward buffers in the French economy that prevent the output to fall strongly when financial stress rises in France. Also the high growth regime France reveals much smaller effects to positive or negative shocks than Germany shows. Overall it appears that France is in some sense more buffered against IMFFSI shocks.[58]

[58] The reason for this might be the presence of a larger State sector in the French economy, where electrical power generation (nuclear), the car industry, utilities, transport are operated with a considerable involvement of the State.

3. During the financial market meltdown, the Italian FSI responded little but the output dropped strongly, as much as in Germany, roughly 2%. There are medium size effects of negative shocks in the high growth regime, and of positive shocks in low growth regimes. In the high growth regime negative shocks have smaller effects, but positive shocks have a large effect in high growth regimes. So, positive shocks from the FSI can break expansion paths significantly in Italy.

4. Before the financial market meltdown of 2007-8 Spain showed an exceptionally long period of very low financial stress. The output fell as much as in Germany and Italy, by roughly 2%, but the financial stress rose very little. This shows also up in the very little recession sensitivity of positive financial stress shocks in the low growth regime. Positive financial stress shocks have also little effect in high growth regimes, and negative shocks have little effects in a high growth regime. Negative shocks have only stronger effects in low growth regimes. Overall, the real economy seems to be more decoupled from financial stress than other economies.[59]

5. Surprising results can be seen for the UK, in terms of the FSI it shows as big a rise in financial stress as the US and a fall in output as Germany and Italy, roughly 2 %, but undertaking the MRVAR estimations, the UK shows little effects of negative or positive financial shocks either on the high nor on low growth regimes. This presumably comes from the fact that the UK financial market is large relative to GDP and the real economy (manufacturing) is relatively decoupled from the financial side. Since its actual manufacturing base, measured by industrial production over GDP has become relatively small.[60] The UK therefore, seems to feel the spillover effects from the EU crisis, and thus output goes down significantly as spillover effect from the EU.

6. The US has the strongest rise of the FSI index, and a strong fall of IP in the meltdown of the years 2007-8. It also shows very strong effects in the low

[59] This may be related to the housing and real estate boom in Spain which was somewhat self-sustained for a longer time period.

[60]

(a) Its industrial production is roughly 11 % of the GDP but its export share to the EU is 57%.

growth regime: positive shocks decrease output significantly. Positive shocks in a high growth regime reduce also output significantly. A low growth regime responds sensitively to negative financial stress shocks (to a reduction of stress) and so does a high growth regime. So the banking sector-output link seems to be strong in the US, as it also holds for Germany.

Overall one might conclude, the stronger the position of an economy in the world economy in terms of its GDP and share of world trade, the more autonomous are the financial stress effects, having direct impact on their economies. Moreover, a large stress reduction in a low growth regimes have a large effect on output improvements in those economies. Yet the smaller the economies are, the more they are dependent on the external dynamics and the spillover effects which significantly seem to impact and modify the effects of their own financial stress shocks. Moreover, as in the case of France, a larger public sector seems to also act as a buffer against to impact of stress shocks.

6 Conclusions

Though most of the historical economic crises ended up as a meltdown of the banking sector, the banking sector has usually exacerbated and amplified the crisis whatever origin it had. To investigate those feedback effects, we have studied the linkage of financial stress, credit spreads and growth regimes. In particular we studied the issue of the instability of the banking sector that is exposed to asset price shocks, credit spread shocks and financial stress. We modeled financial intermediaries as wealth fund that accumulates capital assets, can heavily borrow and has generous payoffs. When the banking sector is exposed to a deterioration of its balances sheets, it turns out that the varying credit spreads and the payouts play an important role for the dynamics of the leverage ratio, the financial stress and the domain of attraction of the stable steady state. Greater credit spreads, larger payout, and larger adverse shocks beyond some thresholds, may generate some instability. Smaller adverse shocks may be mean reverting.

In contrast to previous studies that use the financial accelerator – which is locally amplifying but globally stable and mean reverting – our model admits downward instability as BS (2010) predict. Whereas the financial accelerator leads, in terms of econometrics, to a one-regime VAR, the multi-regime dynamics studied here requires to use a multi-regime VAR (MRVAR). Using the IMF financial stress index and growth regimes, our method of a MRVAR estimate permits us to undertake impulse-response studies for growth regimes.

We show that the shocks have asymmetric effects depending on the regime the economy is in, but we also show that the effects of the shocks are dependent on the size of the shocks.[61] Though there is also significant heterogeneity between countries in the sense that the stress shock effects on output is larger in the bigger economies, for example Germany and the USA, there are also common features. Large positive financial stress shocks in a high growth regime seem to have less of a contractionary effect than in a recessions, but large negative stress shocks – a strong stress reduction– appear to have a stronger expansionary effect in low than in high growth regimes. The latter result seems to us very important for the evaluation of an unconventional monetary policy, since frequently not only the timing, but also the strength of policy actions matter.

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[61] In earlier literature on Keynesian macrodynamics this has been called “corridor stability”: small shocks have only small effects but larger shocks could be destabilizing and have large effects.

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Appendix 1: The Numerical Solution of the Model

We have used the dynamic programming method by Gruene and Semmler (2004) to study the dynamics of the stochastic version of debt-asset ratio with consumption and growth rate of assets as controls. The dynamic programming method can explore the local and global dynamics by using a coarse grid for a larger region and then employing grid refinement for a smaller region. As BS (2010) correctly state, the dynamics should not be studied by first or second order Taylor approximations to solve for the local dynamics, since this will not capture the global instabilities of the model, in particular below the steady state. Instead we use dynamic programming, which can provide us with the truly global dynamics in a larger region of the state space without losing much accuracy (see Becker *et al.*, 2007). In contrast, local linearization, as has been argued there, does not give sufficient information on the global dynamics.

Hence, before going into the model discussion, we start by briefly describing this dynamic programming algorithm and the mechanism by which it enables us to numerically solve our dynamic model variants. The adaptive discretization of the state space feature of the dynamic programming algorithm leads to high numerical accuracy with moderate use of memory. In particular, the algorithm is applied to discounted infinite horizon dynamic decision problems of the type introduced for the study of our search and matching models. In our model variants we have to numerically compute $V(x)$:

$$\begin{aligned} V(x) &= \max_u \int_0^\infty e^{-rt} f(x,u)dt \\ \text{s.t.} &\quad \dot{x} = g(x,u), x(0) = x_0 \in \mathbb{R}^1 \end{aligned}$$

where u represents the decision variable, and x a vector of state variables. Note that one of the components of the vector of state variables, for example x_i, could represent the time index x, as used in sect 2.2.

In the first step, the continuous time optimal decision problem has to be replaced by a first order discrete time approximation given by

$$V_h(x) = \max_{u \in U} J_h(x, u)$$

where $J_h(x, u) = h \sum_{i=0}^{\infty}(1 - \theta h) f(x_h(i), u_i)$, and x_h is defined by the discrete dynamics

$$x_h(0) = x, \;\; x_h(i+1) = x_h(i) + hg(x_h(i), u_i)$$

and $h > 0$ is the discretization time step. Note that U denotes the set of discrete control sequences $u = (u_1, u_2, ...)$ for $u_i \in U$.

The value function is the unique solution of a discrete Hamilton-Jacobi-Bellman equation such as

$$V_h(x) = \max_{u \in U}\{hf(x, u) + (1 - \theta h)V_h(x_h(1))\} \tag{18}$$

where $x_h(1) = x + hg(x, u)$ denotes the discrete solution corresponding to the control and initial value x after one time step h. Using the operator

$$T_h(V_h)(x) = \max_{u \in U}\{hf(x, u_o) + (1 - \theta h)V_h(x_h(1))\}$$

the second step of the algorithm now approximates the solution on a grid Γ covering a compact subset of the state space, i.e. a compact interval $[0, K]$ in our setup. Denoting the nodes of Γ by x^i with $i = 1, ..., P$, we are now looking for an approximation V_h^Γ satisfying

$$V_h^\Gamma(x^i) = T_h(V_h^\Gamma)(x^i) \tag{19}$$

for each node x^i of the grid, where the value of V_h^Γ for points x which are not grid points (these are needed for the evaluation of T_h) is determined by linear interpolation. We refer to Gruene and Semmler (2004) for the description of iterative methods for the solution of (19). This procedure allows then the numerical computation of approximately optimal trajectories.

In order to distribute the nodes of the grid efficiently, we make use of an *a posteriori* error estimation. For each cell C_l of the grid Γ we compute

$$\eta_l := \max_{k \in c_l} \mid T_h(V_h^\Gamma)(k) - V_h^\Gamma(k) \mid$$

More precisely, we approximate this value by evaluating the right hand side in a number of test points. It can be shown that the error estimators η_l give upper and lower bounds for the real error (i.e., the difference between V_j and V_h^Γ) and hence serve as an indicator for a possible local refinement of the grid Γ. It should be noted that this adaptive refinement of the grid is particularly effective for computing steep value functions, non-differential value functions and models with multiple equilibria, see Gruene *et al.* (2004) and Gruene and Semmler (2004). These are all cases where local linearizations are not sufficiently informative.

Appendix 2: Estimating Movements of Credit Spreads

We take the IMF's (2011) FSI as a proxy of the time varying default risk and credit cost. We apply the *Fast Fourier Transformation* (FFT) to the IMFFSI for Germany.[62] The time period is from February 1980 to October 2011 at monthly frequency.

When we apply the FFT for Germany we estimate the periodic components of the actual time series. The estimates of the coefficients are reported in table 3, and the results are then illustrated in figure A1. We estimate a linear combination of sine-cos functions, representing the low frequency components of the actual time series, which then can be used as input into our DP described in Appendix 1. The form of our estimate is

$$x_t = \sum_{i=1}^{n} \Big(a_i \sin\big(\frac{2\pi}{\tau_i}(t - t_0)\big) + b_i \cos\big(\frac{2\pi}{\tau_i}(t - t_0)\big)\Big) . \tag{20}$$

[62] For details see Hsiao and Semmler (2009)

with the coefficients given in table 3.

$i =$	1	2	3	4	5	6
τ_i(month)	120	36	48	60	96	75
a_i	-0.263	0.0854	1.1592	0.1483	1.9493	0.1523
b_i	0.0445	-0.0837	1.2705	0.6065	1.6699	1.2001

Table 3: Coefficients of the harmonic fit of the real bond yield in the equation (20)

Table 3 reports the coefficients of the periodic components and figure A1 the actual and fitted series.

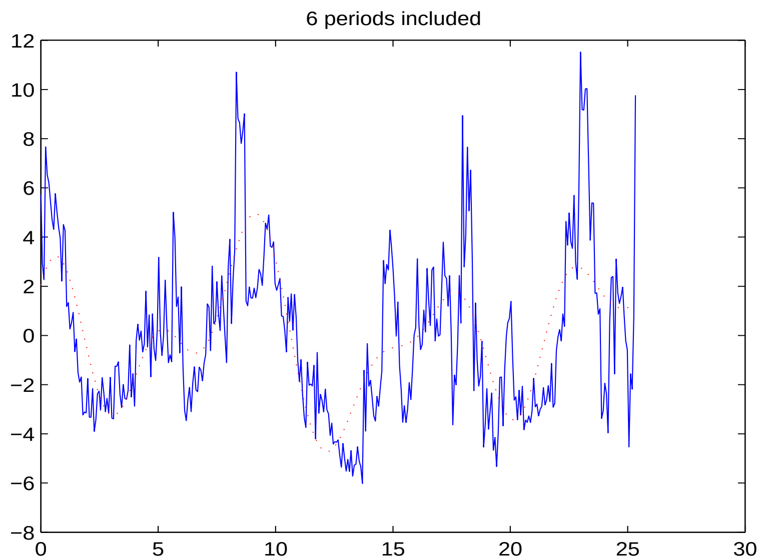

Figure A1:Actual and fitted series of the IMFFSI for Germany

Appendix 3: Figures of the MRVAR Estimations and Impulse-Responses for EU Countries and USA

Information on the VAR and MRVAR estimations for Germany, France, Italy, Spain, UK and the USA are provided in table 2, in sect. 5. There is reported for each country the order of the VAR and MRVAR estimations, the AIC values, the threshold values and the number of observations for the high and low growth regimes.

We report the plots for the smoothed FSI and and IP (Industrial Production) and the impulse-responses of the MRVAR for each EU country and the USA, except for Germany which are reported in sect. 5.

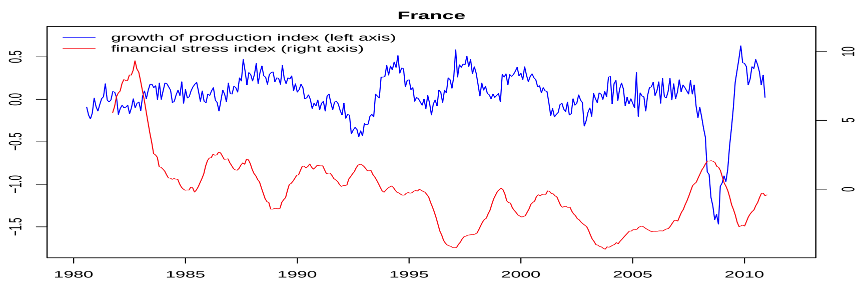

Figure A2: France (smoothed FSI and IP)

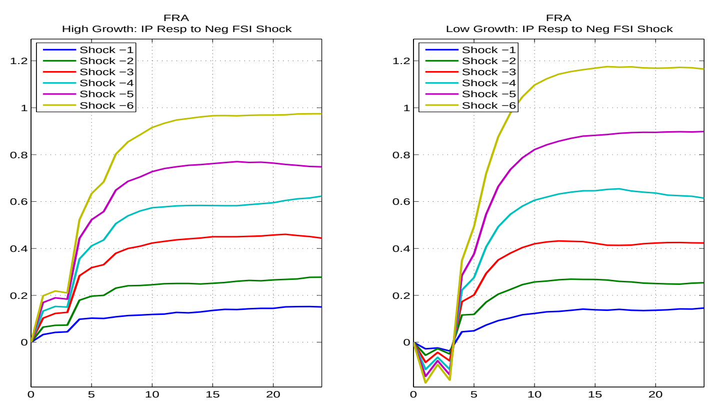

Figure A3: France, impulse-responses, negative shocks

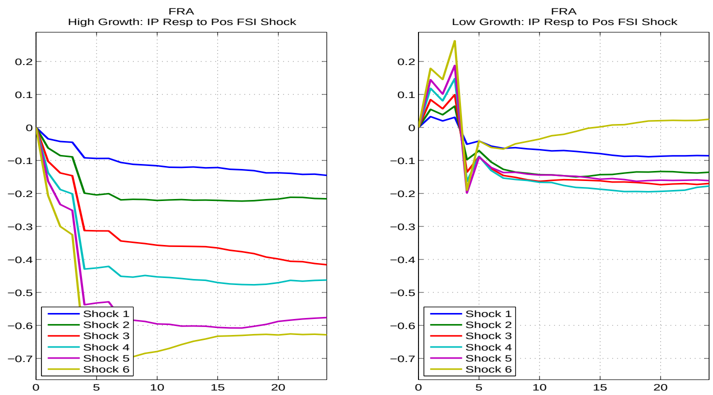

Figure A4: France, impulse-response, positive shock

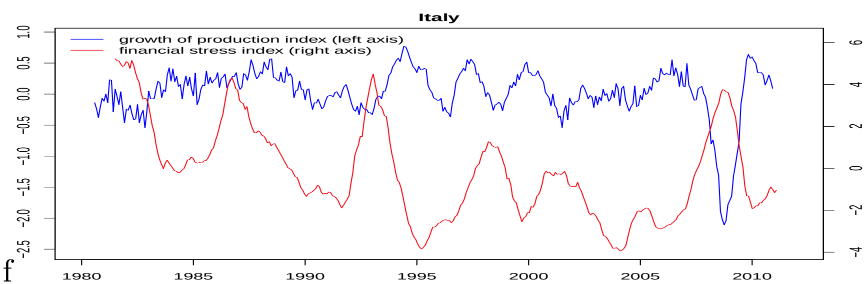

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Figure A5: Italy (smoothed FSI and IP)

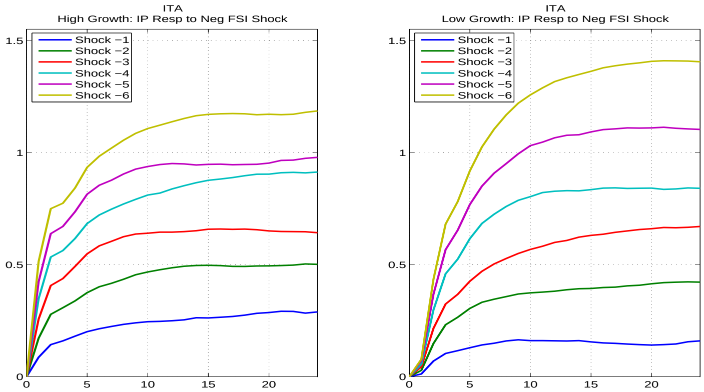

Figure A6: Italy, impulse-response, negative shock

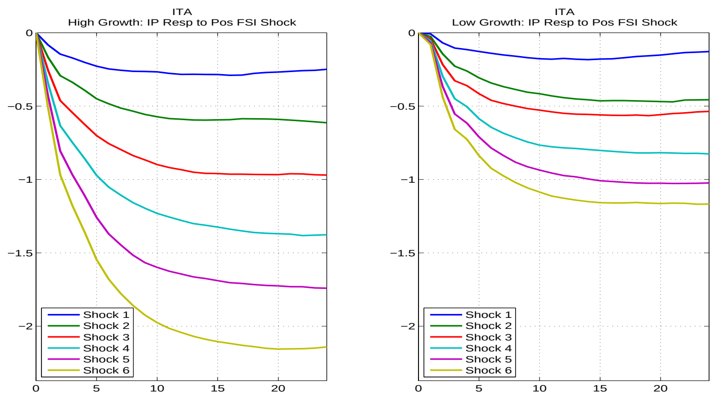

Figure A7: Italy, impulse-response, positive shock

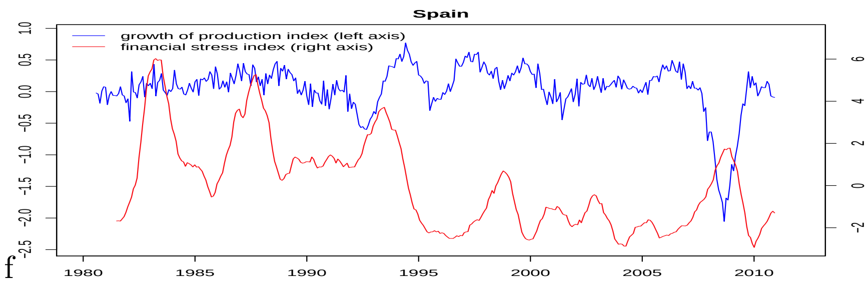

Figure A8: Spain (smoothed FSI and IP)

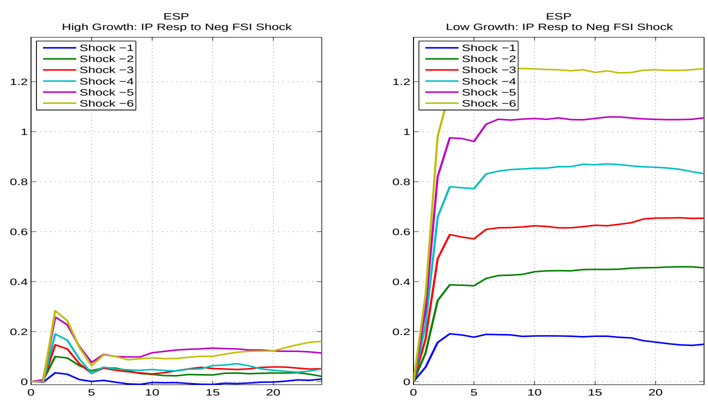

Figure A9: Spain, impulse-response, negative shock

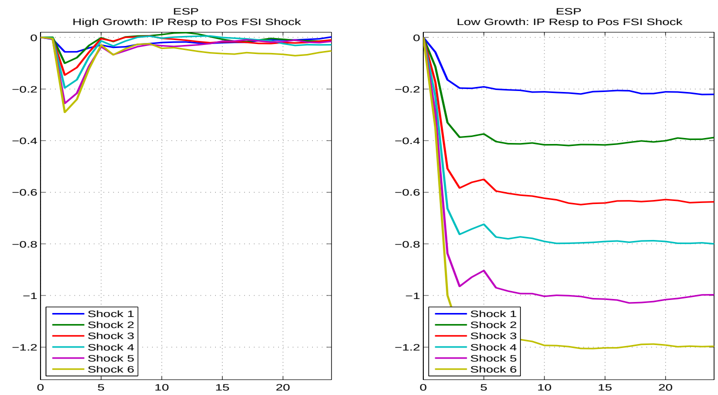

Figure A10: Spain, impulse-response, positive shock

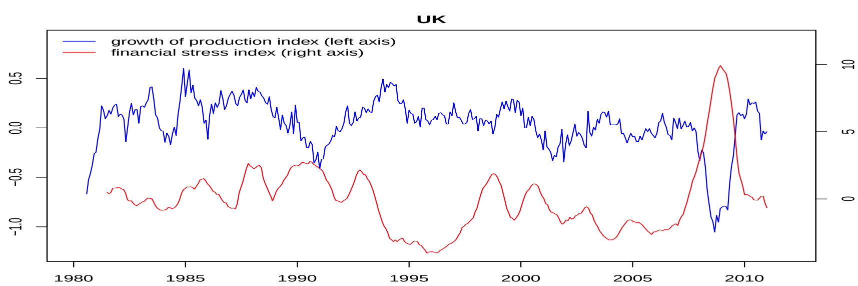

Figure A11: UK (smoothed FSI and IP)

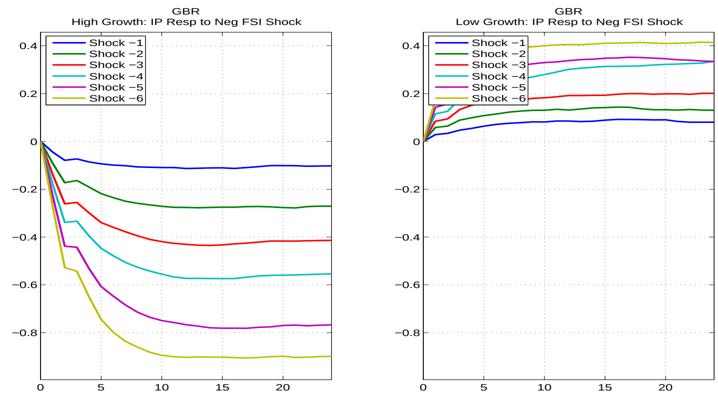

Figure A12: UK, impulse-response, negative shock

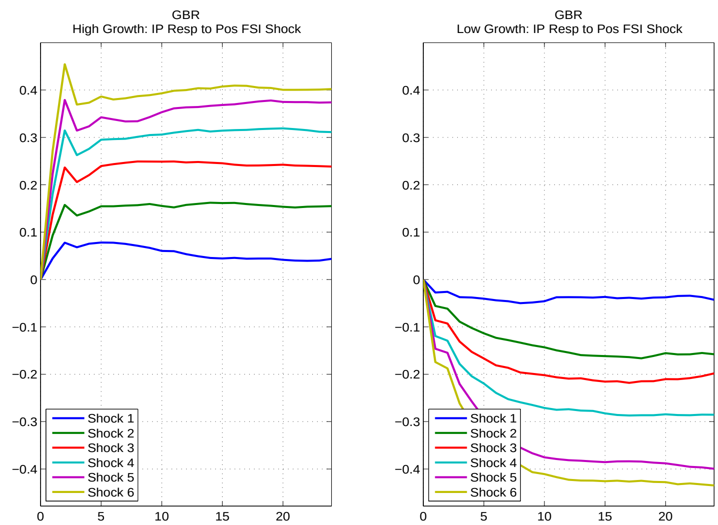

Figure A13: UK, impulse-response, positive shock

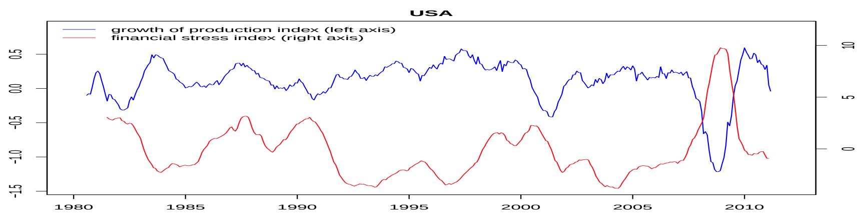

Figure A14: US (smoothed FSI and IP)

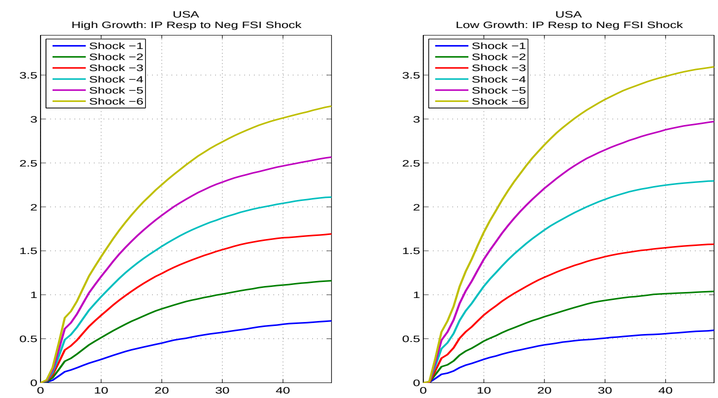

Figure A15: US, impulse-response, negative shock

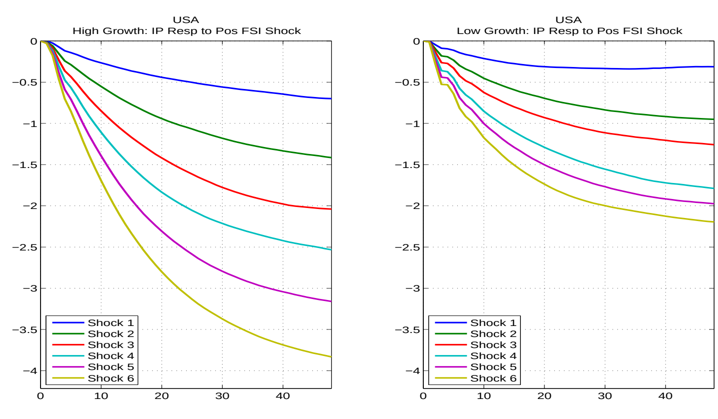

Figure A16: US, impulse-response, positive shock